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Article

Velocity of light through a moving medium applied to the Fizeau experiment

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Abstract: Light travels in a moving medium through densities that differ from the medium at rest. This study derives a formula for light velocity through a moving medium for any direction of light propagation in that medium that, applied to the Fizeau experiment, gives a fringe shift of 0.23541. The Fresnel drag coefficient applied to the Fizeau experiment offers a fringe shift of 0.20466. The mean of the fringe shift observations in the Fizeau experiment of 0.23016 confirms both expectations.

Keywords: velocity of light; moving medium; Fresnel drag coefficient; Fizeau experiment; Hoek experiment

1. Introduction

Fresnel applied the formula of the speed of transverse waves from elasticity theory [1] $v_1 = \sqrt{G_1/\rho_1}$ to the propagation of light through the hypothetical ether where G_1 is the shear modulus, and ρ_1 is the density of the medium at rest through which waves travels. Fresnel approximates that the shear modulus for any ether density in a moving medium has the same magnitude of G_1.

This study considers that light travels in a moving medium through different densities depending on the direction of the light's propagation in that medium.

The physics phenomena observed in inertial frames are similar to those in the frame at absolute rest [2]. Therefore, in this study, the absolute frame is replaced with the inertial frame of the Earth.

Per notations used in this study, points indicated by letters without an index correspond to points seen by an observer in the inertial frame of the moving medium. Points indicated by letters with an index are instances of points from the inertial frame of the moving medium observed in the inertial frame of the Earth. Points with the same index are associated with the same instance. The quantities marked with index a are functions of an angle a.

Section 2 presents the formula for the velocity of light when it travels through the medium at rest, in the same direction as the medium, in the opposite direction to the medium, and at an angle a measured from the direction of the moving medium. Section 3 presents the application of the Fresnel drag coefficient and the formula for the velocity of light through a moving medium obtained in this study to the Fizeau experiment [3].

2. Velocity of light through a moving medium

2.1. Light travels through a medium at rest

Figure 1 illustrates a beam of light from a coherent light source at rest in Earth's inertial frame that travels through a transparent medium with the speed c_1. The source and medium are at rest in Earth's

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inertial frame that travels with velocity v. Light travels through the medium of density ρ_1, through volume V bounded by an infinitesimal cylinder with the circular cross-section S and length $AB = L_1$, and mass $m_1 = \rho_1 V = \rho_1 S L_1$.

The formula for the speed of the transverse waves applied to the speed of light in a medium at rest, as Fresnel did, is the equation

$$c_1 = \sqrt{\frac{G_1}{\rho_1}}. \tag{1}$$

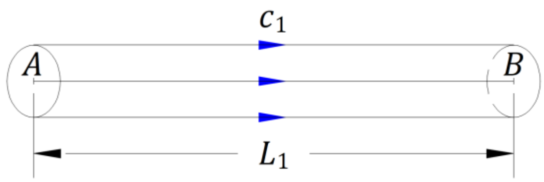

Figure 1. Light travels through the medium at rest.

2.2. Light travels in the same direction as the medium

In Fig. 2, the medium travels with velocity u in the inertial frame of the Earth. The light travels through the moving medium at the velocity c_{2a}, making an angle $\alpha = 0°$ with the velocity u.

In the moving medium, the speed of light

$$c_{2a} = \sqrt{\frac{G_{2a}}{\rho_{2a}}}. \tag{2}$$

The wavefront of the beam of light travels in time t the medium of density ρ_{2a}, length $L = A_1B_2$ through volume V bounded by an infinitesimal cylinder with the circular cross-section S, and mass $m_{2a} = \rho_{2a} V = \rho_{2a} S L$. Simultaneously, the cross-section S from point A_1 travels the distance A_1A_2, and S from point B_1 travels the distance $B_1B_2 = A_1A_2 = ut$.

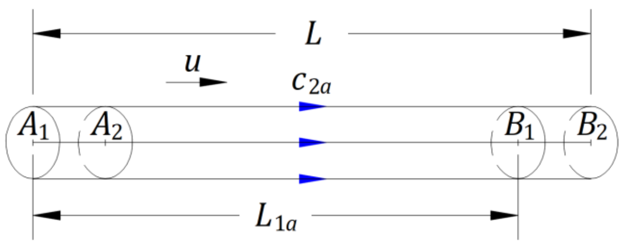

Figure 2. Light travels in the same direction as the medium.

The mass of the mediums bounded by sections A_1 and B_1 at instance one and by sections A_2 and B_2 at instance two is equal to $m_1 = \rho_1 S L_{1a}$, equal to the mass m_{2a}.

The path length $L = L_{1a} + ut \Rightarrow c_{2a}t = L_{1a} + ut \Rightarrow t = L_{1a}/(c_{2a} - u)$. The mass $m_{2a} = \rho_{2a} S L = \rho_{2a} S (L_{1a} + ut) = \rho_{2a} S L_{1a} c_{2a}/(c_{2a} - u)$.

From the equality of the two masses, $m_1 = m_{2a} \Rightarrow \rho_1 S L_{1a} = \rho_{2a} S L_{1a} c_{2a}/(c_{2a} - u)$, that gives the equation

$$\frac{\rho_1}{\rho_{2a}} = \frac{c_{2a}}{c_{2a} - u}. \tag{3}$$

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By introducing ρ_1/ρ_{2a} from Eq. (3) in Eq. (2) and considering $G_{2a} = G_1$, the ratio of Eq. (2) and Eq. (1) is $\frac{c_{2a}}{c_1} = \sqrt{\frac{G_{2a}\rho_1}{G_1\rho_{2a}}} = \sqrt{\frac{\rho_1}{\rho_{2a}}} = \sqrt{\frac{c_{2a}}{c_{2a}-u}}$ that yields the equation

$$c_{2a}^2 - uc_{2a} - c_1^2 = 0. \quad (4)$$

The convenient solution of Eq. (4) is $c_{2a} = \sqrt{c_1^2 + (u/2)^2} + u/2$, which is the speed of the wavefront of the light beam in the inertial frame of the Earth.

In the inertial frame of the moving medium, an observer at point A sees the light traveling the path $A_2B_2 = AB$ with speed $c_{1a} = c_{2a} - u = \sqrt{c_1^2 + (u/2)^2} - u/2$.

2.3. Light travels in the opposite direction to the medium

Figure 3 illustrates light traveling through the moving medium at velocity c_{2a}, making an angle $a = 180°$ with velocity u. The velocity of light c_{2a} is derived in an infinitesimal cylinder, as illustrated in Fig. 2.

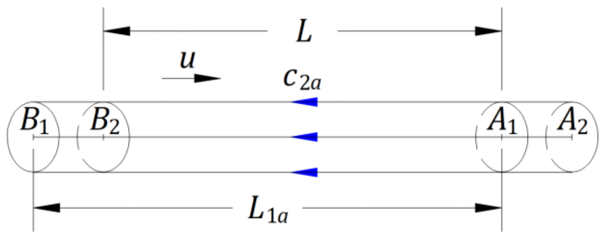

Figure 3. Light travels in the opposite direction to the medium.

With the same reasoning as in the previous subsection,

$$c_{2a}^2 + uc_{2a} - c_1^2 = 0. \quad (5)$$

Solution of Eq. (5), $c_{2a} = \sqrt{c_1^2 + (u/2)^2} - u/2$, is the speed of the wavefront of the light beam in the inertial frame of the Earth.

In the inertial frame of the moving medium, an observer at point A sees the light traveling the path $A_2B_2 = AB$ with speed $c_{1a} = c_{2a} + u = \sqrt{c_1^2 + (u/2)^2} + u/2$.

2.4. Light travels at an angle a measured from the direction of the moving medium

In Fig. 4, the medium travels with velocity u, and the light travels through the moving medium with velocity c_{2a}, which forms an angle a measured from velocity u.

The wavefront of the beam of light travels in time t the medium of density ρ_{2a}, length $L = A_1B_2$ through volume V bounded by an infinitesimal cylinder with the circular cross-section S, and mass $m_{2a} = \rho_{2a}V = \rho_{2a}SL$. Simultaneously, the cross-section S from point A_1 travels the distance A_1A_2, and S from point B_1 travels the distance $B_1B_2 = A_1A_2 = ut$.

For the moving medium, the speed of light

$$c_{2a} = \sqrt{\frac{G_{2a}}{\rho_{2a}}}. \quad (6)$$

The wavefront of the beam of light travels along path L through mass m_{2a} equal to mass m_1 contained in volumes bounded by S at points A_1 and B_1 and at points A_2 and B_2.

Angle $A_1A_2B_2$ is $180° - b$, then $A_1B_2A_2$ is $180° - (180° - b) - a = b - a$. The infinitesimal cylinder, bounded by S at points A_2 and B_2, has the cross-section S_{1a} of oval shape perpendicular to path L_{1a}. Volume bounded by $S_{1a} = S\cos(b-a)$ at points A_2 and B_2 is $V_{1a} = S_{1a}L_{1a}$. The mass of the medium with density ρ_1 contains in volume V_{1a} is $m_1 = \rho_1 V_{1a} = \rho_1 S_{1a} L_{1a} = \rho_1 S L_{1a}\cos(b-a)$, which is equal to m_{2a}.

The equality of the two masses $m_{2a} = \rho_{2a}SL$ and $m_1 = \rho_1 S L_{1a}\cos(b-a)$ gives $\rho_{2a}SL = \rho_1 S L_{1a}\cos(b-a)$ that yields the equation

$$\frac{\rho_{2a}}{\rho_1} = \frac{L_{1a}}{L}(\cos b\cos a + \sin b\sin a)\ . \qquad (7)$$

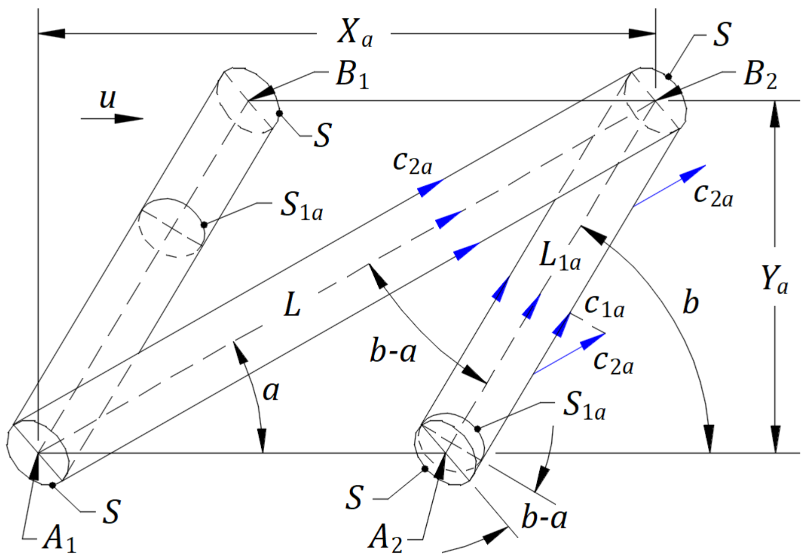

Figure 4. Light travels at an angle a measured from the direction of the moving medium.

From Fig. 4, $\sin a = Y_a/L \Rightarrow Y_a = L\sin a$ and $\sin b = Y_a/L_{1a} \Rightarrow Y_a = L_{1a}\sin b$. Equality of the expressions for Y_a, $L\sin a = L_{1a}\sin b$ offers the equation

$$\sin b = \frac{L}{L_{1a}}\sin a. \qquad (8)$$

$\cos a = X_a/L \Rightarrow X_a = L\cos a$ and $\cos b = (X_a - A_1A_2)/L_{1a} = (X_a - ut)/L_{1a} \Rightarrow X_a = L_{1a}\cos b + Lu/c_{2a}$. Equality of the expressions for X_a, $L\cos a = L_{1a}\cos b + Lu/c_{2a}$ gives the equation

$$\cos b = \frac{L}{L_{1a}}\left(\cos a - \frac{u}{c_{2a}}\right). \qquad (9)$$

By introducing $\sin b$ from Eq. (8) and $\cos b$ from Eq. (9) in Eq. (7), $\frac{\rho_{2a}}{\rho_1} = \frac{L_1}{L}\left[\frac{L}{L_1}\left(\cos a - \frac{u}{c_{2a}}\right)\cos a + \frac{L}{L_1}\sin a\sin a\right]$ yields the equation

$$\frac{\rho_1}{\rho_{2a}} = \frac{c_{2a}}{c_{2a} - u\cos a}. \qquad (10)$$

Introducing ρ_1/ρ_{2a} from Eq. (10) in the ratio of Eq. (6) and Eq. (1), considering $G_{2a} = G_1$ for any angle a, $\frac{c_{2a}}{c_1} = \sqrt{\frac{G_{2a}\rho_1}{G_1\rho_{2a}}} = \sqrt{\frac{\rho_1}{\rho_{2a}}} = \sqrt{\frac{c_{2a}}{c_{2a} - u\cos a}}$ that offers the equation

$$c_{2a}^2 - (u\cos a)c_{2a} - c_1^2 = 0. \qquad (11)$$

The convenient solution of Eq. (11) is $c_{2a} = \sqrt{c_1^2 + \left(\frac{u}{2} cos\, a\right)^2} + \frac{u}{2} cos\, a$.

Equation (11) yields Eq. (4) for $a = 0°$ and Eq. (5) for $a = 180°$. For $a = 90°$ and $a = 180°$ $c_{2a=90°} = c_{2a=180°} = c_1$.

In the inertial frame of the moving medium, an observer at point A sees the wavefront of the light from point A_1 traveling the path $A_2B_2 = AB$ with speed c_{1a} given by the cosine theorem. If the light source is large enough to cover the distance X_a, the observer will see the light traveling the path $A_2B_2 = AB$ with speed c_{1a} given by the cosine theorem.

Along A_1B_2, c_{2a} is the light's propagation velocity. At each point along A_1B_2, the light has the velocity c_{2a}. So, the light's propagation velocity is identical to the light velocity. Along A_2B_2, c_{1a} is the light's propagation velocity. At each point along the light's propagation direction, the light has the velocity c_{2a} [2,4].

2.5. Numerical calculation of the light's shear modulus G_a

From Eq (1), $G_1 = \rho_1 c_1^2$ with the SI units in kg·m^{-3}·m^2·s^{-2} = kg·m^{-1}·s^{-2} = Pa. The density of water at rest is ρ_1 = 997kg/m³, and then $G_1 = \rho_1 c_1^2$ = 5.07264401 60551800E+19 Pa.

For u = 7.059m/s and an angle a, c_{2a} is calculable with solution of Eq. (11) $c_{2a} = \sqrt{c_1^2 + \left(\frac{u}{2} cos\, a\right)^2} + \frac{u}{2} cos\, a$, and $\rho_{2a} = \rho_1 \frac{c_{2a} - u\, cos\, a}{c_{2a}}$ from Eq. (10), then $G_{2a} = \rho_{2a} c_{2a}^2$.

For $a = 0°$, $G_{2a=0°}$ = 5.07264401 60551800E+19 Pa.

For $a = 180°$, $G_{2a=180°}$ = 5.07264401 60551700E+19 Pa.

For $a = 90°$ and $a = 270°$,$G_{2a} = G_1 = \rho_1 c_1^2$ = 5.0726440160551800E+19 Pa.

The ratio $G_1/G_{2a=0°} = G_1/G_{2a=180°} = 1.0000000000000000E+00$. Thus, the approximation that $G_{2a} = G_1$ for a low magnitude of u is correct.

For $G_a \neq G_1$, Eq. (11) becomes

$$c_{2a}^2 - (u\, cos\, a)c_{2a} - \frac{G_{2a}}{G_1} c_1^2 = 0, \tag{12}$$

with the solution $c_{2a} = \sqrt{\frac{G_{2a}}{G_1} c_1^2 + \left(\frac{u}{2} cos\, a\right)^2} + \frac{u}{2} cos\, a$.

For a higher magnitude of u, G_{2a} is calculable through multiple iterations. With c_{2a} calculated from the first iteration with the solution of Eq. (11), the formula $G_{2a} = \rho_{2a} c_{2a}^2$ yields G_{2a}. The solution of Eq. (12) offers speed c_{2a} for the second iteration and the following ones.

3. Fizeau experiment

3.1. Experimental device

Figure 5 illustrates the device schematic of the Fizeau experiment [1,3] at rest in the inertial frame of Earth that is moving with the velocity v. The water in tubes 1 and 2 can flow with speed u.

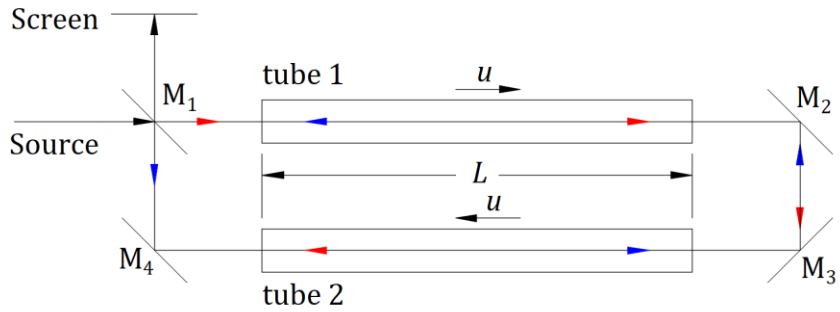

Figure 5. Schematic of the Fizeau experimental device.

A ray of light from the source is split into two rays by the beam splitter M1. The transmitted ray travels to opaque mirrors M2, M3, and M4, back to beam splitter M1, and then to a screen. The reflected ray travels to mirrors M4, M3, and M2, back to beam splitter M1, and then to the screen.

The derivation of the two light paths ignores the distances the light travels through the air because these dimensions cancel one another in the calculation.

In the Fizeau experiment, the water speed in tubes is $u = 7.059$ m/s, and the length of each tube is $L = 1.4875$ m. The speed of light in air/vacuum is $c = 3 \times 10^8$ m/s, and the light has the wavelength $\lambda = 5.26 \times 10^{-7}$m. The refraction index of water is $n_1 = 1.33$, and the speed of light in water at rest is $c_1 = c/n_1$.

When water is stationary in tubes, the two split rays interfere on screen, yielding an interference fringe that shifts when the water travels with speed u. The mean of the fringe shift readings taken by Fizeau in his experiment is 0.23016.

3.2. Fresnel's formula for c_2 applied to the Fizeau experiment

The Fresnel formula is

$$c_2 = c_1 \pm uk, \tag{13}$$

where c_2 is the speed of light in the moving medium and $k = 1 - 1/n_1^2$ is the Fresnel drag coefficient [1,3]. The plus/minus depends on the direction of c_2 and u.

The time t_1 the transmitted ray travels with the speed $c_2 = c_1 + uk$ in tubes 1 and 2 is $t_1 = 2L/(c_1 + uk)$.

The time t_2 the reflected ray travels with the speed $c_2 = c_1 - uk$ in tubes 2 and 1 is $t_2 = 2L/(c_1 - uk)$.

The difference of time $\Delta t = t_2 - t_1$ yields the fringe shift $N = c\Delta t/\lambda = 0.20466$.

3.3. Formula c_2 derived in this study applied to the Fizeau experiment

The solution of Eq. (11) $c_{2a} = \sqrt{c_1^2 + \left(\frac{u}{2} cos\ a\right)^2} + \frac{u}{2} cos\ a$ is $c_{2a=0°} = \sqrt{c_1^2 + (u/2)^2} + u/2$ for $a = 0°$ and $c_{2a=180°} = \sqrt{c_1^2 + (u/2)^2} - u/2$ for $a = 180°$.

The time t_1 the transmitted ray travels with the speed $c_{2a=0°}$ in tubes 1 and 2 is $t_1 = 2L/c_{2a=0°}$.

The time t_2 the reflected ray travels with the speed $c_{2a=180°}$ in tubes 2 and 1 is $t_2 = 2L/c_{2a=180°}$.

The difference of time $\Delta t = t_2 - t_1$ yields the fringe shift $N = c\Delta t/\lambda = 0.23541$.

4. Discussions and conclusions

The velocity of the Earth v is not a part of the formulas that give the velocity c_{2a}. Thus, the Fizeau experiment yields the same fringe shift in any direction on the Earth for a constant magnitude of velocity u, as in the frame at absolute rest. The fringe shift varies proportionally to the magnitude of velocity u.

The dispersion of the fringe shift readings taken by Fizeau in his experiment covers an extensive range from 0.167 to 0.307 fringes with a mean of 0.23016. This result confirms both predictions of 0.23541 and 0.20466.

Light travels in the medium at rest of density ρ_{2a} in the Earth's inertial frame with the corresponding speed c_{2a}. For example, in tube 1 of Fig. 5, transmitted rays travel through tube 1 at speed $c_{2a=0^\circ}$ like in the medium at rest of density $\rho_{2a=0^\circ}$, and reflected rays travel in the opposite direction at speed $c_{2a=180^\circ}$ like in the medium at rest of density $\rho_{2a=180^\circ}$, with no dragging effect of light propagation. So, Snell's law applies to these mediums at rest. $n_{2a} = c/c_{2a}$ for any angle a. $n_{2a=0^\circ}$ varies from 1.330 for $u = 0$ to 1.327 for $u = 10^6$ m/s when speed $c_{2a=0^\circ}$ varies from c_1 for $u = 0$ to $c_{2a=0^\circ} = c_1 + 5 \times 10^5$ m/s for $u = 10^6$ m/s.

Fresnel derives the drag coefficient when the velocity of light c_1 is in the same and opposite direction of the velocity of the moving medium u, so, of the entrained ether. However, he applies it to any angle measured from velocity u the light makes, including 90° and 270° for the transversal directions. The speed of light of Eq. (13) contains terms c_1 and n_1 for any direction.

Hoek performed his experiment [5] in Earth's inertial frame. He expected to see a fringe shift when moving the instrument frame at a constant velocity in Earth's inertial frame. The mechanical phenomena, including emission, propagation, and reflection of light [2], are observed in Earth's inertial frame and any other inertial frame, including the inertial frame created by Hoek, as in the frame at absolute rest. Hook keeps the medium at rest in his inertial frame. Thus, the Hoek experiment does not predict any fringe shift by transferring the instrument from one inertial frame to another, which agrees with Hoek's observations.

The multitude of galaxies with the interstellar medium forms the material Universe. Beyond this Universe that contains matter may be a vacuum Universe that explains galaxies' expansion that travels in a spiral. Interstellar medium and vacuum spaces migrate into each other at the border of the two universes. Vacuum spaces could also be within the material Universe. When a light beam travels through a medium, some waves are reflected outside the light beam, indicating to the human eyes the presence of that light beam. Therefore, a black hole in the Universe may be a vacuum space without stars from which light comes through it to the human eyes. Light from other stars travels visually undetected in black holes. However, it is reflected by the medium surrounding them, forming their bright coronas. Differently, in a space with a medium and multiple stars, rays reflected by the medium create the illuminated interstellar space, and stars get their lighted corona.

References

1. Bărbulescu N. Physics Foundations of Einsteinian Relativity, *St. & Enc. ed.*, **1975**, pp. 30–40, Bucharest, Romania.
2. Filipescu F. D. Emission, propagation, and reflection of light as mechanical phenomena in inertial frames. *Phys. Essays*. **2021**, 34, 587.
3. Fizeau M. H. On the Effect of the Motion of a Body upon the velocity with which it is traversed by Light. *Philos. Mag*. **1860**, 4, 19(127), 245.
4. A Filipescu F. D. Observation of a star's orbit based on the emission and propagation of light as mechanical phenomena. *Phys. Essays*. **2022**, 35, 111.
5. Hoek M. Determination of the Speed with which a Light Wave is Entrained Crossing a Moving Medium. *Verslagen en Mededeelingen der Koninklijke*. **1868**, 2, T, II, 189.