SUPPLEMENTARY (FIGURES AND TABLES)



SF 1: Histogram plot of selected compound flood (CF) drivers

(a)

(b)

(c)

SF 2: Box whisker plot of CF drivers (a) Annual maximum 24-hr Rainfall (mm) (b) Maximum Storm surge (Time interval = $\pm 4$ days) (m) (c) Maximum River discharge (Time interval = $\pm 4$ days) ($m^{3}sec^{-1})$

(a)

(b)

(c)

SF 3: Normal Quantile-Quantile (Q-Q) plots for (a) Annual maximum 24-hr Rainfall (mm) (b) Maximum Storm surge (Time interval = ± 4 days) (m) (c) Maximum River discharge (Time interval = ± 4 days) ($m^{3}sec^{-1}$)

ST 1: Basic summary statistics of the selected compound flooding (CF) drivers

|  |  |  |
| --- | --- | --- |
| Annual Maximum 24-hr Rainfall (mm) | Maximum Storm surge (Time interval = ±4days) (m) | Maximum River discharge (Time interval = ±4 days) ($m^{3}sec^{-1}$) |
| Min. : 33.00  | Min. :-0.0470  | Min. : 800  |
|  1st Qu.: 63.42  | 1st Qu.: 0.1875  | 1st Qu.:1275  |
| Median : 79.50  | Median : 0.3015  |  Median :1840  |
|  Mean : 80.68  | Mean : 0.3025  | Mean :2074  |
| 3rd Qu.: 93.35  | 3rd Qu.: 0.4243  | 3rd Qu.:2405  |
|  Max. :146.00  | Max. : 0.6890  | Max. :5910  |
| Var: 589.0201 | Var: 0.03544252 | Var: 1438211 |

ST 2: Test for homogeneity within individual time series of CF drivers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Flood variables | Pettitt (Estimated p-value) | SNHT test (Estimated p-value) | Buishand (Estimated p-value) | Overall conclusion |
| Annual Maximum 24-hr Rainfall (mm) | 1 > 0.05 (significance level) | 0.7764 > 0.05 (significance level) | 0.8316 > 0.05 (significance level) | Time series is homogenous |
| **Storm Surge (Time interval = ±4days) (m)\*** | **0.02541 < 0.05 (significance level)** | **0.02755 < 0.05 (significance level)** | **0.09325 > 0.05 (significance level)** | **Time series is not homogenous** |
| River Discharge (Time interval = ±4 days) ($m^{3}sec^{-1})$ | 0.5625 > 0.05 (significance level) | 0.8773 > 0.05 (significance level) | 0.5616 > 0.05 (significance level) | Time series is homogenous |
| Note: The p-value has been computed using 20000 Monte Carlo simulations.\*Storm surge events (bold letter with an asterisk) exhibited non-homogenous behaviour, their estimated p-value for the Pettit and SNHT tests is less than p = 0.05 |

(a)

(b)

(c)

SF 4: Qualitative based visual inspection of nonparametric model’s performance fitted to (a) Annual maximum 24-hr Rainfall (mm) (b) Maximum Storm surge (Time interval = ± 4 days) (m) (c) Maximum River discharge (Time interval = ± 4 days) $(m^{3}sec^{-1})$

(a)

(b)

(c)

SF 5: Comparing Probability-Probability (P-P) plots of the nonparametric models fitted to (a) Annual maximum 24-hr Rainfall (mm) (b) Maximum Storm surge (Time interval = ± 4 days) (m) (c) Maximum River discharge (Time interval = ± 4 days) $(m^{3}sec^{-1})$

ST 3: Fitting 2-D parametric copulas in the second tree level (Tree 2) of vine structure constructed in parametric settings for (a) D-vine structure-1 (case-1) (b) D-vine structure-2 (case-2) (c) D-vine structure-3 (case-3)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (a) |  |  |  | Cramer von Mises functional statistics with parametric bootstrap procedure (N = 1000 (No. of bootstrap samples)) |
| Copula family | Parameter estimates ($θ\_{12|3})$ via MPL estimators | Standard Error estimates | MLL (Maximized Log-likelihood) | $$S\_{n}$$ | p-value |
| Clayton | 0.4563 | 0.226 | 2.994 | 0.045034 | 0.1004 |
| Gumbel | 1.326 | 0.137 | 3.885 | 0.0259 | 0.4241 |
| **Frank\*** | **2.925** | **0.886** | **4.651** | **0.018882** | **0.9096** |
| Joe | 1.387 | 0.189 | 2.751 |  |  |
| BB1 (Clayton-Gumbel) | $θ=theta=0.1784$; $δ=delta=1.2295$ | NA | 4.165 | 0.039161 | 0.3651 |
| BB6 (Joe-Gumbel) | $θ=theta=1$; $δ=delta=1.326$ | NA | 3.885 | 0.041983 | 0.3312 |
| BB7 (Joe-Clayton) | $θ=theta=1.2509$; $δ=delta=0.3332$ | NA | 3.956 | 0.047131 | 0.2882 |
| BB8 (Joe-Frank) | $θ=theta=6$; $δ=delta=0.41$ | NA | 4.65 | 0.025064 | 0.6938 |
| Survival clayton | 0.4985 | NA | 2.951 | 0.062309 | 0.1503 |
| Survival Joe | 1.361 | NA | 2.082 | 0.080925 | 0.08242 |
| Survival Gumbel | 1.301 | NA | 3.165 | 0.049017 | 0.2423 |
| Survival BB1 | $θ=theta=0.2346$; $δ=delta=1.1872$ | NA | 3.523 | 0.041955 | 0.3242 |
| Survival BB6 | $θ=theta=1$; $δ=delta=1.301$ | NA | 3.165 | 0.049017 | 0.2512 |
| Survival BB7 | $θ=theta=1.1831$; $δ=delta=0.3829$ | NA | 3.226 | 0.050101 | 0.2313 |
| Survival BB8 | $θ=theta=6$; $δ=delta=0.42$ | NA | 4.732 | 0.023331 | 0.7258 |
| Note: Frank copula (bold letter with asterisk) fitted best in the dependence modelling of conditional flood pair (minimum value of Sn test statistics) in the second tree level (Tree 2) for D-vine structure-1 (case 1) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (b) |  |  |  | Cramer von Mises functional statistics with parametric bootstrap procedure (N = 1000 (No. of bootstrap samples)) |
| Copula family | Parameter estimates ($θ\_{13|2})$ via MPL estimators | Standard Error estimates | MLL (Maximized Log-likelihood) | $$S\_{n}$$ | p-value |
| Rotated Joe 90 degrees | -1.033 | NA | 0.03724 | 0.019008 | 0.8646 |
| Rotated Gumbel 90 degrees | -1.023 | NA | 0.03734 | 0.019269 | 0.8586 |
| Frank | -0.3719 | 0.858 | 0.08094 | 0.021318 | 0.9206 |
| Gaussian (or Normal) | -0.04792 | 0.149 | 0.05882 | 0.02055 | 0.9306 |
| Rotated BB1 90 degrees | Theta = par = -3.438e-08 Delta = par2 = -1.023 | NA | 0.03732 | 0.019269 | 0.8576 |
| Rotated BB6 90 degrees | Theta = par = -1.02, par2 = delta = -1.01 | NA | -6.883e-15 | 0.019159 | 0.8606 |
| Rotated BB7 90 degrees | Theta = par = -1.033; delta = par2 = -6.135e-08 | NA | 0.03724 | 0.019008 | 0.8506 |
| Rotated BB8 90 degrees | Theta = par = -1.2042 Delta = par2 = -0.8715 | NA | 0.2371 | 0.022101 | 0.7797 |
| Rotated BB1 270 degrees | Theta = par = -0.08613Delta = par2 = -1.00000 | NA | 0.2099 | 0.020603 | 0.8397 |
| **Rotated BB6 270 degrees\*** | **Theta = par = -1****Delta = par2 = -1** | **NA** | **-2.442e-15** | **0.018538** | **0.8696** |
| Rotated BB7 270 degrees | Theta = par = -1.0000Delta = par2 = -0.0861 | NA | 0.2099 | 0.020602 | 0.8117 |
| Rotated BB8 270 degrees | Theta = par =-6Delta = par2 = -0.07 | NA | 5.218e-15 | 0.020593 | 0.8167 |
| Note: Rotated BB6 270 degrees copula (bold letter with an asterisk) fitted best in the joint modelling of conditional flood pair (minimum value of Sn test statistics) in second tree level (Tree-2) of the D-vine structure-2 (case-2) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (c) |  |  |  | Cramer von Mises functional statistics with parametric bootstrap procedure (N = 1000 (No. of bootstrap samples)) |
| Copula family | Parameter estimates ($θ\_{23|1})$ via MPL estimators | Standard Error estimates | MLL (Maximized Log-likelihood) | $$S\_{n}$$ | p-value |
| Clayton | 0.6626  | 0.263 | 4.831  | 0.054285 | 0.05844 |
| Gumbel | 1.301 | 0.119 | 3.387  | 0.027878 | 0.2892 |
| **Frank\*** | **2.972**  | **0.946** | **5.483**  | **0.019514** | **0.8776** |
| Joe |  1.324  | 0.147 | 1.882 | 0.052833 | 0.04046 |
| BB1 (Clayton-Gumbel) | $θ=theta=0.5498$; $δ=delta=1.0660$ | NA | 4.901  | 0.053774 | 0.1863 |
| BB6 (Joe-Gumbel) | $θ=theta=1$; $δ=delta=1.301$ | NA | 3.387  | 0.060648 | 0.1643 |
| BB7 (Joe-Clayton) | $θ=theta=1.0000$; $δ=delta=0.6626$ | NA | 4.831  | 0.063413 | 0.1214 |
| BB8 (Joe-Frank) | $θ=theta=6$; $δ=delta=0.4$1 | NA | 5.483  | 0.033341 | 0.492 |
| Survival clayton | 0.4335 | NA | 2.747 | 0.08952 | 0.05544 |
| Survival Joe | 1.531 | NA | 4.078 | 0.073942 | 0.09241 |
| Survival Gumbel | 1.374 | NA | 4.879 | 0.048689 | 0.2343 |
| Survival BB1 | $θ=theta=2.813e-08$ ; $δ=delta=1.374$ | NA | 4.879 | 0.048689 | 0.2363 |
| Survival BB6 | $θ=theta=1$; $δ=delta=1.374$ | NA | 4.879 | 0.048689 | 0.2363 |
| Survival BB7 | $θ=theta=1.4184$; $δ=delta=0.2054$ | NA | 4.415 | 0.057852 | 0.1863 |
| Survival BB8 | $θ=theta=5.77$; $δ=delta=0.44$ | NA | 5.557 | 0.032239 | 0.499 |
| Note: Frank copula (bold letter with an asterisk) fitted best in the joint modelling of conditional flood pair (minimum value of Sn test statistics) in second tree level (Tree-2) of the D-vine structure-3 (case-3) |

ST 4: Fitting 2-D parametric copulas in the second tree level (Tree 2) of vine structure constructed in semiparametric settings (parametric copula with nonparametric marginal pdfs) for (a) D-vine structure-1 (case-1) (b) D-vine structure-2 (case-2) (c) D-vine structure-3 (case-3)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (a) |  |  |  | Cramer von Mises functional statistics with parametric bootstrap procedure (N = 1000 (No. of bootstrap samples)) |
| Copula family | Parameter estimates ($θ\_{12|3})$ via MPL estimators | Standard Error estimates | MLL (Maximized Log-likelihood) | $$S\_{n}$$ | p-value |
| Clayton | 0.4604  |  0.239 | 3.03  | 0.036116 | 0.1983 |
| Gumbel | 1.341  | 0.166 | 4.417  | 0.019725 | 0.7637 |
| **Frank\*** |  **2.898**  | **0.889** |  **4.743**  | **0.015242** | **0.9865** |
| Joe | 1.425  | 0.239 |  3.375  | 0.039103 | 0.1404 |
| BB1 (Clayton-Gumbel) | $θ=theta=0.1479$; $δ=delta=1.2585$ | NA | 4.595 | 0.029008 | 0.5659 |
| BB6 (Joe-Gumbel) | $θ=theta=1$; $δ=delta=1.341$ | NA | 4.417 | 0.031311 | 0.538 |
| BB7 (Joe-Clayton) | $θ=theta=1.2868$; $δ=delta=0.3207$ | NA | 4.415  | 0.035376 |  0.4481 |
| BB8 (Joe-Frank) | $θ=theta=6$; $δ=delta=0.41$ | NA | 4.743 | 0.019621 | 0.8317 |
| Survival clayton | 0.5437  | NA | 3.515  | 0.047034 | 0.2443 |
| Survival Joe | 1.351  | NA | 2.049  | 0.070968 | 0.1164 |
| Survival Gumbel | 1.301  | NA | 3.255  | 0.039981 | 0.4011 |
| Survival BB1 | $θ=theta=0.3195$; $δ=delta=1.1509$ | NA | 3.904 | 0.032153 | 0.504 |
| Survival BB6 | $θ=theta=1$; $δ=delta=1.301$ | NA | 3.255 | 0.039981 | 0.3711 |
| Survival BB7 | $θ=theta=1.1397$; $δ=delta=0.4565$ | NA | 3.681 | 0.038429 | 0.3721 |
| Survival BB8 | $θ=theta=6$; $δ=delta=0.41$ | NA | 4.773 | 0.019717 | 0.8367 |
| Note: Frank copula (bold letter with an asterisk) exhibited minimum value Cramer-Von Mises functional 𝑆n statistics with p-value is greater than 0.05, thus recognized as the most parsimonious bivariate copula in defining bivariate joint dependence structure in Tree 2 for D-vine structure (case-1) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (b) |  |  |  | Cramer von Mises functional statistics with parametric bootstrap procedure (N = 1000 (No. of bootstrap samples)) |
| Copula family | Parameter estimates ($θ\_{13|2})$ via MPL estimators | Standard Error estimates | MLL (Maximized Log-likelihood) | $$S\_{n}$$ | p-value |
|  Rotated Joe 90 degrees | -1.03  | NA | 0.03642  | 0.018955 | 0.8736 |
| Rotated Gumbel 90 degrees | -1.019  | NA | 0.03016  | 0.019115 | 0.8746 |
| Frank | -0.2357 | 0.859 | 0.03313 | 0.021389 | 0.9066 |
| Gaussian (or Normal) | -0.0437 | 0.15 | 0.04946 | 0.022396 | 0.8696 |
| Rotated BB1 90 degrees | Theta = par = -5.383e-08 Delta = par2 = -1.019e+00 | NA | 0.03016  | 0.019115 | 0.8546 |
| Rotated BB6 90 degrees | Theta = par = -1.03, par2 = delta = -1 | NA | -4.441e-15  | 0.018955 | 0.8606 |
| Rotated BB7 90 degrees | Theta = par = -1.030e+00; delta = par2 = -4.136e-09 | NA | 0.03641  | 0.018955 | 0.8516 |
| Rotated BB8 90 degrees  | Theta = par = -1.1328 Delta = par2 = -0.8967 | NA | 0.1476 | 0.021387 | 0.7817 |
| Rotated BB1 270 degrees | Theta = par = --0.081 Delta = par2 = -1.00000 | NA | 0.195  | 0.021287 | 0.7847 |
| **Rotated BB6 270 degrees\*** | **Theta = par = -1****Delta = par2 = -1** | **NA** | **-6.661e-16**  | **0.017848** | **0.8866** |
| Rotated BB7 270 degrees  | Theta = par = -1.0000Delta = par2 = -0.08104  | NA | 0.195  | 0.021289 | 0.7977 |
| Rotated BB8 270 degrees  | Theta = par =-6 Delta = par2 = -0.04 | NA |  8.993e-15 | 0.01939 | 0.8467 |
| Note: Rotated BB6 270 degrees (or r270BB6 Copula) (bold letter with asterisk) exhibited minimum value Cramer-Von Mises functional $S\_{n}$ statistics with p-value is greater than 0.05, thus recognized as the most parsimonious bivariate copula in defining bivariate joint dependence structure in Tree 2 for D-vine structure-2 (case 2). |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (c) |  |  |  | Cramer von Mises functional statistics with parametric bootstrap procedure (N = 1000 (No. of bootstrap samples)) |
| Copula family | Parameter estimates ($θ\_{23|1})$ via MPL estimators | Standard Error estimates | MLL (Maximized Log-likelihood) | $$S\_{n}$$ | p-value |
| Clayton | 0.6005 | 0.258 | 4.211 | 0.04817 | 0.07842 |
| Gumbel | 1.264 | 0.135 | 2.824 | 0.028095 | 0.3262 |
| **Frank\*** | **2.754** | **0.985** | **4.903** | **0.020058** | **0.8616** |
| Joe | 1.278 | 0.174 | 1.475 | 0.052335 | 0.03646 |
| BB1 (Clayton-Gumbel) | $θ=theta=0.5261$; $δ=delta=1.0435$ | NA | 4.244 | 0.053884 | 0.2003 |
| BB6 (Joe-Gumbel) | $θ=theta=1$; $δ=delta=1.264$ | NA | 2.824 | 0.059203 | 0.2023 |
| BB7 (Joe-Clayton) | $θ=theta=1.0000$; $δ=delta=0.6006$ | NA | 4.211 | 0.060295 | 0.1643 |
| BB8 (Joe-Frank) | $θ=theta=6$; $δ=delta=0.39$ | NA | 4.902 | 0.031829 | 0.498 |
| Survival clayton | 0.3986 | NA | 2.628 | 0.080974 | 0.07642 |
| Survival Joe | 1.462 | NA | 3.455 | 0.072261 | 0.09141 |
| Survival Gumbel | 1.336 | NA | 4.312 | 0.047635 | 0.2692 |
| Survival BB1 | $θ=theta=0.03614$ ; $δ=delta=1.31411$ | NA | 4.323 | 0.04636 | 0.2712 |
| Survival BB6 | $θ=theta=1$; $δ=delta=1.336$ | NA | 4.312 | 0.047635 | 0.2702 |
| Survival BB7 | $θ=theta=1.3412$; $δ=delta=0.2268$ | NA | 3.978 | 0.053815 | 0.2273 |
| Survival BB8 | $θ=theta=6$; $δ=delta=0.4$ | NA | 4.918 | 0.032336 | 0.483 |
| **Note:** Frank copula (bold letter with asterisk) exhibited minimum value Cramer-Von Mises functional $S\_{n}$ statistics with p-value is greater than 0.05, thus recognized as the most parsimonious bivariate copula in defining bivariate joint dependence structure in Tree 2 for D-vine structure-2 (case 2). |

(a)

 (b)

(c)

SF 6: Overlapped 2-D scatterplot between simulated flood events (of sample size, N=1000, using D-vine structure in the nonparametric setting) and historical flood events for (a) rainfall and storm surge (b) storm surge and river discharge (c) rainfall and river discharge pair



SF 7: 3-D scatterplot of flood samples generated from best-fitted D-vine structure in the nonparametric settings



SF 8: Vine tree plot of the fitted D-vine structure (case-2) in the nonparametric distribution setting