

Some Comments on a Time Interval Only Description

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Abstract

The time interval description is a natural way to introduce finite intervals, like finite time intervals. This approach depends on results for radiation propagation from star sources, where properties relate to a propagation surface, which is finite for every realistic event and measurement. In contrast the usual vector approach like for Newton's laws depends on introducing an infinite coordinate system. A time interval only approach necessarily has to start from scratch. Properties for time intervals have to be defined with time intervals. Where the first parts of this paper are devoted to time interval set properties, in the discussion part these are applied to quantities and measurements within astronomy. The introduction provides a survey of results.

Key words: time interval, measurement event, Noether charge, structure constants, differentiation properties, star source radiation propagation, gravitation

1. Introduction and survey of results

A finite time interval relates by itself to measurement, even while with approximately zero interference the situation remains, by approximation, 'free' and without interaction. A realistic description for radiation starting from a 'free' situation tends to be finite, when considering star sources and finite velocities. This is independent of time intervals being infinite, termed Newtonian, meaning Hamiltonian H time independent and noninteracting, and time intervals being finite meaning H time dependent. For infinite time intervals the results for the time interval description are the same as for the usual Newtonian vector description like in [1, Arnold]. Defined is time interval Δt to be the relevant event time interval, the reference interval for any measurement events.

Within a 'dialogue' approach, the Heisenberg view of qm is described among others in [2, Beller]: particle properties like positions or paths can only be accessed from sequences of interactions or measurements. With the Heisenberg view the uncertainty relations are the basis of qm [3, Roos]. According to Beller this is an 'operational' view, which rejects causal space-time description in contrast to several of more 'realistic' views which could emphasize the adequacy of unified concepts of qm (terms by Beller). This discussion is not part of this paper however. Measurements and finite time intervals are interrelated, infinite time intervals are not excluded by the time interval description.

Time interval description and t-quantities and Δt -quantities

In this paper within the time interval description, including time interval equilibrium, applied are results from earlier research [ref. 4] and [ref. 5], that include time interval differentiation, commutation and equilibrium relations, where all time intervals, when H time dependent, are finite and asymmetrical. In the time interval description, a quantity can be a t-quantity, variable with one moment time coordinate t during time interval Δt , or a 'neutral' Δt -quantity, variable with Δt , however invariant during Δt , for instance a time interval average. This means linearity in t resp. linearity in Δt due to the specific definition of time interval equilibrium, that is 'mean velocity theorem' (MVT) equilibrium, different from Lagrangian equilibrium when Hamiltonian H is time dependent. Correspondence for t-quantities with Δt -quantities is discussed in paragraph 2 to 5. This allows a time interval only description, where the one moment time coordinates are left out. Properties for the time interval set, like addition, multiplication and differentiation are discussed in the second part of this paper, paragraph 6 to 10. The third part of this paper includes a discussion of star source radiation and gravitation within a time interval only description among other results, paragraph 11 and 12.

Time intervals and the 'infinite regress' problem

The time interval description includes differentiation to time intervals that depend on time interval boundaries that are themselves one moment time quantities. To find a time interval only description, to relate these boundaries to time interval quantities without being time intervals, this is realized in paragraph 2 to 5. This should resolve the 'infinite regress' problem when defining the Hamiltonian being the 'time interval' version of the Legendre transform of the Lagrangian L . With time intervals instead of one moment time quantities this

would mean, due to the derivative in the ‘time interval’ version, time interval boundaries equal time intervals causing the regress.

Star source radiation and gravitation

The third part of this paper concerns application of the time interval description to star source radiation, paragraph 11, and discussion, paragraph 12. The originating process of star source radiation is not investigated. Assumed radiation is in ‘free motion’ without interaction and radiation energy is invariant per time interval due to star source properties with a certain origin mass. Even so, the time intervals related to radiation propagation are always finite, since the wave group-velocity is considered finite. The results for overall propagation energy can be described in terms of gravitational field energy for both zero mass em radiation waves and massive particle waves [4, Hollestelle] and [5, Hollestelle]. Application of time interval only allows an integrated description of these different phenomena, without other assumptions.

Newton’s second law relates kinematics and source related forces, applying vectors and space time coordinates, within a one moment time description [6, Goldstein], [7, Newton I]. In this paper equilibrium is due to a multiple of energies rather than actions. This allows to discard the question of inertial forces. The equivalence principle in general relativity (GR) implies equivalence for gravitational energy and geometry, now a result is equivalence is implied for gravitational energy and radiation energy, ie kinetic propagation surface energy due to the em radiation field. This suggests similarly equivalence can be found for radiation energy and geometry. The subject of action ‘at a distance’ is unresolved within physics. In GR the concept of simultaneous events is proposed, applying a finite velocity of light. In this paper simultaneous radiation energy is the energy of radiation waves ‘arriving’ within Δt at the finite propagation surface.

A finite propagation surface obviously should be asymmetrical due to its time development and propagation away from the source. In terms of fields this means the surface tends to increase distance from the source due to its geometry and asymmetry, and such purely geometric fields have been studied before, [ref. 8]. Curie’s principle seems to be equivalent to Newton’s second law, due to asymmetric propagation including an invariant wave group-velocity related development in time, [9, Curie]. Clearly radiation propagation is away from the source where gravitation as a source for movement is directed towards the mass sources.

The essential property for the time interval set is multiplication $M[\Delta t_1, \Delta t_1] = \Delta t_1$ for any time interval Δt_1 , including relevant event time interval Δt . It means the multiplication inverse for Δt is Δt itself, paragraph 3, allowing energies to be equal in value even while time interval Δt and multiplication inverse Δt_i within the result can still be recovered. In paragraph 12, discussed is for time interval Δt , being finite and asymmetrical, the concepts ‘close’ or ‘remote’, meaning in terms of space intervals the mass sources being localizable or not, and in terms of time intervals, ‘simultaneous’ or ‘not simultaneous’, with regard to measurements.

The time interval only description for em radiation with zero mass wave particles has some simplicity as advantage and can naturally resume non zero mass wave particles. A time interval equilibrium result is, Δt -quantities are linear in Δt , and the time interval set is closed for multiplication, paragraph 9. Twice differentiation to time interval Δt implies scalar multiplication, similar to wave equations in qm, paragraph 10.

Not all results are linear results. Differentiation is nonlinear with respect to scalar multiplication, paragraph 8. Incident radiation energy can change, due to interaction like the photo-electric effect, depending on interaction event time interval Δt and is nonlinear in the variable $\#n$, the number of wave particles, paragraph 11. Other results include derivation of Noether charges, and structure constants, for time interval equilibrium situations, in terms of commutation properties.

2. Commutation relations and differentiation to time intervals

By discussing commutation relations for one moment time t and differentiation to t , the for operators in field theory usual concept of: ‘working to the right’ is introduced for one moment time quantity t . A correspondence relating the one moment time set and the time interval set is defined, applying commutation relations and differentiation to time intervals including the ‘working to the right’ property for time intervals. Correspondence, then, prepares a way to approach the ‘infinite regress’ problem and to derive time interval only set properties.

For t -quantities, the usual description is one where these depend on one moment time t , and Lagrangian equilibrium. The Newtonian kinematic description is a one moment time description. With Newtonian situations however meant are in this paper H time independent situations, ie situations without external interaction.

The usual commutation is defined with $[A1, A2]_u = A1(t) \cdot A2(t) - A2(t) \cdot A1(t)$, indicated with subscript u , for two t -quantities $A1$ and $A2$ at the same one moment time t . With the multiplication \cdot and the usual $+$ and $-$ indications, the following equations are assumed that together define commutation relations for one moment time t . It is assumed that one moment time t is not completely commutation free and multiplication with t is order dependent.

$$1 \quad t \cdot cn' = cn \cdot t$$

$$1/t \cdot cn = cn' \cdot 1/t$$

$$2 \quad [t, cn]_u = t \cdot (cn - cn')$$

$$[1/t, cn]_u = - (cn - cn') \cdot 1/t$$

The cn and cn' are the only commutation quantities related to one moment time, time coordinate, t . From relations 1 it follows the cn and cn' are neutral quantities.

The commutation equations relate the one moment time t to one moment time differentiation to t : d/dt , including differentiation of cn and cn' and differentiation of multiplication $A1 \cdot A2$, meaning $A1$ 'multiplication' $A2$, writing $d/dt [A1 \cdot A2]$ for t -quantities $A1$ and $A2$. Square brackets indicate d/dt 'working to the right' to the bracket. Differentiation d/dt is 'working to the right' on every part of a multiplication within the usual first order differential calculus, [1, Arnold], [6, Goldstein], [10, Newton II], with the following distributive, ie symmetrical second associative, property for differentiation:

$$3 \quad d/dt [A1 \cdot A2] = d/dt [A1] \cdot A2 + A1 \cdot d/dt [A2] = 2 \cdot A1 \cdot d/dt [A2] - [A1, d/dt]_u \cdot A2$$

Trivially however essentially in the second part of this equation differentiation d/dt is 'working to the right' on $A1$ or $A2$ only. The third part is decisive for the interpretation of equations 1 and 2. The differentiation d/dt 'moving to the right' leaving, releasing, at each step, and equation 3 means only one step, a quantity to the left, ie at the first step quantity $A1$, including multiplication with a scalar, that is scalar 2 and a remain, the last part with the minus sign. This is similar to one moment time t or $1/t$ 'moving to the right' leaving to the left cn' or cn respectively in equations 1.

Within the time interval description, multiplication, addition and differentiation can be applied to t -quantities and to Δt -quantities. From equations 1 and 2 similar equations can be derived for the same cn and cn' now regarded to be Δt -quantities instead of t -quantities. Assumed is that the described situation is 'remote' from H time independent: with time interval $\Delta t = [tb, ta]$ asymmetrical and finite such that $|tb| \ll |ta|$. cn and cn' are invariant during Δt and can be regarded Δt -quantities: within the time interval description including MVT equilibrium this means they can be described being linear, meaning linear with Δt , and are not described as t dependent, linear or otherwise.

When changing from t -quantity to Δt -quantity perspective, Δt -quantities are regarded part of the time interval set, since they are linear in Δt , and every Δt -quantity can be written equal to some $\Delta t^3 = a \cdot \Delta t$. For any Δt -quantity being a time interval, a is scalar number. This is discussed in paragraph 13 part IV and paragraph 8, comment 9, the closure theorem for the time interval set. When the considered Δt -quantity can be different from time intervals regarding dimension and argument a can be a scalar different from a number scalar. Part by part the one moment time parameter t is applied less in the following. The indications M and A are applied for multiplication and addition for time intervals. Equations 10, paragraph 5, are commutation relations for the time interval set and without reference to one moment time t and part of a complete and closed time interval only description.

Comment 1. Differentiation of quantity $A1$ to one moment time t within the one moment time description: $d/dt [A1]$, reduces to $D^* [A1] = [1/t, A1]_u = (1/t \cdot A1(t) - A1(t) \cdot 1/t)$, the usual commutation, for differentiation to one moment time t within the time interval description, disregarding a factor $1/2$, [ref. 5]. MVT equilibrium means for any t -quantity $A1 = A1(t)$ linearity in t and $D^* [A1] = +/- A1/t$, the $+$ or $-$ depending on $A1$ increasing or decreasing with t and $A1$ positive or negative. It follows $D^* [A1]$ remains invariant during Δt . This is part of the t -quantity perspective. Addition A for time intervals is evaluated in paragraph 7, multiplication M for time intervals is evaluated in paragraph 8. The time interval description derivative $D^* || \Delta t$ to time interval Δt of quantity $A1$ is defined with the time interval commutator, $D^* || \Delta [A1] = [1/t, A1] || \Delta t = A [1/tb \cdot A1(tb) - 1 \cdot A1(ta) \cdot 1/ta]$, for time interval $\Delta t = [tb, ta]$, and relates directly to equations 1 and 2.

3. Correspondence for one moment time t-quantities with time interval Δt -quantities

The time interval derivative to one moment time t for A_1 positive and increasing, is: $D^* [A_1] = A_1 \cdot 1/t$, comment 1. Due to MVT equilibrium $D^* [A_1]$ remains invariant during Δt . $D^* [A_1]$ can be regarded a t -quantity as well as a Δt -quantity and this allows for the definition of correspondence for t -quantities with Δt -quantities. Applying averages for Δt of A_1 and one moment time t , leaving out subscript $\|\Delta t$: $\langle A_1 \cdot 1/t \rangle = \langle A_1 \rangle / \langle t \rangle$ it follows in terms of t -quantities only:

$$4 \quad A_1 \cdot 1/t = \langle A_1 \rangle / \langle t \rangle$$

This relation is valid for any t -quantity A_1 and follows from MVT equilibrium and $D^*[A_1]$. The average for time interval Δt of parameter t : $\langle t \rangle = \langle t \rangle / \|\Delta t$ depends on the (a-)symmetry of time interval Δt . The following properties for time interval $\Delta t = [t_b, t_a]$ are applied: when H time dependent Δt has finite measure and is asymmetrical with $|t_b| \ll |t_a|$, while for H time independent Δt has infinite measure with $|t_b| = |t_a|$ each infinite. For asymmetric Δt : parameter t average $\langle t \rangle = 1/2(t_a + t_b) = t_0$, with t_0 the “multiplication unit” for one moment time parameters during Δt , meaning $t_0 \cdot t = t$. Left out are factors 2. Similarly, there is time interval ΔU , the “multiplication unit” for the time interval set.

The average for Δt of A_1 is a Δt -quantity and equals $\langle A_1 \rangle = \langle A_1 \rangle / \|\Delta t = D^* [A_1] \cdot t_0$, from equation 4. The derivative of A_1 to one moment time t , $D^* [A_1]$, is similarly a Δt -quantity. The Δt -quantity property of average $\langle A_1 \rangle$, linearity with Δt , and introducing scalar a and time interval multiplication M , means $\langle A_1 \rangle = M [a, \Delta t] = \Delta t^3$. For one moment time t , itself a t -quantity however not a Δt -quantity, one defines correspondence of $\langle t \rangle$ with time interval ΔU : $\langle t \rangle \sim \Delta U$. In this way one can define correspondence, indicated with \sim , for averages of one moment time quantities corresponding with time interval quantities. A_{1i} indicates with subscript i the multiplication inverse for any A_1 . The left side of equation 5 is in terms of averages of t -quantities, the right side in terms of time intervals and Δt -quantities:

$$5 \quad \langle A_1 \rangle / \langle t \rangle \sim M [\Delta t^3, \Delta U_i] = M [M [a, \Delta t], \Delta U_i]$$

Comment 2. Correspondence

For the H time dependent asymmetrical situation, averaging for Δt one moment time t , there is $\langle t \rangle = t_0$, the one moment time ‘multiplication unit’ and different from the ‘multiplication zero’, with $t_0 \cdot t = t$. $t_0 = t$, and t_0 corresponds with Δt_0 in this case being equal to time interval set ‘multiplication unit’ ΔU , and different from the time interval set ‘multiplication zero’ ΔU_0 : $\langle t \rangle / \|\Delta t \sim \Delta U$.

For the H time independent and symmetrical Δt situation with $|t_b| = |t_a|$ there is $\langle t \rangle = t_0$ equals $t + -1 \cdot t = t_0$, with t_0 in this case both the ‘multiplication zero’ and the ‘addition zero’, corresponding with Δt_0 in this case being equal to ΔU_0 both the ‘multiplication zero’ and ‘addition zero’ for the time interval set. In this case $\langle t \rangle$ corresponds with Δt_0 , being equal to ΔU_0 , and not equal to ΔU , because of the symmetry of Δt : $\langle t \rangle / \|\Delta t \sim \Delta U_0$. *The one moment time t average $\langle t \rangle / \|\Delta t = t_0$ corresponds with the specific time interval termed Δt_0 , meaning correspondence of one moment time t_0 with time interval set ‘multiplication unit’ or ‘multiplication zero’ due to H being time dependent or not.*

The corresponding relations for the ‘addition zero’ Δt_0 can be applied to describe a transition from non-zero mass to zero mass radiation. The difference in one moment time t average being $\langle t \rangle / \|\Delta t \sim \Delta t_0$ equal to “multiplication one” or “multiplication zero” for quantities and properties while remaining within physics is interesting in its own.

Comment 3. The defining property for ΔU is: $M [\Delta U, \Delta t_1] = M [\Delta t_1, \Delta U] = \Delta t_1$, and for the multiplication inverse with subscript i of Δt_1 , Δt_{1i} : $M [\Delta t_1, \Delta t_{1i}] = M [\Delta t_{1i}, \Delta t_1] = \Delta U$, for any time interval Δt_1 . One can argue, not regarding uniqueness, ΔU can be identified with Δt , with Δt the relevant event time interval, from $M [\Delta U, \Delta t] = \Delta t$ while also $M [\Delta t, \Delta t] = \Delta t$, a relation derived independently for all Δt_1 from interpretation in paragraph 7. It also follows $\Delta t_0 = \Delta t$ is a solution for Δt_0 . The discussion in paragraph 7 implies some uniqueness properties for Δt .

Comment 4. Where M indicates multiplication, A indicates addition and one defines $A [\Delta t_1, \Delta t_0] = A [\Delta t_0, \Delta t_1] = \Delta t_1$ and $A [\Delta t_1, \Delta t_{1iv}] = A [\Delta t_{1iv}, \Delta t_1] = \Delta t_0$, for Δt_0 ‘addition zero’ for the time interval set and Δt_{1iv} addition inverse with subscript iv for any time interval Δt_1 . Multiplication with scalar -1 of any time interval Δt_1 : $-1 \cdot \Delta t_1$, is not necessarily equal to the addition inverse Δt_{1iv} for Δt_1 , and is not a well-defined time interval when Δt_1 is a time interval and Δt asymmetric due to H being time dependent.

4. Pairs of commutation quantities and pairs of Noether charges

The commutation quantities cn and cn' are t -quantities, invariant during Δt , ie having constant values within their domain (cn -domain or cn' -domain) within Δt , and there is $cn(t) = cn$ and $cn'(t) = cn'$ invariant with t during Δt , where dependence on Δt is described by introducing the positive average scalar density $D(cn, \Delta t)$. For the Δt average for cn , $\langle cn(t) \rangle_{\Delta t} = \int_{\Delta t} dt [cn]$. $1/|\Delta t| = N$. $D(cn, \Delta t)$ with $D(cn, \Delta t) = |cn\text{-domain}|/|\Delta t\text{-domain}|$ a Δt invariant scalar quantity. Applied is the time interval integral to parameter t : $I^* [2] = \int_{\Delta t} dt [2] = 2 \cdot |\Delta t|$, the domain for scalar 2 being the Δt -domain Δt . The t -quantity $cn = \langle cn(t) \rangle_{\Delta t}$ is equal to N only when the cn -domain during Δt has domain measure equal to the Δt -domain measure itself: in this situation there is $D(cn, \Delta t) = |\Delta t|/|\Delta t| = 1$. It is assumed that this is not always the case due to an equilibrium requirement for cn and cn' .

Similar to $D(cn, \Delta t)$ defined is $D(cn', \Delta t)$ for cn' and both densities depend on Δt and on the quantities cn and cn' resp. The equilibrium requirement implies $D(cn, \Delta t) + D(cn', \Delta t) = 1$, both for the same Δt . For two specific time intervals Δt and $\Delta t'$, when the respective domains equal upper limit 1, there is $D(cn', \Delta t') = D(cn, \Delta t) = 1$. With cn and cn' the same sign, and the addition distributive property valid for quantity N , in terms of neutral t -quantities only one finds:

$$6 \quad cn + cn' = N \cdot D(cn, \Delta t) + N \cdot D(cn', \Delta t) = N$$

Rewritten with multiplication M and addition A for time intervals one finds the corresponding Δt -quantity perspective. Equation 6 describes a requirement for $cn = cn(t)$ and $cn' = cn'(t)$ where time interval Δt is the variable. Arguments for this requirement are discussed in comment 6, and paragraph 12, part VI. It exists due to cn and cn' , being not ordinary constants, rather physical quantities defined within the time interval equilibrium description. In support for the requirement, equation 6 and 7, an argument below includes a theorem for time and space averages.

cn and cn' are 'reciprocal' quantities, a change of perspective to Δt - quantities $cn(\Delta t)$ and $cn'(\Delta t)$ and time intervals means the requirement implies $A [cn(\Delta t), cn'(\Delta t)] = \Delta N$, with correspondence $\langle N \rangle = N \sim \Delta N$, invariant with one moment time t and invariant with Δt . From $\langle t \rangle \sim \Delta t_0$ one finds: when $\langle D(cn, \Delta t) \rangle_{\Delta t} \sim \Delta U$ and $cn(\Delta t) = M [\Delta N, \Delta U] = \Delta N$, this implies $\langle D(cn', \Delta t) \rangle_{\Delta t} \sim \Delta U_0$ and $cn'(\Delta t) = M [\Delta N, \Delta U_0] = \Delta U_0$, and the other way around while invariant $|\Delta N|$ remains the upper limit value for $|cn(\Delta t)|$ or $|cn'(\Delta t)|$. For invariants related to one moment time development referred is to [11, Noether], [12, De Wit, Smith]. The equilibrium requirement in Δt -quantity perspective while Δt remains the variable is:

$$7 \quad A [cn(\Delta t), cn'(\Delta t)] = A [M [\Delta N, \Delta U], M [\Delta N, \Delta U_0]] = A [\Delta N, \Delta U_0] = \Delta N$$

Addition $A [A1, A2]$, where $A1$ and $A2$ are Δt -quantities and correspond with t -quantity averages, in this paragraph preliminary applied for addition with ΔU and ΔU_0 , is defined in paragraph 6 and 7. Zero domain is not considered for $cn(\Delta t)$ or $cn'(\Delta t)$ since this means they are not time interval related rather one moment time t related. It can be argued ΔN in equation 7 is an approximation for the complete time interval set average $\langle cn(\Delta t) \rangle_{\text{set}}$, equal to the $cn(\Delta t)$ space average [1, Arnold], considering $cn'(\Delta t) = cn(\Delta t')$ where $\Delta t'$ is the specific time interval different from Δt while $|cn(\Delta t)|$ and $|cn'(\Delta t')|$ equal their upper limit value.

The requirement is supported by the second associative property, equation 3, due to the property for the Noether charge NC , $D^* \int_{\Delta t} [\Delta N] = NC$. Where from equation 7, ΔN relates to addition, ΔN relates to multiplication similarly. This is discussed in paragraph 12, part I and VI. The neutral t -quantities cn and cn' can be regarded t -Noether charges invariant for parameter t during Δt where the corresponding neutral Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$ similarly can be regarded Δt -Noether charges. The usual Noether charges, NC , should relate to the relevant event time interval that is decisive for measurements.

Comment 5. The usual Noether charges NC are the same as the t -Noether charges when limited to the, finite, time interval Δt , the relevant event time interval for measurement and also relevant for deciding on what are constants, ie through measurement. This implies the one moment time description for NC . Otherwise for time interval quantities, the NC are equal to the Δt -Noether charges, similarly limited to finite time interval Δt . The addition of a pair of reciprocal Δt -Noether charges, related through commutation relations for time intervals that correspond with the commutation relations for one moment time t , is invariant with variable Δt , meaning $A [cn(\Delta t), cn'(\Delta t)] = \Delta N$ is an invariant for the overall time interval set, where in fact $cn(\Delta t)$ and $cn'(\Delta t)$ both have the same limit value, and a difference exists only because domain difference depending on the variable, ie Δt . ΔN can be termed a ΔN -Noether charge. ΔN -Noether charges correspond with the usual Noether charges

NC when unlimited infinite time intervals are considered, for instance non interacting situations with H time independent.

Comment 6. For any relevant event time interval Δt , $cn(\Delta t)$ and $cn'(\Delta t)$, being averages and time interval Δt -quantities, resemble for instance $\langle T \rangle_{\Delta t}$ and $\langle V \rangle_{\Delta t}$, ie the kinetic energy and potential energy quantities being invariant during Δt because of averaging however variable with Δt . For kinetic energy T and potential energy V, both t-quantities and not necessarily invariant during Δt and within the usual one moment time description, Lagrangian equilibrium requires total energy $T + V$ remains invariant during Δt just like $\langle T \rangle_{\Delta t} + \langle V \rangle_{\Delta t}$. These quantities are in principle independent, however, like for T and V, equilibrium means some relation applies.

5. Solution of the 'infinite regress' problem due to the definition of time interval boundaries

The usual qm description of em radiation, including propagation surface, waves and wave 'group' velocity and de Broglie complementarity for waves and wave particles, applies the one moment time description, and assumes parallel plane waves and a stationary state energy variance, ie parallel 'ray' propagation [13, Sakurai]. In contrast, star source radiation assumes sphere-like propagation.

Re-introduced are results for star source radiation from [4, Hollestelle] and [5, Hollestelle]. The reference space coordinate q, within both the one moment time and time interval description, indicates a coordinate place at the radiation propagation sphere surface such that coordinate origin q_c is 'close' to q. It is meant $q(t_a) = q$ and $q(t_b) = q_b$, where q_b indicates the star source coordinate place, for $\Delta t = [t_b, t_a]$. The reference space coordinate q implies a possible measurement place. The line of sight from the star source at place q_b to place q at the propagation surface relates to space interval Δq . In this description place q and the 'origin' space coordinate place q_c both are regarded to 'move along' with the radiation propagation surface while the star source remains at the invariant place q_b in space, with distance $|\Delta q| = |q - q_b|$ increasing with propagation development.

The uncertainty relations continue to be part of qm if only because they relate qm subjects, like elementary particles, photons and electrons, to 'Newtonian' experimental subjects, like for measurements [13, Sakurai]. Assuming de Broglie complementarity derived are uncertainty relations within the time interval description, different from the usual uncertainty relations when the Hamiltonian is time dependent [5, Hollestelle].

Any usual Lorentz transformation TU should leave metric distances unchanged. In [5, Hollestelle] it is argued to introduce new Lorentz transformations TL that leave metric propagation surface measure unchanged since star source radiation energy remains unchanged during propagation while being proportional to the metric measure of the propagation surface.

For time interval $\Delta t = [t_b, t_a]$ boundary t_b relates by definition, not regarding causality, to boundary t_a , with $t_b = tb[t_a]$:

$$8 \quad 1/t_b = -1/t_a (1 - (c \cdot q(t_a))^2)$$

The constant c has dimension similar to the multiplication inverse of q, with (c · q) dimensionless. Defined in [4, Hollestelle] are the transformations A and B: A(t) with $A(t_a) = t_b$, and B(t) with $B(t_a) = -1 \cdot t_b$ and its reverse B(-1) for which the Hamiltonian remains invariant. Transformation A defines any time interval Δt with $[t_b, t_a] = [A(t_a), t_a]$. A series of possibly many transformations B(-1) transforms Δt to another $\Delta t'$, and includes a re-scaling of one moment time t, which takes care $(c \cdot q(t_a))^2 \ll 1$ for equation 8 and transformation A to remain valid for defining Δt with t_b and t_a resp. negative and positive sign.

Two different solutions exist: $q(t) = +/- q_0 \cdot 1/t_0 t$ and $q(t) = +/- q_0 \cdot t_0 \cdot 1/t$. The factor (c · q₀) for $q(t) = q_0$ at $t = t_0$ is an invariant, and equal to multiplication M [c, q₀] within the time interval set, where both c and q₀ remain invariant during Δt .

Comment 7. Space- and time coordinates q and t transform to $q + q_L$ and $t + t_L$ resp. when applying a specific Lorentz transformation TL, with q_L and t_L constant space- and time coordinate values. Transformation B(-1) can be interpreted as a Lorentz transformation TL where t to t' corresponds with Δt to $\Delta t'$ with boundaries t_b' and t_a' , where $q' = q(t_a')$ remains well-defined and 'close' to $q_c' = q(t_c')$ like $q = q(t_a)$ 'close' to $q_c = q(t_c)$. When $|t'/t| = |(t + t_L)/t| > 1$ and with equation 8 in terms of transformation A, this means that for, the related to A, specific TL: $t' = TL(t)$ is quadratic in t, [5, Hollestelle]. By applying this TL one derives for the commutation pair $cn(\Delta t)$ and $cn'(\Delta t)$ there is $cn'(\Delta t) = cn(\Delta t')$, starting from cn and cn' , equation 1.

Since time interval $\Delta t = [t_b, t_a]$ includes one moment time boundaries t_b and t_a , due to correspondence of t -quantities and Δt -quantities, from equation 8 and comment 7, it follows equations 9.

$$9a \quad |t_a|/|t_b| = |cn'(\Delta t)|/|cn(\Delta t)|$$

With correspondence indication ~ the one moment time parameter t -quantities on the left side correspond with time interval Δt -quantities on the right side:

$$9b \quad t_a, t_b \sim M [cn'(\Delta t), cni(\Delta t)]$$

Comment 8. from MVT equilibrium follows the relation $D^*[q(t)] = \pm q(t)/t$, still applying one moment time parameter t even within the time interval description like discussed in relation to equation 8 and comment 1, with two solutions, $q(t)$ linear in t and $q(t)$ linear in $1/t$. Due to MVT equilibrium, $D^*[q]$ being neutral, independent of t during Δt , $q(t)$ necessarily is linear in t and the linear in $1/t$ solution can exist only when it is the *same solution*, ie also linear in t .

To evaluate the meaning of the multiplication inverse t_i of t -quantity one moment time t itself, started is from the one moment time 'multiplication unit' t_0 , paragraph 3. For the multiplication inverse for t : $t_i = 1/t$, where t_i however not necessarily belongs to the one moment time set, one finds the relation: $t_i \cdot t = t = t_0$, which leaves possibly many solutions t_i . To give meaning to $t_i = 1/t$, defined is: solution t_i is to belong to the one moment time set. In this definition for one moment time t_1 and t_2 , the multiplication $t_1 \cdot t_2 = t_3$ belongs to the one moment time set in agreement with [4, Hollestelle].

For Δt boundaries t_a and t_b the relation with t_0 is: $t_a + t_b = 2t_0$ and $t_a \cdot t_b = 1/2 (t_a^2 + t_b^2)$ approaching to $t_a \cdot t_b = 1/2 t_a^2$ for $|t_a| \gg |t_b|$, valid for situations with H time dependent, with Δt highly asymmetric. These one moment time t_b and t_a properties correspond with the relevant event time interval Δt property $M [\Delta t_i, \Delta t] = M [\Delta t, \Delta t] = \Delta t = \Delta U$, comment 3. This correspondence is applied to derive equation 9.

The overall time interval set needs a new overall description for its Δt -quantity properties. The multiplication inverse Δt_i for Δt , considered linear in Δt , exists due to the relation $M [\Delta t, \Delta t] = \Delta U$, at least a solution is $\Delta t_i = \Delta t$ itself, comment 3. I^* and D^* within the time interval description refer to integration and differentiation to one moment time t and correspond with $I^*||\Delta t$ and $D^*||\Delta t$ for integration and differentiation to time interval Δt . All Δt -quantities are regarded linear in Δt and can be reduced to time intervals, this is discussed in paragraph 12, part IV. One derives for the Δt -quantity $cn(\Delta t)$ in the time interval description:

$$10 \quad I^*||\Delta t [cn(\Delta t)] = M [\Delta t, A [cn(\Delta t), -1 \cdot cn'(\Delta t)]] = A [cn(\Delta t), -1 \cdot cn'(\Delta t)]$$

$$D^*||\Delta t [cn(\Delta t)] = M [A [cn'(\Delta t), -1 \cdot cn(\Delta t)], \Delta t_i] = A [cn'(\Delta t), -1 \cdot cn(\Delta t)]$$

These equations are time interval only relations, and do not require one moment parameter t . They include multiplication with Δt resp. Δt_i and are the time interval equivalent of equations 1, ie they define commutation relations for time intervals and time intervals 'working to the right'. They include Δt and Δt_i and in this way MVT equilibrium for energies related to $I^*||\Delta t$ and $D^*||\Delta t$ is assured since the results for both in terms of time intervals is the same. In terms of one moment time quantities the results seem opposite since results are similar to those of the one moment time description when H time independent. However, for H time independent, ie a noninteracting situation, both cn and cn' are the same and $cn + -1 \cdot cn'$ equals the 'addition zero' for the one moment time set corresponding with ΔU_0 . *With equations 9 and 10 the 'infinite regress' problem for the definition of time intervals is resolved.*

The meaning of $I^*||\Delta t$ and $D^*||\Delta t$, integration and differentiation to Δt , starts from MVT equilibrium and the linear in Δt property just like I^* and D^* with MVT equilibrium and the linear in t property. Derivation of equation 10 includes the commutation versions, comment 1, for $I^*||\Delta t$ and $D^*||\Delta t$, including one moment time dependence, application of equation 9 and correspondence of t_0 with $\Delta t_0 = \Delta U = \Delta t$ for $I^*||\Delta t$ and linearity in Δt_i for $D^*||\Delta t$, explaining the occurrence of Δt and Δt_i within equation 10.

It is assumed the situation is "remote" from H time independent, ie $|t_a| \gg |t_b|$, and thus $|cn'(\Delta t)| \gg |cn(\Delta t)|$ due to equation 9. For $cn'(\Delta t)$ similar equations can be derived. The indication $*$ is for operators, however applied for the above integration and differentiation by exception. From equation 10 one finds similarity for $I^*||\Delta t$ and $D^*||\Delta t$ in agreement with $\Delta t_i = \Delta t$ and comment 8, except for the order of $cn(\Delta t)$ and $cn'(\Delta t)$. The order difference can be disregarded in this case since from the correspondence t_0 with $\Delta U = \Delta U_i$ it follows $I^*||\Delta t [cn(\Delta t)] = D^*||\Delta t [cn(\Delta t)]$, ie the result in terms of time intervals is in fact the same for both integration and

differentiation, at least for Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$. Since $cn(\Delta t)$ is linear in Δt , and any Δt -quantity can be represented linear in $cn(\Delta t)$, this supports the inference for $I^*||\Delta t$ and $D^*||\Delta t$ that results are exactly similar to equation 10 for all time intervals and Δt -quantities. This is discussed in paragraph 10.

6. Time interval only set properties

Commutation properties are usually defined within a one moment time description and depend on the vector like space time coordinates and quantities that can form combinations like addition or multiplication. In this way differences and change can be included in the one moment time description with differentiation to parameter t . In this paper the time interval description includes both t -quantities and Δt -quantities. However, for a description with time intervals only, specific properties for the time interval set like addition and multiplication are necessary, combinations for time intervals that before did not have meaning yet.

In [4, Hollestelle] two elementary parameters, termed elements, define time interval $\Delta t = [tb, ta]$ and its boundaries tb an ta within a one-dimensional time concept. It is argued these two elements are such that they agree with the way time can be measured or counted and they are independent of, preliminary to, one moment time t . Any time interval is based on these elements where measurement is possibly counting towards ta in the future to count forward to from tb in the past. The definitions for time interval properties and for elements from [4, Hollestelle] remain valid in this paper. Any time interval includes a part of the past tb belongs to and a part of the future ta belongs to. With tb and ta the one moment time t and vector concept remain within the time interval description. In this paper a time interval only description is proposed, starting with the results from paragraph 5.

7. Addition and some fundamental properties

Within the time interval description defined is addition from the introduction of ‘addition zero’ time interval Δt_0 such that addition of Δt_0 with any other time interval Δt_1 leaves Δt_1 invariant. There is $A [\Delta t_1, \Delta t_0] = \Delta t_1$. It is plausible, and discussed for Δt_0 in comment 4, for any Δt_1 there should be Δt_1 ‘addition’ Δt_1 equals Δt_1 . When the time interval description regards events and properties within reality and regards these within any time interval, say Δt_1 , the ‘addition zero’ Δt_0 for the time interval set at least can be relevant event time interval Δt itself. There is $A [\Delta t_1, \Delta t] = \Delta t_1$. This depends on the premise that there is only ‘one’ ‘time’ for the cosmological universe together and all is in the same ‘time’ and nothing is late or early. Time intervals do not exist outside themselves, they remain by themselves and don’t give extra time from outside themselves to themselves:

11 Δt_1 ‘addition’ Δt_1 , to the same time interval, leaves Δt_1 invariant: $A [\Delta t_1, \Delta t_1] = \Delta t_1$

Δt_1 is any time interval within the time interval set. This confirms the interpretation of time interval set addition with domain addition where addition of two identical domains results in the same domain. The time interval description differs in results from the one moment time description when the Hamiltonian H is time dependent [4, Hollestelle], and the relevant event time interval Δt is asymmetric and finite. Time interval Δt includes one moment time boundaries tb and ta : tb differs from -1 . ta , while both tb and ta are finite with opposite sign and $|tb| < |ta|$.

The ‘addition inverse’ Δt_{iv} for a finite relevant event time interval Δt equals Δt , i.e. $A [\Delta t_{iv}, \Delta t]$ equals $A [\Delta t, \Delta t] = \Delta t$. One finds Δt ‘addition’ -1 . Δt does not equal Δt , where -1 . Δt means with opposite signs for the boundaries. The -1 . Δt is not the ‘addition inverse’ for Δt and is not well defined for finite time intervals: it does not belong to the time interval set. One might argue Δt ‘addition’ -1 . Δt equals -1 . Δt from the above definition where Δt_0 equals Δt . Notice that Δt_0 is not a zero-measure time interval in the sense of domain measure equal to zero. The order within addition seems to matter. Addition for any two time intervals Δt_1 and Δt_2 is indicated with $A [\Delta t_1, \Delta t_2]$.

8. Multiplication and the multiplication closure theorem

Similar to addition A one can define multiplication M , with indication $M [\Delta t_1, \Delta t_2]$ for any two time intervals Δt_1 and Δt_2 , from the introduction of “multiplication unit” ΔU , while the relevant event time interval remains Δt . Like with addition it is not clear immediately what multiplication could mean for the time interval set. Several properties for M are now summarized.

Multiplication properties

The result of multiplication is assumed to belong to the time interval set: $M [\Delta t_1, \Delta t_2] = \Delta t_3$ and this implies closure of the time interval set for multiplication M . The resulting Δt_3 is a time interval, confirmed by the time interval set “multiplication unit” ΔU relation $M [\Delta U, \Delta t_1] = \Delta t_1$. This is supported by several of the following properties. The multiplication inverse Δt_{1i} for any time interval Δt_1 is a time interval itself, including the multiplication property: $M [\Delta U, \Delta t_{1i}] = \Delta t_{1i}$. Due to comment 3 the multiplication inverse Δt_i for Δt equals Δt itself, Δt being the relevant event time interval. $M [\Delta t_1, \Delta t_{1i}] = \Delta U$ for any Δt_1 , including those Δt_1 not equal to Δt , assuming there exists at least one multiplication inverse time interval Δt_{1i} for each Δt_1 . With similar argument for validity of equation 11 for addition, multiplication $M [\Delta t_1, \Delta t_1] = \Delta t_1$ is valid for Δt_1 any time interval.

Comment 9. Associative properties. The symmetrical first associative, serie, property for M : $M [M [g_1, g_2], g_3] = M [M [g_1, g_3], M [g_2, g_3]]$ is assumed valid, the symmetrical second associative, parallel, property for M : $M [M [g_1, g_2], M [g_1, g_3]] = M [g_1, M [g_2, g_3]]$ is not necessarily valid, for g_i any three time intervals. The first and second associative property to be similar in validity can be contradictory. The usual derivative of a multiplication applies the second associative property, equation 3. The derivative to one quantity applies commutations in terms of one moment time or time intervals, comment 1. The associative properties are defined for the time interval set, associative properties are known for ordinary sets, for instance for some specific algebras, [14, Jacobson].

For multiplication M the following definition is feasible, relation 12. The integral $I^* || \Delta t_2$ to time interval Δt_2 is introduced, just like the usual integral I^* to one moment time t , with the integral domain equal to the Δt_2 -domain:

Comment 10. Multiplication

$$12 \quad M [\Delta t_1, \Delta t_2] = M [M [< \Delta t_1 > || \Delta t_2, \Delta t_2] = M [< \Delta t_1 > || \Delta t, \Delta t_2] = I^* || \Delta t_2 [\Delta t_1]$$

This definition depends on the order of Δt_1 and Δt_2 . Other relations that possibly are symmetrical in Δt_1 and Δt_2 are possible however not discussed in this paper. With time interval Δt_1 invariant during Δt and $\Delta t_1 = < \Delta t_1 > || \Delta t$ and $\Delta t = \Delta t_0$, there is $< \Delta t_1 > || \Delta t = M [< \Delta t_1 > || \Delta t, \Delta t] = M [< \Delta t_1 > || \Delta t_2, \Delta t_2]$ and $< \Delta t_1 > || \Delta t_2 = M [M [< \Delta t_1 > || \Delta t_2, \Delta t_2], \Delta t_2] = M [M [< \Delta t_1 > || \Delta t, \Delta t], M [\Delta t_2, \Delta t]] = M [\Delta t_1, \Delta t_2]$. This is the meaning of the second and third part of equation 12, including the average $< \Delta t_1 > || \Delta t_2$, and the integral to Δt_2 by definition $I^* || \Delta t_2 [\Delta t_1] = M [< \Delta t_1 > || \Delta t, M [\Delta t, \Delta t_2]] = < \Delta t_1 > || \Delta t_2$. Applied is comment 3 and the symmetrical first associative property for M , comment 9.

With unchanged multiplication order for Δt_1 and Δt_2 , and with $\Delta t_2 = \Delta t$, no changes occur for equation 12 in principle and it immediately follows:

$$13 \quad M [\Delta t_1, \Delta t] = < \Delta t_1 > || \Delta t = I^* || \Delta t dt [\Delta t_1] = \Delta t_1$$

The meaning and difference for averaging and integration to time interval Δt_2 and to Δt , are clearly indicated from the above equations. When time interval Δt_1 and Δt_2 do not extend to outside Δt , ie the common domain for Δt_1 or Δt_2 with Δt equals Δt_1 or Δt_2 , and with one moment time t being continuous within Δt these averages are evident. For $\Delta t_1 = \Delta U$, the ‘multiplication unit’ for the time interval set, one finds:

$$14 \quad M [\Delta U, \Delta t] = I^* || \Delta t [\Delta U] = \Delta t$$

$$15 \quad M [\Delta U, \Delta t_i] = I^* || \Delta t_i [\Delta U] = \Delta t_i$$

The ΔU relations, $M [\Delta U, \Delta t] = M [\Delta t, \Delta U] = \Delta t$, comment 3, and $M [\Delta U, \Delta t] = \Delta U$, equation 13, confirm $\Delta U = \Delta t$ and $\Delta t_i = \Delta t$. From ‘multiplication inverse’ time interval $\Delta t_i = \Delta t$ one finds independent support for equation 12 assuming the definition for time interval multiplication.

Closed multiplication. Closed multiplication implies $M [\Delta t_1, \Delta t_2] = \Delta t_3$ is a time interval, and validity of $\Delta t_3 = a \cdot \Delta t_a$, scalar a itself dependent on time interval Δt_a , implies the solution time interval Δt_a . Not for all scalars a one finds solutions Δt_a , at least for $a = 1$ a solution exists and is $\Delta t_a = \Delta t_3$.

Comment 11. Not all scalar arguments a can be allowed when $M [\Delta t_1, \Delta t_2] = a \cdot \Delta t_a$ belongs to the time interval set, where the future domain part measure exceeds the past domain part measure. When Δt_a is a time interval, -1. Δt_a is not when H time dependent and Δt asymmetric. For any time interval, equation 8 is the defining and

necessary relation for one moment time boundaries t_b and t_a . The existence of Δt_a depends on Δt -quantities being linear with Δt due to MVT equilibrium, paragraph 12, part IV.

The multiplication result Δt_3 can be rewritten with a time interval multiplication $\Delta t_3 = a \cdot \Delta t_a = M [ca, \Delta t_a]$. Due to the symmetrical first associative property: $ca = M [ca, \Delta t] = M [ca, M [\Delta t_a, \Delta t_a]] = M [M [ca, \Delta t_a], \Delta t_a]$, ie a time interval average multiplication $ca = M [< ca > || \Delta t_a, \Delta t_a]$ for any time interval ca . Time interval averages for time intervals, like in equation 12, are time intervals themselves. When applying the first associative property with several steps, one finds: $M [ca, \Delta t_a] = M [M [< ca > || \Delta t_a, a_i \cdot M [ca, \Delta t_a]], \Delta t_a]$ is equal to $M [< ca > || \Delta t_a, a_i \cdot M [ca, M [\Delta t_a, \Delta t_a]]] = M [< ca > || \Delta t_a, a_i \cdot M [ca, \Delta t_a]] = M [< ca > || \Delta t_a, \Delta t_a] = ca$, and there is $ca = \Delta t_3 = a \cdot \Delta t_a$ and $\Delta t_a = \Delta t$ for any allowed scalar a , comment 11. This means a solution is $ca = a \cdot \Delta t$ and $\Delta t_3 = a \cdot \Delta t$ and both are linear in Δt .

These solutions follow directly from $a \cdot \Delta t_a = M [ca, \Delta t_a]$ and property $M [\Delta t_a, \Delta t_a] = \Delta t_a$. Solutions for any Δt_a should exist since the time interval set is linear in Δt , ie 1-dimensional. Due to MVT equilibrium these solutions are found by starting from $ca = e_1 \cdot \Delta t$ and $\Delta t_a = e_2 \cdot \Delta t$, with result $\Delta t_3 = e_1 \cdot e_2 \cdot \Delta t$ linear in Δt and $a = e_1$. Solution $\Delta t_a = e_2 \cdot \Delta t$ is linear in Δt and includes any scalar e_2 , meaning in this way any time interval Δt_a can be a solution. The consideration that any time interval is neutral in Δt and linear in Δt , is discussed in paragraph 12, part IV. For scalar $a = 1$ solution Δt_a equals Δt_3 , and this means a time interval only multiplication exists equal to scalar multiplication at least for $a = 1$: $M [a, \Delta t_a] = M [ca, \Delta t_a]$ with solution $ca = \Delta U = \Delta t$. The scalar a is the resultant for any ca and Δt_a , however with reference to comment 11.

Comment 12. Closure theorem. The relation $\Delta t_3 = a \cdot \Delta t_a = M [ca, \Delta t_a]$ depends on the argument scalar a having 'positive' sign. For argument a with a 'negative' sign the scalar multiplication does not follow the requirements of comment 11. From the first associative property for M with g_1 equal to any scalar a one finds linearity with scalar a for multiplication $M [\Delta t, \Delta t_a]$, with $\Delta t_3 = a \cdot \Delta t_a = a \cdot M [\Delta t, \Delta t_a] = M [a \cdot \Delta t, \Delta t_a]$, equal to $\Delta t_3 = M [ca, \Delta t_a]$ with solution $ca = a \cdot \Delta t$ linear in Δt . From $\Delta t_3 = ca = a \cdot \Delta t$ linear in time interval Δt , ie itself a time interval, it follows the closure theorem for any resultant scalar a , with reference to comment 11: *multiplication M is closed within the time interval set.*

Comment 13. The second associative property for differentiation seems valid if only due to the preserved order of the g_i in series in comment 9. More precisely, this property is related to the following commutation properties for t -quantities $NC(t)$ that are linear in parameter t due to MVT equilibrium. $NC(t)$ is defined from the neutral t -quantities cn and cn' with $D^* [NC(t)] = cn$ and it follows, equation 1,

$$16 \quad t|1. NC(t)|2. cn' = NC(t)|1. t|2$$

$$17 \quad 1/t|1. NC(t)|2 = NC(t)|1. 1/t|2. cn$$

The bar with indication |1 or |2 means the t -quantity at this place depends on parameter t with $t = t_1$ or $t = t_2$. Equations 16 and 17 are not definitions, rather they are derived from the defining relation $D^* [NC(t)] = cn$ and equation 1 for cn and cn' from MVT equilibrium within the time interval description. One finds the order of |1 and |2 is preserved when reversing the order of parameter t and quantity $NC(t)$ within the equations due to commutation. For the overall time interval set and within time interval Δt perspective, cn and cn' correspond with the neutral Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$, linear dependent on Δt , and confirmed is the differentiation second associative property at least for $cn(\Delta t)$ and $cn'(\Delta t)$.

9. Differentiation and the time interval differentiation 'set rule'

Similar like for multiplication M , paragraph 8, one moment time t can be left out for addition A within the time interval set. Proposed is a definition for $A [\Delta t_1, \Delta t_2]$ including multiplication $M [\Delta t_1, \Delta t_2]$ and differentiation to time interval Δt_2 , $D^* || \Delta t_2$. In paragraph 9a and 9b this is applied to $\Delta t_2 = \Delta t$, equation 21 and 22, where the asymmetrical second associative property is assumed for differentiation to Δt , $D^* || \Delta t$, evidently similar with one moment time differentiation d/dt , equation 3.

Comment 14. Addition

$$18 \quad A [\Delta t_1, \Delta t_2] = D^* || \Delta t_2 [M [\Delta t_1, \Delta t_2]]$$

$$19 \quad A [\Delta t_1, \Delta t] = D^* || \Delta t [M [\Delta t_1, \Delta t]] = \Delta t_1$$

$$20 \quad A [\Delta t_0, \Delta t] = D^* || \Delta t [M [\Delta t_0, \Delta t]] = \Delta t_0$$

For Δt equal to Δt_0 , the Δt_0 ‘addition zero’ definition, comment 4, agrees with equation 19 since Δt leaves Δt_1 including $\Delta t_1 = \Delta t_0$ invariant by addition. Equations 19 and 20 result from the identification Δt_2 equals Δt like for multiplication, due to the argument from comment 15. Equation 18 can be derived to be valid for Δt_1 and Δt_2 identified with Δt -quantities $cn(\Delta t)$ and $cn'(\Delta t)$. $cn(\Delta t)$ is linear in Δt due to MVT equilibrium and this supports generalization of equation 18 to the time interval set, discussed in paragraph 12, part IV. The first associative property is applied and cancelling terms are left out. Cancelling terms can be left out due to $A[\Delta t_1, \Delta t] = \Delta t_1$.

Comment 15. The definitions for M and A depend on the existence of a real event where the relevant event time interval is Δt . Only with this Δt the properties for Δt_0 and ΔU can be considered sensible. Multiplication does not make sense with Δt_2 not equal to Δt because it includes a time interval average during Δt_2 , where however the measurement event time interval, the time interval that is also the reference for averaging, is equal to Δt . Multiplication is included in A due to definition equation 18 and therefore also A depends on relevant event time interval Δt .

9a. The asymmetric second associative property and the one moment time differentiation ‘set-rule’

For differentiation of a multiplication $g_1 \cdot g_2$, with g_1 and g_2 arbitrary scalar t-quantities, usually applies: $d/dt [g_1 \cdot g_2] = c_1 \cdot d/dt [g_1] \cdot g_2 + c_2 \cdot g_1 \cdot d/dt [g_2]$, in the one moment time description for the derivative to one moment time t, similar to equation 3 when scalars $c_1 = c_2 = 1$. When, equation 3, both two parts are valued equal, the symmetric second associative, ie distributive, property is valid for the g t-quantities set.

Assume $g_1 \cdot g_2 = a \cdot g_3$ with $g_1 = a$, an invariant scalar, and g_2 any t-quantity. The differentiation ‘set-rule’ defines the result, to release scalar a: $d/dt [a \cdot g_3] = c_1 \cdot d/dt [a] \cdot g_3 + c_2 \cdot a \cdot d/dt [g_3] = a \cdot d/dt [g_3]$, in the symmetric one moment time description, equation 3.

9b. The time interval set and the time interval differentiation ‘set-rule’

Now the perspective is changed to Δt -quantities and the time interval set. The usual properties for addition and multiplication for one moment time sets are based on the vector interpretation for space time coordinates within the one moment time description. They are however not at all evident for A and M within the time interval set and are not to be applied just like this. The usual properties include: $A[a_1 \cdot g_1, a_2 \cdot g_1]$ equals $(a_1 + a_2) \cdot g_1$, for instance with $A[1 \cdot g_1, 1 \cdot g_1] = A[g_1, g_1] = 2 \cdot g_1$, where a_1 and a_2 are scalar numbers and g_1 belongs to the set. These properties decide on the time parameter ‘set rule’. These properties do not exist within the time interval set where $A[\Delta t_1, \Delta t_1] = \Delta t_1$ is essentially different and depending on domain addition instead of vector addition, like for equations 19 and 20 for $g_1 = \Delta t_1$.

Comment 16. The asymmetric second associative property for differentiation to time interval Δt , $D^*||\Delta t [M [g_1, g_2]]$, with g_1 and g_2 any two time intervals, is:

$$21 \quad D^*||\Delta t [M [g_1, g_2]] = A [c_1 \cdot M [D^*||\Delta t [g_1], g_2], c_2 \cdot M [g_1, D^*||\Delta t [g_2]]]$$

with possibly un-equal scalars c_1 and c_2 for asymmetric differentiation and equal scalars for symmetric differentiation of $M [g_1, g_2]$. In the particular case g_1 equals a Δt_1 including any scalar a and time interval Δt_1 , and g_2 identified with Δt , from equation 18 and 21, one finds:

$$22 \quad A [a \cdot \Delta t_1, \Delta t] = D^*||\Delta t [a \cdot \Delta t_1] = A [c_1 \cdot M [D^*||\Delta t [a], \Delta t_1], c_2 \cdot M [a, D^*||\Delta t [\Delta t_1]]]$$

This is not equal to the differentiation to one moment time ‘set rule’: $d/dt [a \cdot g_1] = a \cdot d/dt [g_1]$. The time interval differentiation ‘set rule’, equation 23, is derived from equation 18. Applied is, ‘addition zero’ Δt_0 equals Δt and ‘multiplication unit’ ΔU similarly equals Δt , paragraph 7 and 8. Multiplication with scalar a does not need brackets, $M [a \cdot \Delta t_1, \Delta t] = a \cdot \Delta t_1$. The time interval differentiation ‘set-rule’ is such, that scalar a is placed left and outside of the derivative $D^*||\Delta t$ brackets in equation 22 and not to the right and inside brackets. The ‘set rule’ is non-trivial since $D^*||\Delta t [a]$, for invariant scalar a, is not necessarily equal to ΔU_0 and does not correspond with $d/dt [a]$ equal to the one moment time set ‘addition zero’ and its ‘multiplication zero’. Differentiation provides correspondence for t-quantities with Δt -quantities only when the time interval differentiation ‘set rule’ is introduced.

Comment 17. The existence of scalar multiplication $M [a, \Delta t_2] = a \cdot \Delta t_2$, for any Δt_2 is assured, in the sense of being equal to certain time interval Δt_3 , with invariant scalar a, not a time interval. For scalars the usual one moment time indications and definitions for M and A like \cdot and $+$ apply. Due to the closure theorem, comment

12, for any scalar a with positive sign, the result for $M [a, \Delta t, \Delta t_2] = a \cdot \Delta t_2 = \Delta t_3$, is a time interval. With solution $\Delta t = \Delta t_2$ and Δt_3 belonging to the time interval set, this allows the application of M and A and $I^* \|\Delta t_2$ and $D^* \|\Delta t_2$ to these Δt_3 , a linearity or subset property, paragraph 12, part IV. Time interval derivative $D^* \|\Delta t_2 [a]$ exists independent of this for any scalar a .

It follows from direct evaluation of A and M , applying derivatives $D^* \|\Delta t [\Delta t_1] = [1/t, \Delta t_1] \|\Delta t$, and $D^* [A1] = [1/t, A1]u$, for time interval Δt_1 including one moment time boundaries t_b and t_a and one moment time quantity $A1$, comment 1, $D^* \|\Delta t [a, \Delta t_1]$ does not have to be linear in scalar a and a remain $\text{Rest}(a)$ is added, equation 23. This is derived without applying the second associative property constants c_1 or c_2 for equation 22.

Comment 18. The time interval differentiation 'set rule'

$$\begin{aligned} 23 \quad D^* [a, \Delta t_1] &= A [a, D^* [\Delta t_1], \text{Rest}(a)t] \\ D^* \|\Delta t [a, \Delta t_1] &= A [a, D^* \|\Delta t [\Delta t_1], \text{Rest}(a) \|\Delta t] \\ \text{Rest}(a)t &= [1/t_b, a]u \cdot \Delta t = M [D^* [a], \Delta t] \\ \text{Rest}(a) \|\Delta t &= M [D^* \|\Delta t [a], \Delta t] \end{aligned}$$

It should be for any 'set rule', scalar a is released from $a \cdot \Delta t_1$ within the derivative, say $D^* \|\Delta t [a, \Delta t_1]$, and returns to the left of the derivative of Δt_1 without a , say $D^* \|\Delta t [\Delta t_1]$, while remain $\text{Rest}(a)$ is added to the right. This is in agreement with the interpretation of $D^* \|\Delta t$ 'moving to the right', like one moment time t or time interval Δt , equation 1 and 10, leaving a commutation quantity to the left. The remain $\text{Rest}(a)$ is invariant during Δt , and therefore can be considered a t -quantity or a Δt -quantity. $\text{Rest}(a)$ is derived by applying equations 4 and 5 and correspondence relations, paragraph 3.

The properties for t_0 , including $t \cdot t_0 = t_0 \cdot t = t$ for any t , are defined in [4, Hollestelle]. The usual commutation $[1/t_b, a]u$ has to be evaluated carefully. For $a = 1$ there is $[1/t_b, 1]u$ equals: $t_b \cdot 1 - 1 \cdot t_b$, with $t_b = 1/t_b$, indeed part of the one moment time set, comment 3. However, this is not necessarily equal to $(1 - 1) \cdot t_b$, the 'multiplication zero' equal to the 'addition zero' for the one moment parameter set, due to nonzero commutation involved, and since the specific scalar 1 is 'multiplication unit' for the scalar set one finds $t_b \cdot 1 - 1 \cdot t_b = t_b + (t_b)iv$ only, ie equal to the one moment time 'addition zero' only, when for all t , one moment time addition inverse $tiv = -1 \cdot t$. The usual commutation can be resolved by rewriting it like a time interval commutation that corresponds, equation 4, with time interval differentiation, and regains due to MVT equilibrium zero commutation, comment 1. Applied is time interval operator t^* leaves scalar a invariant and the derivation includes a transformation of scalar a by multiplication with parameter t . Operator $t^* [A1] = M [D^* \|\Delta t [M [t, A1]] \cdot \Delta t]$ for entity $A1$, [5, Hollestelle]. In this way it follows $[1/t_b, a]u$ equals $D^* [a]$.

From equation 4 and comment 4, with one moment time average $\langle t \rangle = t_0$, $D^* [a]$ equals $\langle a \rangle / \langle t \rangle = a \cdot t_0i$, for scalar a positive and invariant with t . Assuming H time dependent one moment time 'multiplication unit' t_0 corresponds with time interval 'multiplication unit' ΔU . Recall $\Delta U_i = \Delta U = \Delta t_0$, from comment 3, and Δt_i equals Δt , from comment 21. It follows $D^* \|\Delta t [a]$ and $\text{Rest}(a) \|\Delta t$ both equal $a \cdot \Delta U$. $\text{Rest}(a) \|\Delta t$ is a time interval related Δt -quantity, implying equation 23 is meaningful as a time interval only relation.

For any scalar a , $\text{Rest}(a)t = M [a, t_0i, \Delta t]$ corresponds with $a \cdot M [\Delta U, \Delta t] = a \cdot \Delta U$. This equals $\text{Rest}(a) \|\Delta t = M [a, \Delta U, \Delta t] = a \cdot \Delta U$, equal to ΔU for $a = 1$. In this case, for $a = 1$, one finds $D^* [a, \Delta t_1] = a \cdot D^* [\Delta t_1]$ and $D^* \|\Delta t [a, \Delta t_1] = a \cdot D^* \|\Delta t [\Delta t_1]$ as it should be.

Comment 19. $\text{Rest}(a)t$, the equation 23 third, right side, part with derivative $D^* [a]$, can be derived independently from the second part with the usual commutation $[a, 1/t_b]u$. This means, the second part due to the derivation above, is confirmed by the third part. This is discussed in paragraph 12, part III.

Comment 20. $M [\Delta U_0, \Delta t_1] = \Delta U_0$, with ΔU_0 the time interval set 'multiplication zero', corresponding with the scalar set 'multiplication zero'. The domain measure for ΔU_0 is zero and $\Delta U_0 = 0 \cdot \Delta t_1$, for any finite Δt_1 including $\Delta U_0 = 0 \cdot \Delta U$. Being of zero domain measure, ΔU_0 is not a proper time interval, and resembles a one moment time parameter and does not belong to the time interval set. Time intervals with zero domain measure are not included in the time interval description in this paper.

Comment 21. The multiplication inverse Δt_i for Δt , is not well defined yet to belong to the time interval set. When $M[\Delta U, \Delta t] = \Delta t$ and $M[\Delta t_i, \Delta t] = \Delta U = \Delta t$ one finds Δt is a solution for Δt_i : $M[\Delta t, \Delta t] = \Delta U$. The solution $\Delta t_i = \Delta t$ is well defined and belongs to the time interval set.

9c. The differentiation asymmetric second associative property and arguments c1 and c2

With c_1 and c_2 scalars defining the asymmetrical second associative property for differentiation $D^*||\Delta t [M[\Delta t_1, \Delta t_2]]$ with equation 21, one finds, from $D^*||\Delta t_2 [\Delta t_1] = \Delta t_1$ for Δt_2 equal to Δt , equation 19, and $a = 1$,

$$24 \quad D^*||\Delta t [M[\Delta t_1, \Delta t]] = A [c_1 \cdot M[\Delta t_1, \Delta t], c_2 \cdot M[\Delta t_1, \Delta t]]$$

Within the last part, $M[\Delta t_1, \Delta t] = \Delta t_1$, due to $\Delta t = \Delta U$ and $\Delta t = \Delta t_0$ for the time interval set.

$$25 \quad A[\Delta t_1, \Delta t] = D^*||\Delta t [M[\Delta t_1, \Delta t]] = A [c_1 \cdot \Delta t_1, c_2 \cdot \Delta t_1] = \Delta t_1$$

This is the time interval set relation equivalent to the one moment time set relation where traditionally $c_1 = c_2 = 1$. When both c_1 and c_2 equal 1 it follows: $A[\Delta t_1, \Delta t_1] = \Delta t_1$. This is the same result derived from interpretation, paragraph 7. Recall that c_1 and c_2 are not any scalars, they have the meaning of second associative property arguments for $D^*||\Delta t [M[\Delta t_1, \Delta t_2]]$, equation 21, and are possibly asymmetric with c_1 different from c_2 . Independent of equation 22, that depends on the second associative property for multiplication M , comment 9, instead equation 23 introduces $Rest(a)$, without associative property arguments. When both c_1 and c_2 equal 1 and with $\Delta t_1 = \Delta t$, the time interval relation seems similar to the traditional one moment time relation: $d/dt [t \cdot t] = c_1 \cdot (1 \cdot t) + c_2 \cdot (t \cdot 1) = 2 \cdot t$, however it is not. The time interval set properties differ from the usual one moment time set properties also in this way.

10. Time interval set equilibrium and a theorem for time intervals including wave equations

With the introduction of averages and time interval set commutation to define time interval differentiation and integration to time intervals, one moment time boundaries remain included within the time interval description, comment 1. Time interval set commutation and one moment time t can be left out by defining integration and differentiation to time intervals in terms of M and A only, equation 12. For $c_n(\Delta t)$ and $c_n'(\Delta t)$ equations 26 and 27 are valid due to the results of paragraph 5, and inferred is validity for any two time intervals Δt_1 and Δt_2 .

$$26 \quad I^*||\Delta t_2 [\Delta t_1] = M[\Delta t_1, \Delta t_2]$$

$$27 \quad D^*||\Delta t_2 [\Delta t_1] = A[\Delta t_1, \Delta t_2]$$

Comment 22. In the time interval description operators are indicated with $*$, depending on a ‘operator quantity’ for their interpretation. Operator quantities are indicated with $|operator^*|$, for example the time operator t^* relates to quantity $|t^*| = t$. Such operator quantities are not defined for I^* and D^* and $I^*||\Delta t_2$ and $D^*||\Delta t_2$, and, even while ‘working to the right’ like operators, these are not proper operators.

It follows from $I^*||\Delta t_2$ and $D^*||\Delta t_2$ having equal results for $\Delta t_2 = \Delta t$, equation 10 and discussion, that similarly M and A have equal results in this case, at least for $c_n(\Delta t)$ and $c_n'(\Delta t)$. This does not depend on the perspective: integration or differentiation to one moment time t or to time interval Δt , since M and A are time interval multiplication and addition for both perspectives and, paragraph 9b, with $\Delta t_2 = \Delta t$ the relevant event time interval, $I^*||\Delta t$ and $D^*||\Delta t$ correspond exactly with I^* and D^* .

M and A depend on time interval properties resembling common domain and combined domain respectively. The difference for M and A , from $I^*||\Delta t [c_n(\Delta t)]$ and $D^*||\Delta t [c_n(\Delta t)]$, equation 10, depends on the order for $c_n(\Delta t)$ and $c_n'(\Delta t)$ and does not return in the result itself in terms of time intervals. To clarify how $I^*||\Delta t$ and $D^*||\Delta t$, and M and A can have the same results: this follows from $D^* [M] = A$ and $D^* [M] = M$ with $A = A[\Delta t_1, \Delta t] = \Delta t_1$ and $M = M[\Delta t_1, \Delta t] = \Delta t_1$ which is valid for any Δt_1 due to the time interval set property $\Delta t = \Delta t_0$ and $\Delta t = \Delta U$. From equations 26 and 27 for $c_n(\Delta t)$ and $c_n'(\Delta t)$ one finds equations 28 to 33.

Comment 23. theorem. Equations $I^*||\Delta t_2 [\Delta t_1] = M[\Delta t_1, \Delta t_2]$ and $D^*||\Delta t_2 [\Delta t_1] = A[\Delta t_1, \Delta t_2]$ can be applied to any two Δt -quantities or any two time intervals Δt_1 and Δt_2 from the time interval set.

The following does not apply the definitions for multiplication or addition. The inference follows directly from the definitions for M and A however these definitions have validity only from assumption. Applied are MVT equilibrium and the time interval derivative. The proof for the closure theorem, comment 12, is slightly different.

Due to a specific Lorentz transformation TL there is $cn'(\Delta t) = cn(\Delta t')$ defining $\Delta t'$ from Δt . The specific TL and change of Δt to $\Delta t'$ such that the Hamiltonian H remains unchanged is derived in [4, Hollestelle]. For other changes of Δt to say Δt_1 , it follows H can be variable. For transformation TL with Δt to $\Delta t'$, the theorem is valid due to the results of paragraph 5. For all other transformations Δt to Δt_1 the linearity of any Δt_1 with Δt is assured, even when the linearity constants do not agree with the specific transformation TL and $cn(\Delta t)$ and $cn'(\Delta t)$ do not agree with Δt and Δt_1 . From linearity of Δt_1 with Δt it follows the argument for $\Delta t'$ from Δt is valid for any Δt_1 from Δt within the time interval set.

From equation 1 defined are $cn(\Delta t') = cn'(\Delta t)$. Meanwhile $cn(\Delta t)$ and $cn(\Delta t')$ are linear in the relevant event time interval, ie Δt and $\Delta t'$ respectively, implying a translation Δt to $\Delta t'$. This relation can be reversed unless for instance $D^*||\Delta t$ for $cn(\Delta t)$ or $cn'(\Delta t)$ equals the time interval set 'addition zero' $\Delta U0$. In the reverse case $\Delta t'$ is linear in $cn(\Delta t)$ and thus $\Delta t'$ is linear in Δt . In fact, for any time interval, the implied translation from Δt then results in a linear multiplication of Δt .

For any Δt_2 the transformation Δt_2 from Δt_1 is the same as a time interval addition applying a nonspecific Lorentz transformation, ie a transformation without the property H remains unchanged. A translation exists due to this addition, and the theorem does apply for these Δt_2 since a translation means a linear multiplication result, due to MVT equilibrium. Similarly, when Δt_2 linear in Δt_1 a translation does exist, this is discussed in paragraph 12, part IV. Thus, not only for $cn(\Delta t)$ and $cn'(\Delta t)$, for any time interval Δt_1 and Δt_2 , and for any Δt -quantities being linear in Δt , the theorem is valid.

An interpretation of change for Δt is necessary. For the changed situation the validity of the following is to be ensured: $M' [\Delta t_1, \Delta t'] = M'' [\Delta t_1, \Delta t''] = \Delta t_1$, ie with the situation change the relevant event time interval $\Delta t'$ changes to $\Delta t''$ while specific properties for M and A do not change and are defined with the relevant event time interval indication Δt , one of these properties $\Delta t = \Delta U$. It depends on the situation present: before change $\Delta t' = \Delta t$, after change $\Delta t'' = \Delta t$, where situations for $\Delta t'$ or $\Delta t''$ share properties, however not all properties, to assure change.

The no-reverse case.

In this case a change for Δt does not change cn or cn' (recall cn' is defined from equation 1, with invariant Δt): $cn(\Delta t') = cn(\Delta t'')$ and cannot be reversed at all. For a change $\Delta t'$ to $\Delta t''$ with $\Delta t' \neq \Delta t''$, and where for now $\Delta t' = \Delta t''$ is not considered, with $\Delta t' = \Delta t$ for the situation before change, and the factor $+/-1$ not yet specified while $+1$ for positive increasing $cn(\Delta t')$ etc, the complete equation for invariant $cn(\Delta t')$ is $A [cn(\Delta t''), cn(\Delta t')iv] = M [+/-1. D^*||\Delta t [cn(\Delta t')], A [\Delta t'', \Delta t'iv]] = \Delta U0$. For $D^*||\Delta t [cn(\Delta t')] = \Delta U0$ valid, one finds $cn(\Delta t') = cn(\Delta t'') = \Delta U0$. The time interval $\Delta U0$ is both 'multiplication zero' and 'addition zero' for the time interval set and has time interval domain measure zero and is not a proper time interval, and $cn(\Delta t)$ and $cn'(\Delta t)$ are not well defined.

For $D^*||\Delta t [cn(\Delta t')] = \Delta U0$ not valid one finds:

I. Above, starting from $\Delta t' = \Delta t$ a change to $\Delta t'' = \Delta t$ means: $A [cn(\Delta t''), cn(\Delta t')iv] = M [+/-1. D^*||\Delta t [cn(\Delta t')], A [\Delta t'', \Delta t'iv]] = \Delta U0 = \Delta t0$ equal to the time interval "addition zero" since $cn(\Delta t'') = cn(\Delta t')$.

II. From I: $cn(\Delta t'') = A [M [+/-1. D^*||\Delta t [cn(\Delta t')], \Delta t''], cn(\Delta t')] = A [M [cn(\Delta t''), \Delta t''], cn(\Delta t'')] = cn(\Delta t')$. These equations follow from $\Delta t' = \Delta t$ and $\Delta t'iv = \Delta t'$ for the situation before change. Applied is $+/-1. D^*||\Delta t [cn(\Delta t')] = M [cn(\Delta t'), \Delta t'] = cn(\Delta t')$ for all $\Delta t'$ including $\Delta t''$, ie the validity of equations 26 and 27 for cn and cn' including similarity of results for M and A, depending on equation 10.

III. From II: $A [cn(\Delta t''), cn(\Delta t')iv] = M [cn(\Delta t'), A [\Delta t'', \Delta t'iv]] = \Delta U0$. A solution is $cn(\Delta t') = \Delta t'$ for all $\Delta t'$ including $\Delta t''$ where $\Delta t' = \Delta t$. When $cn(\Delta t') = M [cn(\Delta t'), \Delta t']$ one similarly finds the solution $cn(\Delta t') = \Delta t'$ from $M [\Delta t', \Delta t'] = \Delta t'$ for all $\Delta t'$. This applies the first associative property.

The no-reverse case, with $D^*||\Delta t [cn(\Delta t')] = \Delta U0$ not valid for any $\Delta t'$, means $cn(\Delta t')$ is linear in $\Delta t'$, and this does not agree with $cn(\Delta t')$ invariant. *This completes the proof for theorem comment 23.*

$$28 \quad I^*||\Delta t [cn(\Delta t)] = A [cn(\Delta t), A [\Delta U, cn(\Delta t)]] = A [cn(\Delta t), cn(\Delta t)]$$

$$29 \quad I^*||\Delta t [cn'(\Delta t)] = M [cn'(\Delta t), \Delta t] = A [cn'(\Delta t), A [\Delta U, cn'(\Delta t)]] = A [cn'(\Delta t), cn'(\Delta t)]$$

$$30 \quad A [a. \Delta t_1, \Delta t] = D^*||\Delta t [M [a. \Delta t_1, \Delta t]] = D^*||\Delta t [a. I^*||\Delta t [\Delta t_1]] = A [a. D^*||\Delta t [I^*||\Delta t [\Delta t_1]], Rest(a)||\Delta t]$$

Equation 30 is valid also with $cn(\Delta t)$ or $cn'(\Delta t)$ instead of Δt . Not necessarily $cn(\Delta t)$ follows the requirement of time interval asymmetry like Δt itself for situations when H time dependent, and $-1 \cdot cn(\Delta t)$ is valid however $-1 \cdot \Delta t$ is not. From equation 10 for $I^*||\Delta t$ it follows:

$$31 \quad \begin{aligned} a. cn(\Delta t) &= D^*||\Delta t [a. I^*||\Delta t [cn(\Delta t)]] = A [a. D^*||\Delta t [I^*||\Delta t [cn(\Delta t)]], Rest(a)||\Delta t] = A [a. D^*||\Delta t [A [cn(\Delta t), -1. cn'(\Delta t)], Rest(a)||\Delta t] \\ a. cn'(\Delta t) &= D^*||\Delta t [a. I^*||\Delta t [cn'(\Delta t)]] = A [a. D^*||\Delta t [I^*||\Delta t [cn'(\Delta t)]], Rest(a)||\Delta t] = A [a. D^*||\Delta t [A [cn(\Delta t), -1. cn'(\Delta t)], Rest(a)||\Delta t] \end{aligned}$$

For scalar $a = 1$, $\Delta t1 = cn(\Delta t)$ and $\Delta t2 = \Delta t$ the remain $Rest(a)||\Delta t$ is nonzero and equal to $cn(\Delta t)$. Scalar a can be released from $I^*||\Delta t$ without any nonzero remain, however only released from $D^*||\Delta t$ with 'set rule' equation 23 including remain $Rest(a)||\Delta t$. Due to property $A [\Delta t1, \Delta t1] = \Delta t1$ for any time interval $\Delta t1$ including $\Delta t1 = cn(\Delta t)$, the remain can be left out and one finds $D^*||\Delta t$ and $I^*||\Delta t$ to be exactly the same.

$$32 \quad \begin{aligned} a. cn(\Delta t) &= D^*||\Delta t [a. I^*||\Delta t [cn(\Delta t)]] = A [a. D^*||\Delta t [I^*||\Delta t [cn(\Delta t)]], Rest(a)||\Delta t] \\ a. cn'(\Delta t) &= D^*||\Delta t [a. I^*||\Delta t [cn'(\Delta t)]] = A [a. D^*||\Delta t [I^*||\Delta t [cn'(\Delta t)]], Rest(a)||\Delta t] \end{aligned}$$

For $a = 1$ and with Δt -quantity $cn(\Delta t)$ linear with Δt , one finds:

$$33 \quad \begin{aligned} cn(\Delta t) &= D^*||\Delta t [I^*||\Delta t [cn(\Delta t)]] = A [D^*||\Delta t [I^*||\Delta t [cn(\Delta t)]], cn(\Delta t)] \\ cn'(\Delta t) &= D^*||\Delta t [I^*||\Delta t [cn'(\Delta t)]] = A [D^*||\Delta t [I^*||\Delta t [cn'(\Delta t)]], cn'(\Delta t)] \end{aligned}$$

Wave equations and structure constants

These equations being second order resemble quite closely wave equations. From the discussion below equation 7 it follows ΔN equals both addition or multiplication of the reciprocal pair of commutation quantities. The derivative $D^*||\Delta t [\Delta N] = A [cn(\Delta t), cn'(\Delta t)] = \Delta N$ and similarly the second derivative $D^*||\Delta t [D^*||\Delta t [\Delta N]] = \Delta N$ and equations 33 apply to ΔN . These equations are due to the derivatives being time interval MVT equilibrium derivatives. Recall ΔN is an invariant for the overall time interval set. ΔN , like any Δt -quantity, is part of the linearity 'subset' for $cn(\Delta t)$ or $cn'(\Delta t)$. This is discussed in paragraph 12, part IV.

From combinations of commutations, being multiplications within the generator set, one finds the set structure constants from the second derivatives, [12, De Wit, Smith], [15, Veltman]. Structure constants for the time interval set depend on $D^*||\Delta t [D^*||\Delta t [M]]$, where due to property equation 11 for A and the similar property for M , paragraph 8, multiplication within the generator set equals multiplication within the time interval set, and one finds $M = \Delta N$. The second derivative invariants are Lorentz transformation TL invariants, due to TL being a surface measure preserving transformation, unlike the usual TU. Due to this the overall TL invariant ΔN can be interpreted directly to be a structure constant.

From equations 1 and 10 one finds the pair-like commutation properties of the one moment time set and the time interval set respectively. The commutation constants form a multiple and the existence of an equilibrium overall invariant, in this case $\Delta N = M [cn(\Delta t), cn'(\Delta t)]$, means for structure constants:

Comment 24. *The dimension of the set decides on the number of structure constants to form a set multiple.*

Comment 25. *For the time interval set the multiple is a pair, due to time and the time interval set, with all time intervals linear in Δt due to closure, being one dimensional.*

Paragraph 11. Star source wave propagation

Radiation from a star source described with propagation sphere surfaces is different from radiation with stationary parallel propagation. The metric surface measure, not meaning the usual line element dependent metric, for propagation surfaces is invariant with change of relevant event time interval Δt that indicates time development within the time interval description for qm fields [4, Hollestelle]. This description still assumes de Broglie complementary and depends on the interpretation of radiation existing of waves or wave particles of zero or non-zero mass with a group velocity $c(\Delta t)$, c_{light} for the usual c for photons. An effort to describe the resulting folded, wrinkled, radiation propagation surface for changing Δt is the following.

Propagation surface A related to radiation time interval Δt , can be described with a regular sphere $Q(\Delta t)$ that however does not fulfill the above measure invariance. The realistic propagation surface A, can be approximated with surface P, applying a construction from reference [16, Hocking, Young].

For a set of dense disjoint open surface parts, constructed from an indecomposable continuum surface, the overall sum of metric surface part measures can be different from the metric continuum surface measure. The set of parts P_i , $i = 1$ to n , n the number of parts, can be termed a distribution for the continuum surface Q, and their union termed P. There is $mP = \sum mP_i$, leaving out n , with m indicating metric surface measure. The construction allows mP to be different from mQ . When $mA = mP$ for P an approximation for A, mA can be invariant with Δt , needed to describe propagation surfaces.

Comment 26. The construction of P to describe the propagation wave surface A does not mean it is implied the propagation surface is divided into photon paths P_i . The discussion of propagation starting from the concept of photon paths in qm is not the subject of this paper.

It is possible a distribution exists for indecomposable metric continua with a finite number of metric continua parts. For the propagation distribution the number of parts n is variable with Δt , while for the reference distribution n remains constant and the parts are developed with a variable t^* , where the reference parts Q_i represent the construction result for the limit of infinite t^* . This allows for any width factor v , $v \cdot mQ_i = mQ$. The propagation surface distribution is described within the time interval perspective. The surface parts P_i depend on, develop with, the relevant time interval Δt , are interpreted to be Δt -quantities, and P includes iteration of additions $uP_{i+1} = A [P_{i+1}, uP_i]$, and $P = uP_i$, for $i = n$, where n itself is variable with Δt . For any number n the parts P_i are assumed dense, disjoint and open in Q and $mP_i = mQ$. For all i and all n , the part P_i equals Q_i and the metric area $mP_i(v) = v \cdot mP_i$, by construction proportional with, ie multiplied with an average part width v , itself possibly depending on n and Δt . It follows for any n , $mP(v) = n \cdot v \cdot mQ_i = n \cdot mQ$.

During propagation development when the relevant event time interval Δt changes to $\Delta t'$, the regular sphere Q similarly changes to Q' with its surface area measure proportional to radius squared. The invariant regular metric cancels from the following.

$$34 \quad mQ'/mQ \sim (|c'(\Delta t')| \cdot |\Delta t'|)^2 / (|c(\Delta t)| \cdot |\Delta t|)^2$$

Surface measure mA can be approximated by mP including a metric m for P. The requirement of invariant mA , due to invariant radiation propagation surface energy E, means: $mA(\Delta t')/mA(\Delta t) = mP(\Delta t')/mP(\Delta t)$ remains invariant and equal to one. With v dependent on n and Δt and with for all $P_i(v)$ the same metric area measure proportional with v , change Δt to $\Delta t'$ means a change A to A', with n to n' and v to v' accordingly, implying equation 35. From mA'/mA invariance it follows the *metric requirement for maintaining E invariance*:

$$35 \quad 1 = mA'/mA = n' \cdot mQ'/n \cdot mQ$$

One can change description from Δt to $\Delta t'$ while n'/n changes proportionally with $|\Delta t'|/|\Delta t|$. For $n'/n > 1$, this means more 'close' to the star source, or in time interval perspective, more shortly, 'earlier' after emission when $|\Delta t'| < |\Delta t|$ from the perspective of a specific Δt , and for $n' = 1$ a one-part union P. For $n'/n < 1$, this means more 'remote' from the star source, more 'to the future', 'later' from the specific Δt perspective. In [5, Hollestelle] it is argued the propagation 'surface' metric requirement for A does not apply the linear metric, line element, measures, even when the metric surface measure of a regular sphere traditionally relates to the metric radius measure squared. The parts $P_i(v)$ being 'linear'-like, due to decreasing v , does not imply the ordinary line element. With 'folded, wrinkled' surface is meant a change of n , v , P and A from a situation with $n = 1$ and $mA = mQ$, due to Δt to $\Delta t'$, to $n' > 1$ and $mA' > mQ'$.

The other way around when Δt remains invariant, n remains invariant however un-decided. When n is independent it supports a degree of freedom. For any n , each of the n parts P_i is dense in Q and the measure for the complete union P remains $mP = n \cdot mQ = mA$.

One interpretation is, energy does not change with n , and a change of n implies a symmetry transformation. Any $n \geq 1$ is realistic for propagation surface A, approximated with P the union of the parts P_i , to remain a covering for the regular sphere Q. A second interpretation is, implied is a situation with zero temperature. A description for star source cloud radiation exists, including temperature for the source cloud and for the star sources themselves [5, Hollestelle]. One finds $NC = (mA)^2 = E^2$. A zero temperature means the average space

interval Δq_{bi} for star source i relative to their average $\langle \Delta q_{bi} \rangle$ is near zero which has implications for the symmetry for the star source collective in space.

A is assumed to be an indecomposable continuum. This means no partition includes parts with an interior, even when n equals 1, due to a result from topology [16, Hocking, Young]. This agrees with the concept of propagation surface A being the limit surface for radiation waves during Δt and part of the time interval only description without one moment time however with some structure from the P_i . Simultaneity defined from the complete propagation surface A needs an interpretation, it depends on ‘timely’ time intervals for radiation measurements, which is assured by Δt , the relevant event time interval, being ‘timely’. The term ‘timely’ is introduced for time intervals, say Δt_1 , in [4, Hollestelle] meaning: a time interval Δt_1 is measurable with one measurement with result $|\Delta t_1|$. Simultaneous measurements, in the time interval description, are possible within Δt , which agrees with Δt being the relevant event time interval. The concept of change, ie for star source radiation propagation, is linked to finite time intervals Δt and to measurements and H being time dependent. The propagation surface energy is integrated for the inner part of the surface, from the star source to the propagation surface, that is for all energy with simultaneous propagation, all energy ‘arriving’ at A during Δt , [5, Hollestelle].

Comment 27. Distribution theorem. A 1-dim. space-like radius does not allow a distribution from radius parts. A distribution for the radius or Δq implies a distribution for time interval Δt . The metric requirement is applicable for the 2-dim overall propagation surface A and a distribution for A is possible only because it does not imply a distribution for relevant event time interval Δt .

Star source wave propagation with zero or non-zero mass

When one defines vacuum with total energy H_0 value zero, and with kinetic radiation energy $E = \#n \cdot h \cdot \nu$, for photon number $\#n$ and photon energy $h \cdot \nu$, one can re-write $H = H_0 + \Delta H$, to agree with the specific energy quantity ΔH introduced in [4, Hollestelle]. Within the time interval description, a time dependent $H = H_0 + \Delta H = E + V + \Delta H$ is the ‘time interval’ version of the Legendre transform of Lagrangian $L = E - V$, with E kinetic energy and V potential energy and ΔH depending on the t -quantity $\#n$, possibly variable due to interaction. The discussion in this paragraph depends on earlier results, applying de Broglie complementarity and the time interval description for star source radiation propagation with relevant event time interval $\Delta t = [t_b, t_a]$, and there is for radiation energy E :

$$36 \quad E(t_b) = \#n \cdot h \cdot \nu = \#n \cdot M [h_+, \Delta t_i]$$

The function h_+ is defined with: $h_+ = 1/2 (\Delta^* p \cdot \Delta^* q + \Delta^* q \cdot \Delta^* p)$, applying the usual one moment time description indication \cdot and $+$ for one moment time t -quantities and the indication Δ^* for variances, different from indication Δ for intervals. Even so, one moment time t_b and t_a occur. Energy E , without interaction and wave function collapse during Δt , is an invariant and neutral Δt -quantity.

The following properties are valid within the time interval description. Δt -quantities can be written linear with Δt due to MVT equilibrium. Δt -quantities are parameter t independent quantities, however can change with a change of Δt . The time interval description, including MVT equilibrium, for t -quantities depends on linearity in t . A t -quantity due to multiplication quadratic in t reduces to a neutral quantity, ie a $t^{(2)}$ quantity equals a $t^{(0)}$ Δt -quantity, comment 4. In contrast, a t -quantity, ie linear in t , remains t -quantity due to addition, with linearity implying addition, and remains possibly variable during Δt . A $t^{(-1)}$ quantity equals a $t^{(+1)}$ t -quantity, comment 4.

Assume a non-zero mass m_1 for material wave particles complementary to radiation waves that emerge from a star source and introduce a certain mass m_2 . Then it follows E includes multiplication $M [m_1, m_2]$ and is inverse linear with $|\Delta q| = |c(\Delta t)| \cdot |\Delta t|$ with $c(\Delta t) = M [\Delta q, \Delta t_i]$, the wave group-velocity, [5, Hollestelle]. This can be interpreted due to its similarity with Newtonian gravitational energy for situations without external interaction. In this energy view propagation wave energy, ie kinetic energy E , equals gravitational energy E due to the above dependences. The meaning of dispersion free propagation energy is unrestrained wave kinetic ‘movement’, the meaning of gravitation energy is field energy for the propagation surface depending on the star source assuming masses m_1 and m_2 . From the definition for $c(\Delta t)$, paragraph 11, one finds with $\Delta q_i = M [\Delta t_i, c(\Delta t)_i] = c(\Delta t)_i$,

$$38 \quad E = \#n \cdot A [1/|c(\Delta t)|, M [m_1, m_2]], \text{ equal to } \#n \cdot A [1/|c(\Delta t)|, m_2] \text{ for } m_1 \text{ ‘multiplication unit’}$$

$$39 \quad E = \#n. M [1/\Delta q], M [m1, m2] = \#n. D^*|\Delta q [M [m1, m2]] = \#n. D^*|\Delta t [M [1/c(\Delta t)], M [m1, m2]]$$

With equation 38 and 39 one finds a definition for non-zero mass $m2$, that can be identified with the source mass. D^* or $D^*|\Delta t$ of an invariant does not have to be zero. With $1/c(\Delta t) = D^*|\Delta q [m1] = M [m1, \Delta qi]$, there is $c(\Delta t)$ equals the multiplication inverse for the propagation wave particle mass density. There is for photons $E = \#n. A [\Delta qi, m2]$, equation 38, for $m1$ the ‘multiplication unit’ mass quantity, and E can be interpreted to be the source mass apparent density. This suggests wave particle energy $h. v$ is dependent only on multiplication $M [m1, m2] = m2$, the source mass.

For $c(\Delta t) = M [\Delta q, \Delta ti]$ invariant, where Δq is not part of the time interval set and $c(\Delta t)$ is not part of the time interval set, $c(\Delta t)$ is a Δt -quantity linear in Δt including a non-scalar multiplication, ie Δq . The complete ‘volume like’ propagation surface A from star source to propagation surface, paragraph 11, depends on Δt and $c(\Delta t)$. A variable non zero mass $m1$ or an ‘addition zero’ mass $m1$ can both be applied due to $m1$ and $m2$ not being scalars rather Δt -quantities where an ‘multiplication unit’ mass $m1$ corresponds with $\Delta t0 = \Delta U$, with both $A [m1, m2] = m2$ and $M [m1, m2] = m2$ while E does not reduce to zero. Then, zero mass wave particles, ie photons, relate to mass $m1$ being for mass quantities both ‘addition zero’ and the ‘multiplication unit’, different from a ‘multiplication zero’ mass in the sense of corresponding to scalar ‘multiplication zero’ or to $\Delta U0$, and for any mass $m2$ there is:

$$40 \quad A [m1, m2] = A [m2, m1] = m2 \\ M [m1, m2] = M [m2, m1] = m2$$

Including $\#n$ the number of photons, non-interacting radiation propagation surface energy $E = E(tb) = \#n. h. v$ is proportional with $\#n$, however with $m = \sum_i m_i = m1$, $i = 1$ to $\#n$, the overall photon mass m remains invariant and equal to “addition zero” $m1$ for any $\#n$.

A change of radiation energy E during Δt for invariant v due to external interaction and measurements including wave packet reduction like the photo-electric effect, depends non-linear on the variable $\#n$ where $\#n(tb) = \#nb$ from before interaction: $\#n. h. v = + 3/2 (2 \#nb - 3)^{-1} (\Delta H(ta) - \Delta H(tb)) = + 3/2 (2 \#nb - 3)^{-1} M [D^* [\Delta H], \Delta t]$, [4, Hollestelle]. This is different from the situation where v is variable, due to internal interaction with a variable frequency v and variable wave particle mass and number, however without external interaction and where $E = \#n. h. v$ remains invariant, [5, Hollestelle].

Since wave energy $h. v$ remains positive it follows a change of sign for the difference $\Delta H(ta) - \Delta H(tb)$ is possible from a variable $\#n$, $\#n$ itself remaining a positive t -quantity, and when $\#nb$ is 1 or 2 there is $\Delta H(ta) - \Delta H(tb)$ is negative or positive. For a situation within the infinite limit of increasing $\#n$, considering $\Delta H(t) = H(t) - E(tb)$ and with $\Delta H(tb)$ equal to zero, $H(t)$, a t -quantity, equals $\Delta H(t)$ apart from the constant $E(tb)$, and the difference $\Delta H(ta) - \Delta H(tb) = H(ta)$, a t -quantity, is proportional with $\#n. E$.

Paragraph 12. Discussion

The time interval only description provides an alternative to the one moment time description and to vector calculus to describe differences and change. A time interval only description and solution for the ‘infinite regress’ problem, is presented in paragraph 2 to 5. Some results that depend on time intervals only were already obtained in [4, Hollestelle] and [5, Hollestelle]: one result is gravitational energy and radiation propagation energy for both zero and nonzero mass wave particles can be described integrated in one approach.

The description in this paper depends on time interval commutation quantities. Due to the considered set dimension, these quantities form multiples, for the time interval set, multiples are pairs of quantities. Where the number of degrees of freedom increases due to these multiples, equilibrium induces constraints and reduction of degrees of freedom. The time interval equilibrium requirement means invariance, for the multiplication of all quantities within a multiple, in this way introducing Noether charges, and in the same way, structure constants, for the relevant set. In this paper the relevant set is the time interval set, a one-dimensional set.

Radiation propagation in the usual qm field theory setting applies fields that extend to all infinite spacetime. This is due to radiation interpreted being parallel and stationary. Infinite spacetime is not very suitable for the definition of measurement events. When a star source is assumed, radiation propagation includes sphere surface limits unlike the parallel situation. Measurement and interaction, with H time dependent, allows for the introduction of a finite relevant event time interval. In the following discussed are some of the results.

1. The equivalence principle and simultaneous descriptions

Assuming a situation without interaction, radiation energy E , interpreted in paragraph 11 and 12 with kinetic radiation energy, is an invariant. Radiation propagation implies dispersion free noninteracting radiation complementary with free moving wave particles, ie de Broglie complementarity. According to general relativity and the equivalence principle, this kinematic energy is 'equal' to gravitational energy when considering a kinetic energy space time coordinate system accelerated in the gravitational energy space time coordinate system and it is not possible to decide from measurement [6, Goldstein], [18, Einstein]. Like reciprocal quantities, paragraph 3, defined is a simultaneous pair of energies, E_s , kinetic, and E_e , gravitational. To follow the arguments for reciprocal quantities the energy pair is related to each other by a reference transformation. The energies E_s and E_e have the same invariant value each from their own reference, not meaning the respective space time coordinate system, rather like complementarity of wave and wave particle within qm. A relation for E_s and E_e references differs from the usual Newtonian one equating kinematics and dynamics, since radiation and gravitation are considered from energy, not action. Due to MVT equilibrium, $I^*|\Delta t$ and $D^*|\Delta t$ being 'equal' is assured, since the results for both in terms of time intervals are the same, equation 10.

From equation 38 and 39, $E_s = \#n. A [1/c(\Delta t)]$, $A [m_1, m_2]$, and $E_e = \#n. M [1/c(\Delta t)]$, $M [m_1, m_2]$. The E_s reference is 'addition' or differentiation, the E_e reference is 'multiplication' or integration. Both results agree with Newtons second law from differentiation of equations 38 and 39, E_s due to differentiation to Δt and coordinate Δq being positive and decreasing and E_e due to differentiation to Δq , [4, Hollestelle], [6, Goldstein] and [7, Newton I]. In particular differentiation to resp. Δt or Δq implies opposite sign. The mass quantities for both equations 38 and 39 being regarded identical is the basis for the interpretation of matter properties and of the equivalence principle [1, Arnold], [17, Newton III], [18, Einstein]. Now it is tried to apply the arguments of the description of reciprocal quantities, paragraph 3.

Following the discussion below equation 8 for a pair of Noether charges, introduced is a simultaneous reference transformation TR, where pair-part for E_e is the transform of pair-part for E_s and vice versa. Still, energies E_s and E_e are Δt -quantities that allow application of A , M and $D^*|\Delta t$ while remaining within the same pair-part.

The proposed pair transformation TR implies for the energies: $A [E_s, \Delta t]$ to $M [E_e, \Delta t]$, from addition to multiplication and reverse, where evidently the value for E_s and E_e remains the same. Both pair-parts depend on energy invariance, valid for propagation surface energy due to noninteraction and for gravitation energy due to the time interval property $D^*|\Delta t [E_e] = E_e$.

With $D^*|\Delta t [E_s] = E_s$ and $D^*|\Delta t [\Delta t] = \Delta t$, since $\Delta t = \Delta U$ there is $E_s = M [\Delta t, E_s] = M [E_s, \Delta t]$ and it follows $E_s = D^*|\Delta t [M [E_s, \Delta t]] = A [c_1. M [E_s, \Delta t], c_2. M [E_s, \Delta t]] = A [c_1. E_s, c_2. E_s]$. A reference transformation results in $E_e = M [c_1. E_e, c_2. E_e]$. This also means: $D^*|\Delta t [E_s] = A [c_1. E_s, c_2. E_s]$ and $D^*|\Delta t [E_e] = M [c_1. E_e, c_2. E_e]$.

The pair E_s and E_e relate due to the specific reference transformation, just like a multiple of reciprocal quantities that relate due to a certain equilibrium requirement that can be described with a transformation, say $cn(\Delta t)$ to $cn'(\Delta t)$ and a specific TL, such that $cn(\Delta t') = cn'(\Delta t)$.

Comment 23. The time interval densities for $cn(\Delta t)$ and $cn'(\Delta t)$ decide on the second associativity factors c_1 and c_2 for differentiation $D^*|\Delta t [M]$ of a multiplication $M = [A_1, A_2]$, like for instance $M = E_e$ proportional with $M [m_1, m_2]$, due to $D^*|\Delta t [cn(\Delta t)] = A [c_1. cn(\Delta t), c_2. cn(\Delta t')]$ for Δt -quantity $cn(\Delta t)$, one pair-part of the multiple $cn(\Delta t)$ and $cn'(\Delta t)$.

The m_1 and m_2 represent the Δt and $\Delta t'$ sources and star source for E_e respectively. The differentiation constants c_1 and c_2 are recognized to relate to multiple Δt -quantities like Δt -Noether charges due to their multiplication being invariant, in this case equal to ΔN , and themselves are properties changing with Δt , ie propagation and time development.

It is inferred that differentiation as an expression for geometry is directly related to source interaction like gravitational energy. Since differentiation is part of the usual wave equation description of qm, the inference means geometry and gravitation can be integrated with qm through the second associative factors for differentiation. Indeed, one way to derive qm wave functions is from Noether charges.

This inference is a time interval only result. It is expected that similarly differentiation can be generalized to space-time intervals and second associativity property factors related to Noether charges that describe properties changing with space-time intervals. However, there is a difference for time intervals and space intervals, already encountered in [4, Hollestelle] and [5, Hollestelle]: time intervals and time do not allow to 'turn backwards' where 3-dimensional space intervals do, and, the propagation sphere surface requirement of invariant metric

area only emerges from Δt , not from Δq . This difference is also discussed when introducing structure constants, paragraph 10, and the distribution theorem, paragraph 11, comment 27.

II. Radiation energy and wave particle number

The time interval description of time development applies MVT equilibrium instead of Lagrangian equilibrium. A multiplication of with q and p corresponding quantities, within the context of the time interval description of star source radiation, recovers the uncertainty relation $h^+ = 1/2 (\Delta^* p \cdot \Delta^* q + \Delta^* q \cdot \Delta^* p)$, paragraph 12, reducing to Planck's constant h for H time independent [5, Hollestelle].

Resuming the de Broglie complementary wave energy $h \cdot \nu$, one finds: $M [h^+, \Delta t_i] = M [M [\Delta p, \Delta q], \Delta t_i] = M [h, \nu]$ and without interaction the invariant radiation energy equals $E_s = \#n \cdot h \cdot \nu$ for $\#n$ wave particles or photons. It is argued Δ^* variances are equal to Δ variations except for a minus sign, and when including interaction, the variable t -quantity is $\#n$ even while the relevant event time interval $\Delta t = [t_b, t_a]$ starts or ends with a noninteracting situation. Then, radiation energy $E_s(t_a) = \#n \cdot E_s(t_b)$ for $E_s(t_b) = \#n \cdot h \cdot \nu$ depends nonlinear on $\#n$ however linear on frequency ν , paragraph 11. *This relation can provide independent confirmation from measurements for the de Broglie Einstein relation for the interdependence of kinetic energy and frequency, $E = h \cdot \nu$ for wave particle and wave respectively.*

III. The 'set rule' and Stokes theorem. The meaning of 'close' and 'remote'

Comment 24. Scalar multiplication $M [\Delta t_1, \Delta t_2] = \Delta t_3 = a$. Δt_a can be evaluated by applying transformation T_1 and 'scale' transformation T_2 , termed B and C in [4, Hollestelle] and discussed below equation 8. T_1 includes the specific transforms $T_1 [t_b] = t_b$ and $|T_1 [t_0]| \ll |t_0|$ and $T_1 [t_a] = -1 \cdot t_b$, and T_2 is a scale transformation transforming $\Delta t = [t_b, t_a]$ to $T_2 [\Delta t] = [C \cdot t_b, C \cdot t_a]$ with C a scalar and boundary parameters t to $T_2 [t] = C \cdot t = t_0 \cdot (T_2 [t_0])^{(-1)}$, t , for a certain given $T_2 [t_0]$, except for $t = t_0$, since t_0 is not a possible boundary parameter. T_1 and T_2 transform time intervals to time intervals while remaining within the time interval set. Δt_b within commutator $[1/t_b, A] \|\Delta t_b$, is the result of repeated application of T_1 and T_2 to $\Delta t = [t_b, t_a]$.

From equation 23 for 'set rule' remain $\text{Rest}(a)|t$, the third part is derived independently and confirms the second part: derivative $D^* [a] = D^* \|\Delta t [a]$ equals the ordinary commutator $[1/t_b, a]u$. This indicates that situations 'remote' from and 'close' to with respect to situations with H time independent are also more similar than the terms suggest. The situations 'remote' from and 'close' to Newtonian, with Newtonian meaning: with H time independent, can be related by interpretation from MVT equilibrium of the commutators $[1/t, a] \|\Delta t$ and $[1/t, a]u$ for any quantity a and any Δt .

A change of the relevant time interval Δt from $\Delta t'$ to $\Delta t''$ corresponds with a change of the interval boundaries. The time interval derivative to one moment time parameter t_b then can be defined to be equal to $[1/t_b, a] \|\Delta t_b$ with $\Delta t_b = [t_b', t_b'']$ and similarly for t_a with $\Delta t_a = [t_a', t_a'']$. Recall that these Δt_b , and Δt_a , are for all t_b' and t_b'' well defined time intervals, due to T_1 and T_2 transforming proper time intervals to proper time intervals. One derives $[1/t_b, a] \|\Delta t_b$ is equal to $[1/t, a] \|\Delta t$, and thus corresponds with $D^* \|\Delta t [a]$. For this to confirm Stoke's theorem, in a time interval set version, it should be $D^* \|\Delta t [a] = a$. This is valid for a equal to a multiplication like ΔN , paragraph 10.

The derivatives $D^* [a]$ and $D^* \|\Delta t [a]$ being equal is discussed below 'set rule' equation 23. One also derives $[1/t_b, a] \|\Delta t_b \sim 1/2 [1/t_b, a]u$ with both correspondences only valid for 'close' to Newtonian, H time independent, situations. It follows $D^* \|\Delta t [a] \sim 1/2 [1/t_b, a]u$, which both are proper quantities, for 'close' to Newtonian situations meaning Δt is 'close' to, ie asymptotically, symmetrical, while the independent confirmation, comment 19, of this correspondence for both expressions from equation 23 is derived for all situations, H time dependent or time independent. This implies some questions concerning what means 'close' to time independent for H and what means 'close' to symmetrical for Δt . In the description of star source radiation 'close' to or 'remote' from symmetrical for Δt means 'remote' from or 'close' to for the propagation surface from the star source [5, Hollestelle]. The above relativity of the concept of measure and asymmetry in terms of the relevant time interval Δt can be an introduction to space time itself and to the resolution of the problem of action at a distance.

IV. Scalar multiplications and scale transformations. Linear subsets within the time interval set. Closure theorem

From $M[\Delta t_1, \Delta t_2] = \Delta t_3$, per definition a time interval, and since any time interval is neutral in Δt , there is assumed Δt_3 is linear in Δt : $\Delta t_3 = e \cdot \Delta t$ for some scalar e . Transformation T_2 with scale factor $C = a$ provides meaning for scalar multiplication with a time interval result: $a \cdot \Delta t_a = \Delta t_3 = M[\Delta t_1, \Delta t_2]$ with $a \cdot \Delta t_a = T_2[\Delta t_a]$ remains a proper, ie well defined, time interval. Now write Δt_a itself linear in Δt . With $ca = \Delta U$ and $a = 1$ multiplication $\Delta t_3' = M[\Delta U, \Delta t_a] = e' \cdot \Delta t = \Delta t_a$. All $a \cdot \Delta t_a$ can be expressed with time intervals $e \cdot \Delta t$ linear in Δt . By choosing $\Delta t_a = \Delta t$ one finds specific ca and a where $M[\Delta t_1, \Delta t_2] = e \cdot \Delta t = ca$.

Then $a \cdot \Delta t_a = M[ca, \Delta t_a]$ and $a \cdot \Delta t_a = a \cdot M[\Delta U, \Delta t_a] = M[a \cdot \Delta U, \Delta t_a]$. This means multiplication with 'multiplication unit' ΔU or with ca both leave the linear subset for Δt_a invariant. Indeed, it follows $M[\Delta t_1, \Delta t_2]$ is linear in Δt_a . The linear subset for Δt_a is the part of the time interval set defined from, time interval or scalar, multiplication with Δt_a . It follows $M[\Delta t_1, \Delta t_2] = a \cdot e' \cdot \Delta t = e \cdot \Delta t$ with scalar $e = a \cdot e'$ and part of the linear subset for Δt . When Δt_2 equals Δt there is $M[\Delta t_1, \Delta t] = \Delta t_1$ and this confirms the assumption: any time interval Δt_1 from the time interval set can be written like a multiplication including Δt , and is linear in Δt , one of the properties for M , paragraph 8. This confirms in terms of linear subsets the time interval set closure theorem, comment 12.

V. Linear and non-linear events to part I

Introduced are two different quantities within the time interval description, t -quantities and Δt -quantities. It is possible to relate non-linear events, including Δt -quantities like Noether charges that depend on multiplication properties, with linear events, with t -quantities like one moment time t coordinates that depend on addition properties, by applying transformations 'working to the right' like multiplication with t or $1/t$. This means, non-linear events are related to linear events within the time interval description by applying one of these transformations.

VI. Structure constants

Usually, structure constants are defined from the multiplication properties for the set or group elements writing these like exponentials of generator set elements. From assuming the canonical property for the set elements, it follows the commutator for two generator elements can be assumed to be linear within the generator set, ie gives another generator set element for result, including a scalar multiplication. These scalars are termed structure constants. Taking care of the multiplication Taylor series depends on writing out all higher order ordinary commutators of generators that should reduce to first order ordinary commutators which provides a linear result, depending on the structure constants for the set. For the time interval set, multiplication includes a linear result with $M[\Delta t_1, \Delta t_2] = \Delta t_3 = a \cdot \Delta t_a$ providing the linearity constants and in this case this is enough to find the structure constants, without applying the generator set.

For any set, structure constants themselves are independent of the set representation. Indeed, for the time interval set the structure constants are independent of the number of necessary and different Δt_a , ie whether the subset of Δt_a is reducible or not. Similarly, the Δt_a subset is not completely determined by structure constant properties. There being only one independent subset for the time interval set, confirms the time interval set being 1-dimensional, and the existence of only one independent structure constant. A structure constant is to relate to the essential properties or quantities of the set, in this case these are the group velocity $c(\Delta t)$ and the differentiation arguments c_1 and c_2 , that together introduce the specific relations for space-time and time development, like propagation.

There is $c(\Delta t) = M[\Delta q, \Delta t_i] = D^* \Delta t[\Delta q] = [\Delta t, \Delta q] \Delta t$ is equal to ordinary commutator $[\Delta t, \Delta q]u$ which is independent of t within Δt . For Δq a neutral Δt -quantity, $M[c(\Delta t), \Delta t] = c(\Delta t) = M[M[c(\Delta t), \Delta t_i], \Delta t]$ provides the linearity constant for Δt , and ordinary commutator $[\Delta t, \Delta q]u = M[M[c(\Delta t), \Delta t_i], \Delta t]$ with $M[c(\Delta t), \Delta t_i] = c(\Delta t)$ a structure constant for the time interval set. However, $c(\Delta t)$ does not have scalar dimension.

When measuring $c(\Delta t)$ the time interval Δt is the relevant event time interval, which also is the relevant event time interval for measuring Δq . This can be resolved with the quantity h_+ . One of the assumptions for non-interaction star source wave propagation [4, Hollestelle] is $|\Delta^*q| = |\Delta q|$ and $|\Delta^*p| = |\Delta p|$ and this implies, h_+ is a Δt -quantity linear in $|\Delta q|$, $|\Delta p|$, and due to being equal to $M[E, \Delta t]$, linear in Δt , paragraph 12. $E = M[h_+, \Delta t_i]$ for E invariant during propagation and without interaction. Both these relations are equal to ordinary commutator relations and both h_+ and Δq are linear in Δt since they are Δt -quantities. It follows $E = M[h_+, \Delta t_i] = [\Delta t, h_+]u$ and this ordinary commutator equals $E = [E, \Delta t] = M[M[E, \Delta t_i], \Delta t]$ with $M[E, \Delta t_i] = E$ a structure constant for the time interval set like $c(\Delta t)$. E and h_+ are only introduced to find that from quantities with

different dimensions the structure constant for a set remains the same, the number of structure constants depending just on the dimension of the set, and the time interval set is 1-dimensional.

Interaction can be included with $h_+ = h_+(\Delta t)$ or with $\Delta q = \Delta q(\Delta t)$ and time interval differentiation to Δt without applying parameter t . The wave velocity $c(\Delta t)$ and energy E , Δt -quantities, can be interpreted to be structure constants disregarding dimension. Now introduce the following invariant velocity and energy quantities c_{-}/c_{+} and h_{-}/h_{+} that render dimensionless multiplication results with $c(\Delta t)$ and E respectively. *It follows* $sc = M [c(\Delta t), (c_{-}/c_{+})_i]$ and $se = M [E, (h_{-}/h_{+})_i]$ are structure constants for the time interval set.

The time interval set is different from the usual groups considered in relation with structure constants, ie within elementary particle field theory. One difference is the canonical property usually assumed for these groups to derive the multiplication results for the generators, which however is not valid for the time interval set, due to the existence of the non-zero remain $\text{Rest}(a)$. The structure constants being provided by the addition quantities ΔN , it is inferred that there is only one pair of $cn(\Delta t)$, $cn'(\Delta t)$, since equation 1 includes indeed only one corresponding pair cn and cn' , and there is only one independent structure constant for the time interval set itself, with $cn(\Delta t)$ and $cn'(\Delta t)$ Δt -quantities. This means $sc = se$, they are same structure constant. Similar arguments are valid for interval quantities with a dimension different from the time interval set, for instance Δq -quantities and the space interval set. Due to equation 23 the remain $\text{Rest}(a)$ is included in Noether charge NC , that can be derived by assuming both the overall time interval and space interval averages for $cn(\Delta t)$ are the same, [ref. Arnold] and equal to ΔN . One finds the time interval derivative to Δt of the complete space integral ΔN to be $NC = A [M [c(\Delta t), \Delta N], c(\Delta t)]$, depending on the first associative property for multiplication and applying similarity for M and A , paragraph 10. This is equal to the time interval derivative of overall time interval average $\Delta N = M [cn(\Delta t), cn(\Delta t')]$, ie equal to $D^*||\Delta t [M [cn(\Delta t), cn(\Delta t')]] = A [a. D^*||\Delta t [\Delta ta], \text{Rest}(a)|\Delta t] = NC$, for $a = c(\Delta t)$ and $\Delta ta = \Delta N$ and $\text{Rest}(a)|\Delta t = c(\Delta t)$. From paragraph 10, $D^*||\Delta t [\Delta N] = \Delta N$ meaning $\Delta N = NC$ and $NC = M [c(\Delta t), NC]$ with solutions $c(\Delta t) = NC$ or $c(\Delta t)$ equals 'multiplication unit' ΔU .

Comment 25. the non-linear property of the time interval set is included in Noether charge NC , with $NC = sc. M [c_{-}/c_{+}, \Delta Ni]$ where sc the structure constant for the time interval set depending on $c(\Delta t)$, and with $NC = M [\text{Rest}(c(\Delta t)), \Delta N] = M [c(\Delta t), \Delta N]$.

Comment 26. considered only are time intervals with domain measure non zero. Not considered are time intervals with the domain measure zero.

VII. Equation-sign and the addition commutator

The equation $A [\Delta U_0, \Delta t_0] = \Delta t_0$ from $\Delta U_0 = (1 - 1) \cdot \Delta t_0 = A [\Delta t_0, -1 \cdot \Delta t_0]$ seems contradictory, unless one considers comment 20 for ΔU_0 , and indicates there is an extra freedom when quantities can be subject to 'moving to the other side of the equation-sign' by multiplication with scalar -1 . It means introducing an addition commutator. When commutation quantities are added the related multiple changes to include these quantities and similar for the Noether charge and structure constants, and the number of degrees of freedom. The equation-sign side seems to matter and have different value, depending on the order of the involved time intervals. This is subject for another paper, however the time interval set seems to be a good start to investigate this. A similar freedom seems to reside with Newton's laws and equilibrium definitions.

The confusion exists when one applies: $-1 \cdot \Delta t_0$ to mean Δt_0iv within the above addition A , and interpreting addition to mean a difference due to scalar -1 , and can be avoided by 'moving to the other side of the equation-sign'. Differences are part of the one moment time description since this description depends on vectors. However, in the time interval only description the addition inverse is defined from $A [\Delta t_2, \Delta t_2iv] = \Delta t_0$ for any time interval Δt_2 .

References

- [1] Arnold, V. I. (1974): *Mathematical Methods of Classical Mechanics*, Russian/English translation, Vogtmann K. and A. Weinstein, Springer. 1989
- [2] Beller, M. (1999): *Quantum Dialogue*, Univ. Chicago Press
- [3] Roos, W. J. (1978): *Grondslagen van de quantummechanica (Foundations of quantum mechanics)*, graduate course, Univ. of Utrecht
- [4] Hollestelle, H. (2020): *Time, Equilibrium, and General Relativity*, Int. J. of Science and Tech.
- [5] Hollestelle, H. (2021): *Some Results for a Time Interval Approach to Field Theory and Gravitation*, Int. J. of Science and Tech.

- [6] Goldstein, H. (1950): *Classical Mechanics*, Addison-Wesley
- [7] Newton, I. (1686) (I): *The Mathematical Principles of Natural Philosophy*, Latin/English translation A. Motte, Benjamin Motte Editors, 1729, Book I, pg 19, Axioms or laws of motion, Law II
- [8] Hollestelle, H. (2017): *Thermodynamics of a Set of Closed Curved Surfaces and Noether's Theorem*, *Int. Jnl. of Science and Tech.*, 5, 1
- [9] Curie, P. (1894): *Sur la symétrie dans les phénomènes physiques, symétrie d'un champ électrique et d'un champ magnétique*. *J. Phys. Theor. Appl.*, 3 (1), published Hal archives-ouvertes
- [10] Newton, I. (1686) (II): *The Mathematical Principles of Natural Philosophy*, Book II, pg 251 (open access not available), Theorem II case I, includes the principles of differentiation by infinitesimal limit
- [11] Noether, E. (1918): *Invariant Variation Problems*, German/English translation, Tavel, M. A., *Transport Theory and Statistical Physics*, 1971
- [12] De Wit, B. and J. Smith (1986): *Field Theory in Particle Physics*, Vol. I, North Holland Physics/Elsevier
- [13] Sakurai, J. J. (1978): *Advanced Quantum Mechanics*, Addison-Wesley
- [14] Jacobson, N. (1974): *Basic Algebra I*, Freeman
- [15] Veltman, M. (1974): *Representations of Groups and Applications to Elementary Particle Physics*, graduate course, Univ. of Utrecht
- [16] Hocking, G., S. Young (1961): *Topology*, Addison-Wesley
- [17] Newton, I. (1686) (III): *The Mathematical Principles of Natural Philosophy*, Latin/English translation A. Motte, Benjamin Motte Editors, 1729, Definition I, includes the definition of the quantity of matter, related to acceleration, proportional to mass known by its weight.
- [18] Einstein, A. (1923): *The Foundation of the General Theory of Relativity*, in Lorentz, H., Einstein, A., Minkowski, H. and H. Weyl: *The Principle of Relativity*, (reprint of the original publication, Methuen and Company, translated by Perrett, W. and G. Jeffery, Dover Publishers, 1952)