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Article

Nonsingular Phantom Cosmology in Five Dimensional $f(R,T)$ Gravity

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Abstract: We obtain exact solutions to the field equations for 5 dimensional LRS Bianchi type-I spacetime in $f(R,T)$ theory of gravity where specifically the following three cases are considered: (i) $f(R,T) = \mu(R+T)$, (ii) $f(R,T) = R\mu + RT\mu^2$ and (iii) $f(R,T) = R + \mu R^2 + \mu T$ where R and T respectively the Ricci scalar and trace of the energy-momentum tensor. It is found that the equation of state (EOS) parameter w is governed by the parameter μ involved in the $f(R,T)$ expressions. We fine-tune the parameter μ to obtain effect of phantom energy in the model, however we also restrict this parameter to obtain a stable model of the universe. It is noted that the model isotropizes at finite cosmic time.

Keywords: Phantom energy; LRS Bianchi type-I; $f(R,T)$ theory; 5d spacetimes

1. Introduction

Researchers have found it impossible to prevent the addition of dimensions and the unification of forces in nature. It is notable that time can be seen as a fourth component of special relativity, according to Minkowski []. In a similar way, Maxwell combined the theories of electricity and magnetism []. The next step in this regard was to combine electromagnetism with general relativity (GR). Over the year many researchers have been trying to construct unified field theories which geometrize all the fundamental forces of nature. The geometrization of gravity by the general theory of relativity (GR) motivated [1,2] to propose a five-dimensional theory that can unify gravitation and electromagnetism. The gravity and electromagnetism are coupled via an additional dimension in this five-dimensional general relativity model.

In their interesting work Chodos and Detweiler [3] showed the evolution of a 5d vacuum universe into a cogent 4-dimensional one. Alvarez and Gavela [4] advocated for a cosmological scenario that produces ample entropy in the universe because of the dynamical compactification of the higher dimensions. Further, they pointed out the possibility of solving flatness and horizon problems in this scenario.

Under the Kaluza-Klein (KK) theory, Marciano [5] investigated time variation of the fundamental constants. He derived the relationships between the low-energy couplings as well as masses and propounded that a time variation in any of these parameters can render proof for higher dimensions. Furthermore, he reviewed experimental bounds and urged for new measurements. Gegenberg and Das [6] constructed 5d cosmological models with a real massless non-self interacting scalar field source. They pointed out that non-trivial solutions to the field equations occur only when the homogeneous and isotropic 3-space has non-positive constant curvature. However, Lorenz-Petzold [7] obtained exact solutions to the higher-dimensional field equations in a vacuum as well as perfect fluid case along with a non-vanishing cosmological constant.

Wesson [8] considered the higher-dimensional spacetimes with a new challenge by pointing out that "the space part of its metric varies with time in the same way as the de Sitter solution of the conventional four-dimensional theory" and formulated his so-called five-dimensional gravitational theory. Under this theory Grøn [9] successfully obtained vacuum, radiation and matter-dominated cosmological models. These models describe an inflationary universe in the variable rest-mass theory as was proposed by Wesson.

Several authors [10–14] have been discussed KK extension of the FRW cosmological models. In higher dimensions, anisotropic generalizations of these models are available in literature [15–17] whereas inhomogeneous cosmologies in $5d$ have been studied by other authors [18–22]. A few exact solutions to the Einstein field equations in KK spacetime obtained by various authors and showed that those reproduce as well as extend the known solutions of the 4-dimensions [23–27]. The exact solution to the Einstein field equations which is Ricci and Riemannian flat in $5d$ was obtained by Liko and Wesson [28]. Interestingly, this solution in 4d represents a cosmological model for the early vacuum-dominated universe. Some noteworthy works where variable G and Λ have been studied [29,30] have immense consequences in KK cosmology and higher dimensional geometry, e.g. Pahwa [31] constructed a homogeneous, anisotropic $4+d$ cosmological model and hence studied the late-time acceleration of the universe.

Higher dimensional cosmology in various alternative theories of gravity can also be found in the literature which originated due to a few drawbacks of Einstein's general relativity (especially GR has been failed to explain the late time cosmic acceleration phenomena) and hence to comply with the observational evidences. Therefore, one possible technique to justify the observational data [32–38] is the modification of GR. Harko et al. [39] obtained the gravitational field equations in the metric formalism and the equations of motion for the test particles. This theory is called as $f(R,T)$ theories of gravity where the Lagrangian is an arbitrary function of R and T being the Ricci scalar and trace of the energy-momentum respectively. Under the $f(R,T)$ gravity various authors have studied different mathematical aspects as well as physical applications of the theory [40–65].

We design the present article as follows: we provide the basic equations as well as the Einstein field equations under the cosmological system in Sec. 2. In Sec. 3 we have the exact solutions sets under three specific cases, whereas in Sec. 4 the behavior of the models are presented and analyzed. The results and their discussion are presented in Sec. 5 to provide some concluding remarks along with salient features.

2. Einstein's Field Equations

In their theory Harko et al. [39] considered three explicit functional form of $f(R,T)$ as follows:

$$f(R,T)=\begin{cases} R+2f(T), \\ f_1(R)+f_2(T), \\ f_1(R)+f_2(R)f_3(T). \end{cases} \tag{1}$$

One can therefore obtain several theoretical models for each choice of $f(R,T)$. However, in the present work we consider the second and third case, i.e., $f(R,T)=f_1(R)+f_2(T)$ and $f(R,T)=f_1(R)+f_2(R)f_3(T)$ for constructing cosmological models through the $5d$ metric in the form

$$ds^2=dt^2-A(t)^2dx^2-B(t)^2(dy^2+dz^2)-F(t)^2dn^2, \tag{2}$$

where A, B and F are functions of the time coordinate only.

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Now, the gravitational field equations can be provided as [39]

$$f_R(R,T)R_{ij} - \frac{1}{2}g_{ij}f(R,T) + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = \\ 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \tag{3}$$

where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$ and $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ are partial derivative with respect to R and T, respectively, $\Box = \nabla_i\nabla^i$, ∇_i denotes covariant derivates and $\Theta_{ij} = -2T_{ij} - g_{ij}P$.

The field equations now have the following form:

$$f_R(R,T)R_{ij} - \frac{1}{2}g_{ij}f(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}. \tag{4}$$

Here, we consider the source of gravitation as the perfect fluid. Therefore, the energy-momentum tensor is taken as

$$T_{ij} = (P+\rho)u_iu_j - Pg_{ij}, \tag{5}$$

together with the comoving coordinates

$$g_{ij}u^iu^j = 1. \tag{6}$$

In the above equations P, ρ and u_i are the isotropic pressure, energy density and five-velocity vector of the cosmic fluid distribution respectively.

3. Solutions to the Field Equations

3.1. $f(R,T) = \mu R + \mu T$

Let us consider here the second case, i.e. $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \mu R$ and $f_2(T) = \mu T$ whereas μ is an arbitrary constant. Now Eq. (4) becomes

$$G_{ij} = \left[\frac{8\pi+\mu}{\mu}\right]T_{ij} + \left[P + \frac{T}{2}\right]g_{ij}. \tag{7}$$

Here $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor. For the line element (2), the explicit form of the field equations (7) using (5) and(6) can be obtained as

$$-\frac{2\mu A_4B_4}{AB} - \frac{\mu A_4F_4}{AF} - \frac{2\mu B_4F_4}{BF} - \frac{\mu B_4^2}{B^2} + \mu P - \frac{3}{2}\mu\rho - 8\pi\rho = 0, \tag{8}$$

$$\frac{2\mu B_{44}}{B} + \frac{2\mu B_4F_4}{BF} + \frac{\mu B_4^2}{B^2} + \frac{\mu F_{44}}{F} - 2\mu P - 8\pi P + \frac{1}{2}\mu\rho = 0, \tag{9}$$

$$\frac{\mu A_{44}}{A} + \frac{\mu A_4B_4}{AB} + \frac{\mu A_4F_4}{AF} + \frac{\mu B_{44}}{B} + \frac{\mu B_4F_4}{BF} + \frac{\mu F_{44}}{F} - 2\mu P - 8\pi P + \frac{1}{2}\mu\rho = 0, \tag{10}$$

$$\frac{\mu A_{44}}{A} + \frac{2\mu A_4B_4}{AB} + \frac{2\mu B_{44}}{B} + \frac{\mu B_4^2}{B^2} - 2\mu P - 8\pi P + \frac{1}{2}\mu\rho = 0. \tag{11}$$

Here and what follows, the suffix '4' after a field variable represents an ordinary differentiation with respect to the time 't'.

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In order to derive the exact solution of the field equations (8 - 11), we take the following scale transformations [66],

$$\begin{aligned} A(t) &= e^{\alpha(\tau)}, \\ B(t) &= e^{\gamma(\tau)}, \\ F(t) &= e^{\lambda(\tau)} \\ dt &= AB^2Cd\tau. \end{aligned} \tag{12}$$

Now, the field equations (8 - 11) using (12) reduce to

$$2\mu P-(3\mu+16\pi)\rho+e^{-2(\alpha+2\gamma+\lambda)}[-2\mu\lambda^{'}(\alpha^{'}+2\gamma^{'})-2\mu\gamma^{'}(2\alpha^{'}+\gamma^{'})]=0 \tag{13}$$

$$\mu(\rho-2e^{-2(\alpha+2\gamma+\lambda)}[\lambda^{'}(\alpha^{'}+2\gamma^{'})+\gamma^{'}(2\alpha^{'}+\gamma^{'})-2\gamma^{''}-\lambda^{''})]-4(\mu+4\pi)P=0 \tag{14}$$

$$e^{2(\alpha+2\gamma+\lambda)}[\mu\rho-4(\mu+4\pi)P]+2\mu[\alpha^{''}-\alpha^{'}(2\gamma^{'}+\lambda^{'})+\gamma^{''}-\gamma^{'}(\gamma^{'}+2\lambda^{'})+\lambda^{''}]=0, \tag{15}$$

$$\mu(\rho-2e^{-2(\alpha+2\gamma+\lambda)}[-\alpha^{''}+\lambda^{'}(\alpha^{'}+2\gamma^{'})+\gamma^{'}(2\alpha^{'}+\gamma^{'})-2\gamma^{''})]-4(\mu+4\pi)P=0, \tag{16}$$

where the prime stands for $\frac{d}{d\tau}$.

It is to be noted that there are five unknowns α, γ, λ, P and ρ involved in the above four equations. Therefore for obtaining exact solutions of Eqs. (13 - 16) we need to consider some interplaying relationships between any two parameters, such as [67–69]

$$\lambda = m\gamma, \tag{17}$$

where $m \neq 0$ is a parameter the value of which can be chosen suitably depending on the physical situation.

Now solving Eqs. (13 - 16), we get the solutions as

$$\gamma(\tau) = k_1\tau + k_2, \tag{18}$$

$$\lambda(\tau) = m(k_1\tau + k_2), \tag{19}$$

$$\alpha(\tau) = k_3\tau + k_4, \tag{20}$$

$$\rho = -\frac{2\mu(3\mu+8\pi)(c_1e^{-2\tau c_2})}{(\mu+8\pi)(5\mu+16\pi)}, \tag{21}$$

$$P = -\frac{4\mu(\mu+4\pi)(c_1e^{-2\tau c_2})}{(\mu+8\pi)(5\mu+16\pi)}, \tag{22}$$

where $c_1 = k_1[2k_1m + k_1 + k_3(m+2)]$ and $c_2 = k_1(m+2) + k_3$, however without affecting physical property one can consider $k_1 = k_3 \neq 0$ and $k_2 = k_4 = 0$.

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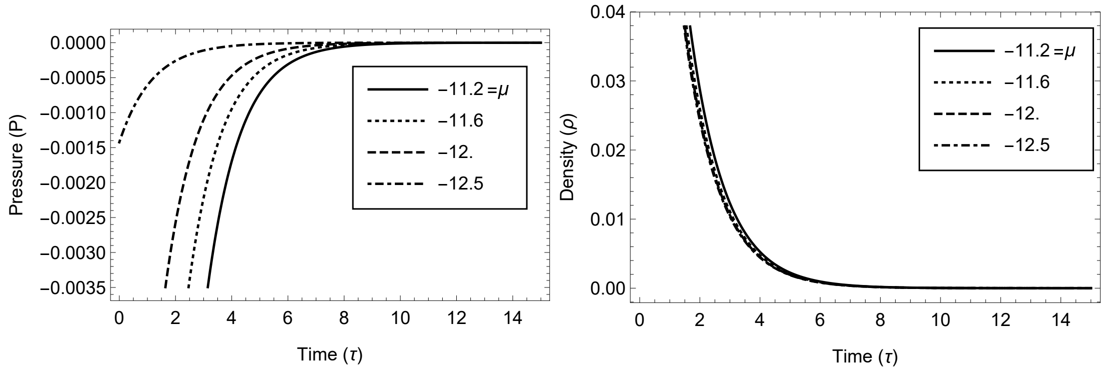

Figure 1. Variation of the pressure and density w.r.t. time (Case-I). Here we have considered the following parametric values: $k_1 = 0.13$, $k_3 = 0.1$ and $m = 0.5$ which will also be followed in all other plots.

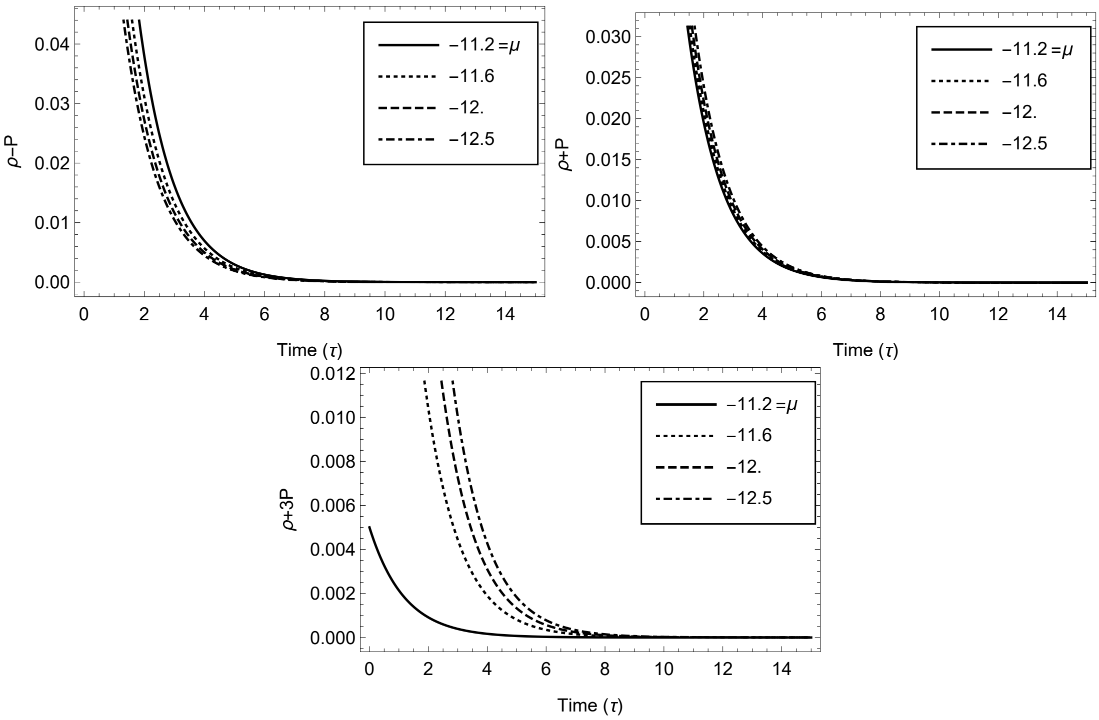

Figure 2. Variation of (a) $\rho - P$ (b) $\rho + P$ and (c) $\rho + 3P$ w.r.t. time (Case-I).

3.2. $f(R,T) = R + \mu R^2 + \mu T$

In this case, we consider $f_1(R) = R + \mu R^2$ and $f_2(T) = \mu T$. Therefore, Eq. (4) becomes

$$G_{ij} + 2\mu R R_{ij} - \frac{1}{2}\mu R^2 g_{ij} = (8\pi + \mu)T_{ij} + \left(P + \frac{T}{2}\right)\mu g_{ij}. \tag{23}$$

For (2), the field equations in $f(R,T)$ theory using (12) and (23) are

$$\begin{aligned} 4\mu(-\alpha'' - 2\gamma'' + 2\gamma'\lambda' + (\gamma')^2 - \lambda'')(-\alpha'' - 2\gamma'' + 3\gamma'(\gamma' + 2\lambda') - \lambda'') + \\ 12\mu(\alpha')^2(2\gamma' + \lambda')^2 - 2\alpha'(2\gamma' + \lambda')(4\mu(2(\alpha'' + 2\gamma'' + \\ \lambda'') - 3\gamma'(\gamma' + 2\lambda')) + e^{2(\alpha+2\gamma+\lambda)}) - 2\gamma' e^{2(\alpha+2\gamma+\lambda)}(\gamma' + 2\lambda') + \\ e^{4(\alpha+2\gamma+\lambda)}(-3\mu\rho + 2\mu P - 16\rho\pi) = 0, \end{aligned} \tag{24}$$

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$$-2e^{2(\alpha+2\gamma+\lambda)}(\alpha'(2\gamma'+\lambda')-2\gamma''+2\gamma'\lambda'+(\gamma')^2-\lambda'')+4\mu(-\alpha''+\alpha'(2\gamma'+\lambda')-2\gamma''+2\gamma'\lambda'+(\gamma')^2-\lambda'')(\alpha''+\alpha'(2\gamma'+\lambda')-2\gamma''+2\gamma'\lambda'+(\gamma')^2-\lambda'')+e^{4(\alpha+2\gamma+\lambda)}(\mu\rho-4P(\mu+4\pi))=0, \tag{25}$$

$$2(2\mu(-\alpha''+\alpha'(2\gamma'+\lambda')+2\gamma'\lambda'+(\gamma')^2-\lambda'')(-\alpha''+\alpha'(2\gamma'+\lambda')-2\gamma''+2\gamma'\lambda'+(\gamma')^2-\lambda'')+e^{2(\alpha+2\gamma+\lambda)}(\alpha''-\alpha'(2\gamma'(\tau)+\lambda')+\gamma''-\gamma'(\gamma'+2\lambda')+\lambda''))+e^{4(\alpha+2\gamma+\lambda)}(\mu\rho-4P(\mu+4\pi))=0, \tag{26}$$

$$-2e^{2(\alpha+2\gamma+\lambda)}(-\alpha''+\alpha'(2\gamma'+\lambda')-2\gamma''+2\gamma'\lambda'+(\gamma')^2)+4\mu((-\alpha''+\alpha'(2\gamma'+\lambda')-2\gamma''+2\gamma'\lambda'+(\gamma')^2)^2-(\lambda'')^2)+e^{4(\alpha+2\gamma+\lambda)}(\mu\rho-4P(\mu+4\pi))=0. \tag{27}$$

We note that in the present case, the field equations (24) - (27) yield the same solution (18 - 20) as obtained in subsection (3.1). However, the other two physical parameters can be provided as

$$\rho=\frac{\left(2c_1\mu(7\mu+24\pi)-e^{2c_2\tau}(3\mu+8\pi)\right)\times 2c_1e^{-4c_2\tau}}{(\mu+8\pi)(5\mu+16\pi)}, \tag{28}$$

$$P=\frac{\left(c_1\mu(3\mu+8\pi)-e^{2c_2\tau}(\mu+4\pi)\right)\times 4c_1e^{-4c_2\tau}}{(\mu+8\pi)(5\mu+16\pi)}. \tag{29}$$

Let us find out the expression for the ratio of the pressure and density (i.e., the EOS parameter w), which is

$$\frac{P}{\rho}=\frac{2\left(c_1\mu(3\mu+8\pi)-e^{2c_2\tau}(\mu+4\pi)\right)}{2c_1\mu(7\mu+24\pi)-e^{2c_2\tau}(3\mu+8\pi)}. \tag{30}$$

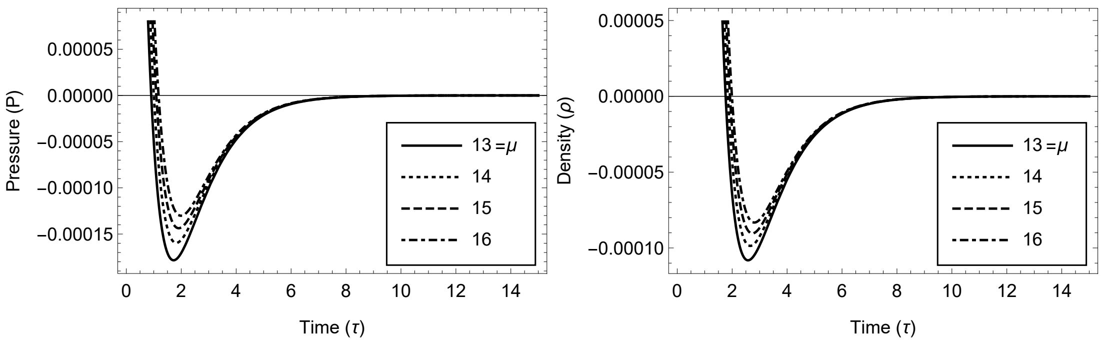

Figure 3. Variation of the pressure and density w.r.t. time (Case-II).

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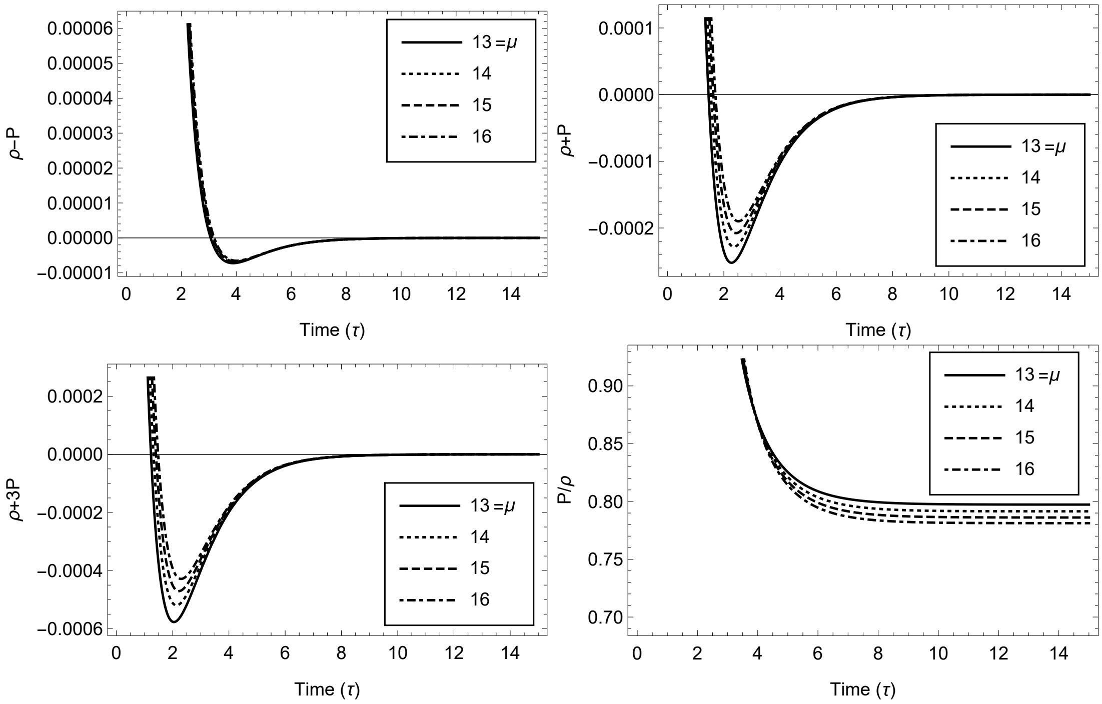

Figure 4. Variation of (a) $\rho - P$, (b) $\rho + P$, (c) $\rho + 3P$ and (d) P/ρ w.r.t. time (Case-II).

3.3. $f(R,T) = R\mu + RT\mu^2$

We consider here the third case, i.e. $f(R,T) = f_1(R) + f_2(R)f_3(T)$ where $f_1(R) = f_2(R) = \mu R$ and $f_3(T) = \mu T$. Now Eq. (4) becomes

$$G_{ij} = \frac{\mu P R g_{ij}}{\mu T + 1} + \frac{T_{ij}\left(\mu^2 R + 8\pi\right)}{\mu(\mu T + 1)}. \tag{31}$$

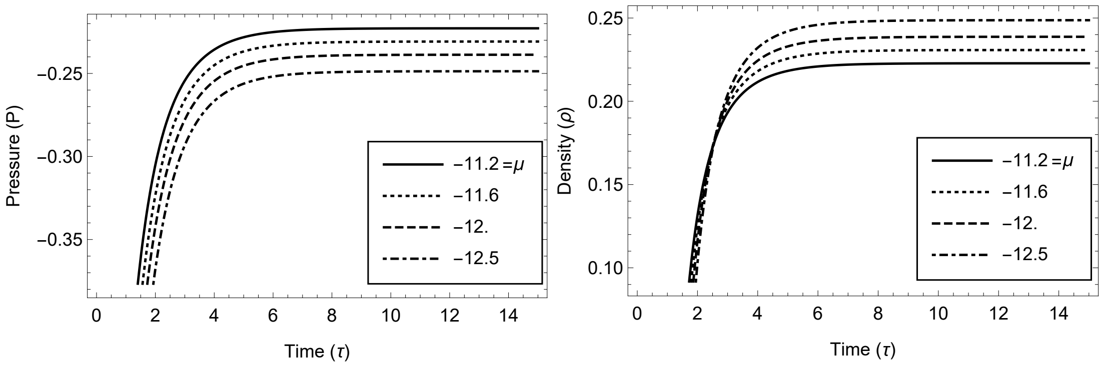

Figure 5. Variation of the pressure and density w.r.t. time (Case-III).

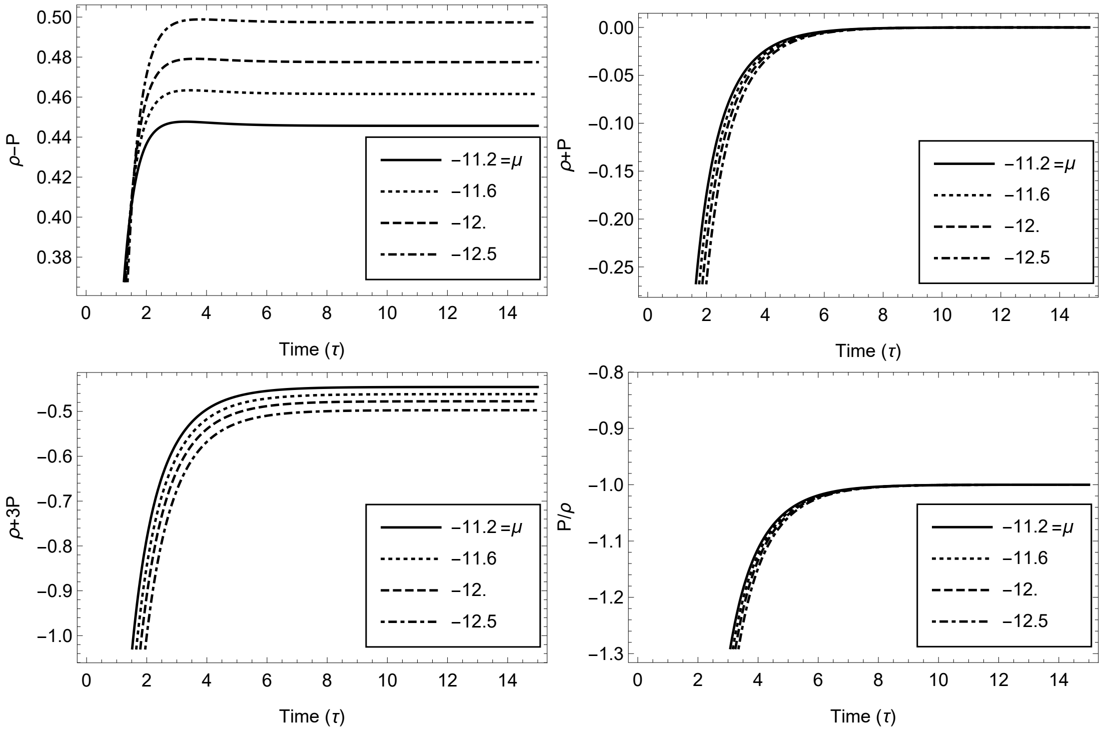

Figure 6. Variation of (a) $\rho - P$ (b) $\rho + P$, (c) $\rho + 3P$ and (d) P/ρ w.r.t. time (Case-III).

For Eq. (2), the field equations in $f(R,T)$ theory using Eqs. (12) and (31) are

$$\begin{aligned}
&-(\mu+16\rho\pi)e^{2(\alpha(\tau)+2\gamma(\tau)+\lambda(\tau))}-2(\mu(2\mu(P+\rho)-1)+1)(\alpha^{''}(\tau)+2\gamma^{''}(\tau)+\lambda^{''}(\tau))+\\
&2\alpha^{'}(\tau)(\mu(\mu\rho+6\mu P-2)+1)(2\gamma^{'}(\tau)+\lambda^{'}(\tau))+4\gamma^{'}(\tau)\lambda^{'}(\tau)(\mu(\mu\rho+6\mu P-2)+1)+\\
&2\gamma^{'}(\tau)^2(\mu(\mu\rho+6\mu P-2)+1)=0,
\end{aligned} \tag{32}$$

$$\begin{aligned}
&2(\alpha^{''}+2\gamma^{''}+\lambda^{''}-\mu(\alpha^{''}+\mu(4P-\rho)(2\gamma^{''}+\lambda^{''}))+\alpha^{'}(2\gamma^{'}+\lambda^{'})(\mu^2(4P-\rho)-1)+\\
&2\gamma^{'}\lambda^{'}(\mu^2(4P-\rho)-1)+(\gamma^{'})^2(\mu^2(4P-\rho)-1))+e^{2(\alpha+2\gamma+\lambda)}(\mu-16P\pi)=0,
\end{aligned} \tag{33}$$

$$\begin{aligned}
&2(\alpha^{''}+2\gamma^{''}+\lambda^{''}+\mu(-\gamma^{''}-\mu(4P-\rho)(\alpha^{''}+\gamma^{''}+\lambda^{''}))+\alpha^{'}(2\gamma^{'}+\lambda^{'})(\mu^2(4P-\rho)-1)+\\
&2\gamma^{'}\lambda^{'}(\mu^2(4P-\rho)-1)+(\gamma^{'})^2(\mu^2(4P-\rho)-1))+e^{2(\alpha+2\gamma+\lambda)}(\mu-16P\pi)=0,
\end{aligned} \tag{34}$$

$$\begin{aligned}
&-2(\mu-1)\lambda^{''}+2(\mu^2(4P-\rho)-1)(-\alpha^{''}+\alpha^{'}(2\gamma^{'}+\lambda^{'})-2\gamma^{''}+2\gamma^{'}\lambda^{'}+(\gamma^{'})^2)+\\
&e^{2(\alpha+2\gamma+\lambda)}(\mu-16P\pi)=0.
\end{aligned} \tag{35}$$

In this case also, the field equations (32) - (35) admit the same solutions set (18 - 20) as obtained in subsection (3.1), however the rest of the parameters are

$$\rho=\frac{c_1e^{2c_2\tau}\left(5\mu^3-16\mu\pi+8\pi\right)-4\mu\pi e^{4c_2\tau}+2c_1^2\mu^2(4\mu-5)}{2\left(5c_1^2\mu^4-20c_1\mu^2\pi e^{2c_2\tau}+32\pi^2e^{4c_2\tau}\right)}, \tag{36}$$

$$P=\frac{c_1^2\mu^3+2\mu\pi e^{4c_2\tau}-4c_1\pi e^{2c_2\tau}}{5c_1^2\mu^4-20c_1\mu^2\pi e^{2c_2\tau}+32\pi^2e^{4c_2\tau}}, \tag{37}$$

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$$\frac{P}{\rho} = \frac{2\left(c_1^2\mu^3 + 2\mu\pi e^{4c_2\tau} - 4c_1\pi e^{2c_2\tau}\right)}{c_1 e^{2c_2\tau}\left(5\mu^3 - 16\mu\pi + 8\pi\right) + 2c_1^2\mu^2(4\mu - 5) - 4\mu\pi e^{4c_2\tau}}. \tag{38}$$

Thus, the five dimensional cosmological model in $f(R,T)$ theory of gravity corresponding to the solutions of subsections (3.1 - 3.3) can be uniquely presented as

$$dS^2 = d\tau^2 - e^{2k_3\tau}dX^2 - e^{2k_1\tau}(dY^2 + dZ^2) - e^{2mk_1\tau}dN^2. \tag{39}$$

4. Some Physical and Geometrical Properties

In this section, we study some physical and geometrical properties of the models obtained in the preceding subsections under the five dimensional cosmological model in $f(R,T)$ theory of gravity.

4.1. Status of the Model

The spatial volume (V), scalar expansion (θ), Hubble parameter (H), shear scalar (σ^2) and redshift (z) for the model are given by

$$V = e^{c_2\tau}, \tag{40}$$

$$\theta = c_2 e^{-c_2\tau}, \tag{41}$$

$$H = \frac{1}{4}c_2 e^{-c_2\tau}, \tag{42}$$

$$\sigma^2 = \frac{1}{8}e^{-2c_2\tau}(k_1^2(m(3m-4)+4) - 2k_1k_3(m+2) + 3k_3^2), \tag{43}$$

$$z = \frac{1}{\sqrt[4]{e^{c_2\tau}}} - 1, \tag{44}$$

$$\frac{\sigma^2}{\theta^2} = \frac{k_1^2(m(3m-4)+4) - 2k_1k_3(m+2) + 3k_3^2}{8c_2}. \tag{45}$$

From the above solutions set we notice that at $\tau = 0$, $V = 1$ and as $\tau \to \infty$, $V \to \infty$. Therefore it can be inferred that our model is free from the initial singularity. We also note that the pressure and density are finite at $\tau = 0$, which decrease as τ increases and tend to zero when $\tau \to \infty$. This means at infinite time, our model leads to a vacuum model. Further, as $\frac{\sigma^2}{\theta^2} \neq 0$, so our model is anisotropic throughout the evolution. Eq. (44) exhibits the expansion of the spacetime in the universe, when $\tau \to \infty$, however in the present model $q = 3$, which means that the universe is in decelerating phase.

4.2. Stability of the Model

The stability of the model is obtained by considering the ratio $\frac{dp}{d\rho}$ which can be shown equivalent to C_s^2. If C_s^2 is positive then the model is stable, whereas if C_s^2 is negative the model is unstable. In our case $\frac{dp}{d\rho} = 1 - \frac{\mu}{3\mu+8\pi}$. From this relation, we notice that C_s^2 is positive for $\mu > -4\pi$ and thus provides a stable model under this restrictive condition.

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4.3. EOS Parameter (w)

In the present model EOS parameter is governed by the parameter μ. One can note that different values of the parameter lead to a different model in the $f(R,T)$ gravity. Caldwell and coworkers [70,71] pointed out that $w < -1$ is a better fit for the observed astrophysical data. But this violates the weak energy condition (WEC) $\rho \geq 0$. This type of matter is called phantom [72]. Therefore, the matter-energy field in our model behaves like a phantom dominated universe when $\mu < -3.2\pi$.

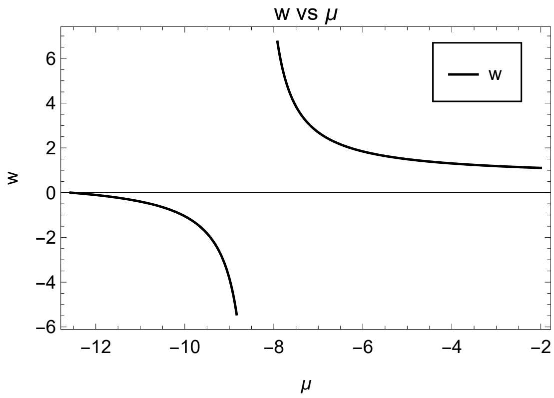

Figure 7. Variation of w vs. μ (Case-I).

5. Discussion and Conclusion

In the present work our motivation was to obtain exact solutions to the Einstein field equations for $5d$ LRS Bianchi type-I spacetime in $f(R,T)$ theory of gravity. We have presented cosmological models under the following three specifications: (i) $f(R,T) = \mu(R+T)$, (ii) $f(R,T) = R\mu + RT\mu^2$, and (iii) $f(R,T) = R + \mu R^2 + \mu T$. The solutions sets under these models via the graphical plots exhibit that the EOS parameter w is completely governed by μ. Fine tuning of the parameter μ provides the effect of phantom cosmology. Moreover, imposing restriction upon this parameter we are able to obtain a stable model of the universe which isotropizes at finite cosmic time.

Some other salient and characteristics features of the cosmological models are as follows:

(1) We notice that the model is free from the initial singularity and hence physically viable. This feature is obvious as for $\tau = 0$ we get $V = 1$ and for $\tau \to \infty$ one can obtain $V \to \infty$.

(2) The fluid pressure and matter density of the cosmic distribution are finite at $\tau = 0$. The physical quantities decrease as τ increases and tend to zero when $\tau \to \infty$. Thus at infinite time our presented model leads to a vacuum cosmological solution.

(3) As $\frac{\sigma^2}{\theta^2} \neq 0$, so the model is anisotropic throughout the evolution. Again, $\tau \to \infty$ exhibits the expanding universe, however $q = 3$ dictates that the universe is decelerating.

(4) The stability of the model is obtained by considering the ratio $\frac{dp}{d\rho}$ which is positive for $\mu > -4\pi$ to yield a stable model.

(5) The EOS parameter is governed by the parameter μ and its value can be found as $\mu < -3.2\pi$. This is related to $w < -1$ which behaves like a phantom energy inspired cosmology. This type of phantom cosmology allows to account for dynamics and matter content of the universe tracing back the evolution to the inflationary epoch [73].

Thus an obvious issue is here: how to incorporate an accelerating phase of the universe, which is the present cosmological scenario, along with the decelerating phase in our phantom type dark energy model. However, following Capozziello et al. [74] one can make an endeavor to get a transition from deceleration to acceleration phase of the universe. Therefore, this issue can be addressed in a future project.

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