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# The Origins of Seljuk Ornamental Art in Anatolia

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**Abstract**

The Seljuks, who came from the Central Asian prairies, invaded Asia Minor towards the end of the 11th century. The land had been settled by then mainly by the Christian Eastern Romans and Armenian peoples. Seljuks were Moslems; they built monumental structures, some of which have survived the natural disasters of several centuries to the present day. Most of these architectural marvels contain extraordinary decorations in the form of ornaments, friezes and rosettes. I have studied periodic ornaments and classified them into 17 mathematical *wallpaper groups* according to their symmetry properties that reveal their global structure. On the other hand, the local details of the ornaments, the motifs, show a clear variation from simple geometric patterns to complicated and refined forms. Seljuk art was originally influenced by Persian styles, later influenced by the Christian population in Asia Minor, and finally represents the impact of Islamic culture.

**Keywords:** Ornaments; Symmetry; Wallpaper Group; Rum Seljuks; Islamic Art

## 1 Introduction

Ornaments, especially regular mosaic-like patterns, are a collection of intelligent signs created by people of a cultural group. They are characteristic of the group of craftsmen and express their state of mind, which has developed and accumulated over time. Therefore, the ornaments can be studied to identify an ethnic or cultural group and the influences acting on them.

Symmetry brings out some fascinating features in works of art. Two-dimensional periodic patterns can be described by their point-group and translational symmetries. Historically, the decisive achievement was the realization by Georg Pólya that the symmetry properties of ornaments can be explained by 17 *wallpaper groups* of mathematical group theory, otherwise used in crystallography [1]. This type of categorization made it possible to extract invariant features of artworks that seemed to be very different at a perceptual level [2]. Therefore, the classification of planar ornaments using the mathematical tools turns out to be a rigorous method that allows to identify their intrinsic features.

Beyond the identification of cultural groups by their artistic practices manifested in their wallpapers, the distribution of wallpaper groups allows a comparison between different civilizations with the aim of finding cultural interactions between such groups. I presented the idea of calculating a statistical correlation function for the wallpaper distributions of some medieval groups: Armenia, Byzantium (Eastern Romans), the Moslems of Arabia, the Great Seljuks, and the Seljuks of Anatolia [3]. I have found close similarities between the Seljuks of Anatolia and the Moslems of Arabia, and between Armenian and Byzantine art. Later, we have refined this work by computing a hierarchical dendrogram that groups cultures in clusters with similar art practices [4].

The heart of an ornament is its repeating unit, the crystallographic *unit cell*, which consists of a *motif* that rotates, mirrors or glide reflects in compliance with the associated symmetry-group properties, and finally completes the entire ornament surface by translation in two dimensions. The motifs are the fine structures of the unit cells. Even if ornaments belong to the same symmetry group, their motifs can be very different. For the symmetry group of *p4mm* on the floor ornaments of the Basilica of San Marco in Venice, I have found 40 different motifs [5]. This fact gives a special touch to the ornamental building blocks.

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Here I will first deal with the Seljuks in Anatolia, their origin and fate. Then I will apply the mathematical methods to their artistic creations. I will give examples of crystallographic classification of ornaments. In Section 3 I will analyze in detail three selected planar ornaments from the favorite group of Seljuks $p4mm$. The next Section is devoted to an ornament with several star-shaped polygons. In the last Section I will reflect on the origins of the observed diversity of Seljuk design.

# 2 Seljuk Turks

## 2.1 Origins

The Seljuks were descendants of Turkic peoples, the Göktürks, who lived as nomadic herders on the prairies between southern Mongolia and northern China [6]. The Göktürks had their own script, which is used in the 7th-century Orhun monuments in what is now Mongolia. The Göktürks and other early Turkic tribes mostly adhered to animism and shamanism as well as to Buddhism, while they were mostly subject to slavery. They converted to Islam during the early Arab conquests in the Moslem Khalifate Period towards the end of the 7th century. Under the Umayyads, most Turkic tribes worked as domestic servants. Several records indicate that they were later trained as soldiers and became the most feared enemies of their neighbors. They even served as military leaders in the Abbasid army.

One of the Turkish officers, Seljuk Bey, assumed military power by taking over Turkish provincial groups and moved across Afghanistan to Persian Khorasan around the year 1000. This date is considered the birth of the Great Seljuk Sultanate. They conquered all of Persia and established a powerful state that threatened the Islamic Abbasid Dynasty and the Eastern Roman Empire in the west. In the new sultanate, the Turks were a successful military power and the Persians superior organizers of cultural and governmental affairs.

The Seljuks made Isfahan their capital and conquered Baghdad in 1055. Their cultural communication was in Persian. Alpaslan was one of the successful army leaders who defeated his rebellious uncle, killed him and threw his sons into dungeon. With Alpaslan, the Seljuks moved west and began their first advances into Anatolia. They were soon confronted by the Roman army and won the Battle of Manzikert in 1071. This success opened the gates of Anatolia for them, where there were already some Arab settlements.

Alpaslan returned to his capital, Isfahan, but was killed a year later. Alpaslan's nephew Suleyman benefited from the lack of leadership, escaped from prison, gathered a group of Turkmen followers, and moved west to infiltrate Byzantine Anatolia. In 1081, they declared Nicaea (Iznik) in the northwest as their capital. This was the first seed of Turkish settlements in Anatolia, and these Seljuks were called Rum Seljuks, *rum* meaning *west* in Turkish. Around 1096, the first Crusaders arrived in Asia Minor on their way to Palestine and expelled the Seljuks from Nicaea. Iconium (Konya) was their next capital.

The Seljuks of Rum brought the Perso-Islamic culture to Anatolia. The Christian and Greek-speaking population began to change into a predominantly Moslem and Turkish/Persian-speaking one. The new Seljuks were not great warriors like their Seljuk ancestors, but settled as civilized peoples within a full-fledged sultanate [7].

The glorious Seljuk Dynasty of Rum lasted until the Mongol invasion of Anatolia. In 1243, the Mongols defeated the Seljuks at the Battle of Köse Dağ and became the new rulers of Anatolia [8]. Especially after 1277, the Turks became vassals of the Ilkhan, a successful Mongol dynasty that established its own empire under Hulagu Khan, the grandson of Genghiz Khan. Towards the end of the 13th century, some Turkmen tribes became independent principalities called *beylik*, reducing the influence of the Seljuks. They even turned away from mainstream Islam and joined mystical and revolutionary sects such as the *Alevits* and *Bektashis*. This development marked the end of the Seljuk dynasty in Anatolia before 1300.

## 2.2 Art practices

The Seljuks were influenced by Persian civilization in many ways and became bearers of Persian culture. Thus, the newcomers of Anatolia represented the Persian-Islamic culture, and intermarriages with the native Christian population led to the transformation of the nomadic tribe from Central Asia into a civilized, art-producing group. They built mosques, mausoleums, caravanserais, medreses, and even a shipyard in Alaiye (Alanya).

The outstanding examples of Seljuk architecture are found throughout Anatolia. They are richly decorated with ornaments, friezes and rosettes that serve as an inexhaustible source for classical studies [9, 10, 11]. A recent publication similarly deals with Seljuk ornamentation in Anatolia [12]. These studies classify the planar ornaments into groups according to their local geometric features. Gert Schneider, in his monumental archive, has classified 440 Seljuk ornaments into 50 such arbitrary groups [11]. I have used the planar ornaments from his book and divided them instead into 17 wallpaper groups [13]. This classification scheme, commonly used by crystallographers, is scientific and thus objective and rigorous. The results are presented in Figure 1.

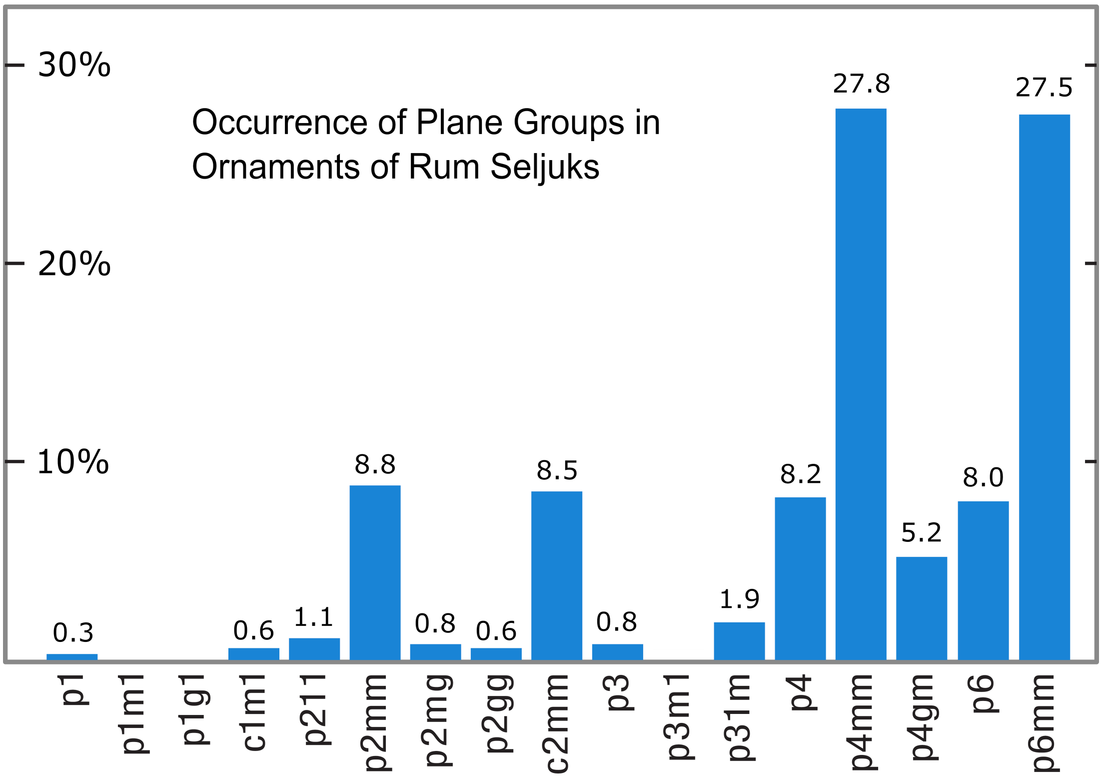


Figure 1: The classification of Seljuk Rum ornaments into 17 plane groups. The numbers above the bars show the percentage frequency of each symmetry group as indicated by the ordinate. 364 individual occurrences of ornaments are included in this distribution. All examples are from the collection of Gerd Schneider [11].

An ornament is formed by *translational* repetition of a *unit cell* in two principal directions. The repeating unit is either a *primitive* cell or has an additional element in the *center* of the cell; hence a $p$ or a $c$ is used in the group designation. The number represents the rotational symmetry $n$ of the unit cell, where $n$ is the fraction of the total circle $2\pi$. In the group symbol, $m$ stands for a *mirror reflection*, $mm$ for reflections in two main directions of the ornament. The letter $g$ stands for a glide reflection. The Seljuk wallpapers in Figure 1 show that the groups $p4mm$ and $p6mm$ together dominate the distribution. Moreover, the groups with double-mirror properties $mm$ occur more frequently in each rotation class. Previously, I counted the occurrence of a symmetry group in the whole Seljuk creation in Anatolia for all monuments. This type of counting resulted in a total number of 1067 artifacts [3]. Here each pattern is counted as one, even if it occurs in different places. The number is thus reduced to 364.

# 3 Ornaments in $p4mm$ symmetry

## 3.1 Unit cells

We note that the symmetry group $p4mm$ is most commonly used in Seljuk monuments. In fact, this group is the preferred one for several Middle Eastern medieval civilizations [3]. I now give three examples for the group $p4mm$ found in Anatolia and examine this symmetry group in detail.

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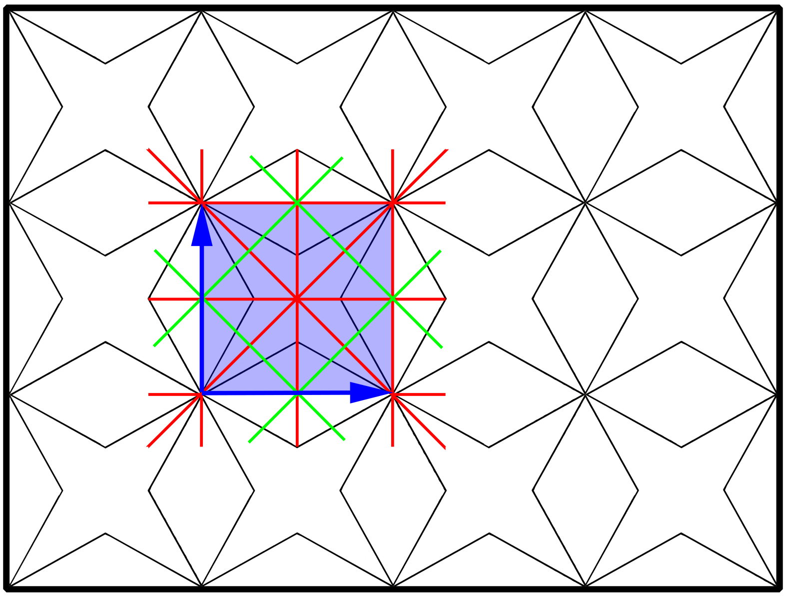

Figure 2: An ornament from Schneider's archive [11], which belongs to the group *p4mm*. It consists of regular four-pointed stars stacked periodically in two directions. Blue are the translation vectors defining the stacking. The unit cell in square shape is highlighted. The four corners and the center of the unit cell are axes of fourfold symmetry. Red are the mirror reflection lines, green the glide reflection lines. The latter pass through the axes of the twofold symmetry. The axes of the rotational symmetry are not shown in order not to overload the figure.

Figure 2 is a fourfold symmetrical ornament found on mosaic tiles in the Buruciye Medrese in Sivas, which was built in 1271. The medrese was used by Muzaffer Burucerdi for education in natural sciences. He was from Borujerd in western Iran. The ornament is number 159 of Group 15, *Additive Ornaments I*, in Schneider's archive [11].

We can interpret the pattern as composed of rhombi with alternating orientations, four rhombi and a white star forming a regular octagon. However, it is crystallographically convenient to think of the fourfold-symmetric stars as white areas periodically repeating in two directions and covering the surface. The building blocks of this ornament are thus quite simple.

Figure 3 is another fourfold symmetrical ornament of Seljuk design. It is located in Allaeddin Mosque built 1223 in Niğde. It is carved in stone and is located in the prayer niche. Schneider classifies it as number 300 of his 28th group, *Ornaments of eightfold stars with horizontal, vertical and diagonal rays*. This design is two distinct units, two squares of different size and orientation. The clever superposition of the squares turned against each other results in eightfold stars. In this respect, this ornament is a simple development of the simple case shown in the previous figure. Instead of one, there are two different shapes that make up the pattern. Nevertheless, both patterns have the same global fourfold symmetry.

Figure 4 shows yet another fourfold symmetrical ornament of Seljuk civilization. The main symmetry properties are sketched in the figure without explicitly showing the rotational axes. This ornament is number 429 in the 47th group *Different systems of solar and stellar rays of the same pattern* of Schneider's archive [11]. It is located at the main entrance of the Ağzıkara Han in Aksaray. It is carved in stone and dates from 1242/43. The pattern shown in the figure was generated using the corresponding ornament from Brian Wichmann's archive as a template [14].

This last ornament is fundamentally different from the previous two. Instead of repeating elementary shapes that fit into the unit cell, we have an abundance of polygons. In crystallographic terms, the unit cell is huge. It contains some regular polygons: 1 twelvefold star and 1 regular octagon, which form the fourfold-rotation axes of the *p4mm* group. There are 2 tenfold stars containing the twofold-symmetry axes of the unit cell. There are another 4 ninefold and 16 fivefold stars, in addition to several distorted fivefold stars, the starfish, arrows, ivy leaves and petal shapes.

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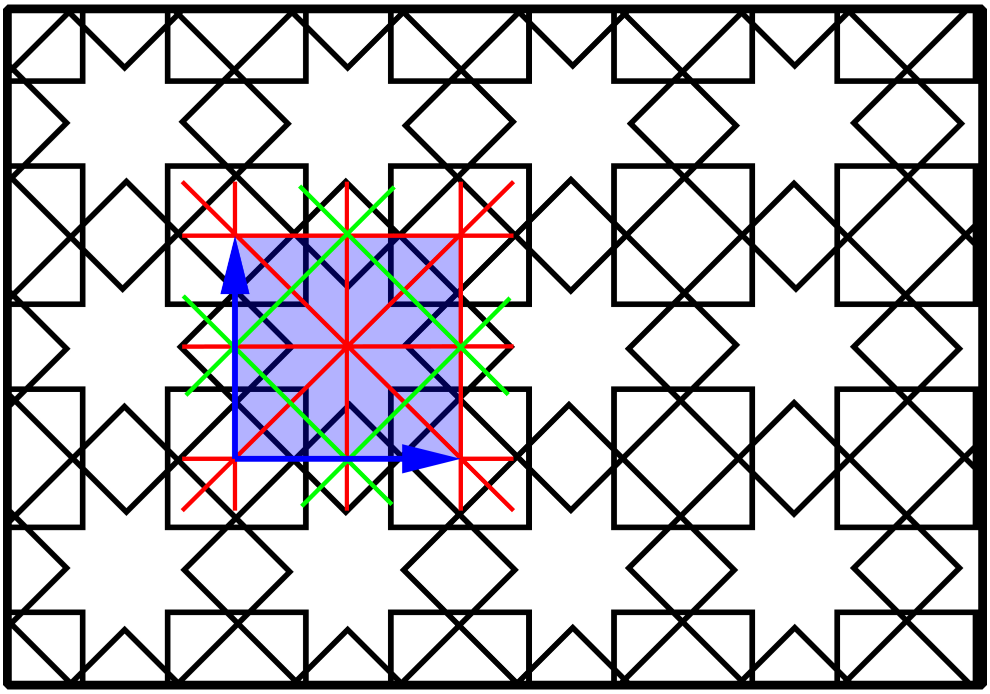

Figure 3: Another ornament from Schneider's archive [11], which belongs to the group *p4mm*. It consists of two groups of periodically stacked squares, one group consisting of larger squares than the other and rotated by 45°. Shown in blue are the translation vectors that define the stacking. Highlighted is a square unit cell whose corners and center are axes of fourfold symmetry. Red are the mirror reflection lines, green are the glide reflection lines. The glide lines cross at the axes of twofold rotational symmetry. The axes of rotational symmetry are not shown explicitly for simplicity.

## 3.2 Motifs and beyond

The repeating unit of a two-dimensional periodic ornament is the square unit cell, as we have seen in Figs. 2 – 4 for three examples of the symmetry class *p4mm*. The first example is a simple geometric shape repeated over the entire surface. About one-tenth of Seljuk ornaments have such a simple design [11], while almost any geometric shape can be used for this purpose. The ornament shown in Figur 3 consists of two squares of different sizes. Although this design is similarly simple, it gives the ornament an attractive appearance. There are quite a few combinations of regular shapes to construct such two-part ornaments: Triangles, squares, hexagons, octagons and often their combinations. About one-third of Seljuk ornaments feature this type of construction. More than half of the ornaments contain one or more stars with a number of five to twenty arms [11]. These ornaments have a large unit cell.

Figure 5 shows the unit cells of the above ornaments. The motif of an ornament is the smallest element that forms the unit cell when the proper symmetry operations are applied. In the figure, the motif of each unit cell is the lighter part. This motif, together with the horizontal reflection along its vertical side, which is a mirror-reflection line, occupies a quarter of the unit cell. This element completes the unit cell by four rotational additions of 90° each. The motif of the simple pattern on the left is a single line separating two areas. The motif of the ornament consisting of two squares contains three lines. The motif of the pattern with multiple polygons shows an eighth of each shape, an octagon and a dodecagon, a quarter of a decagon and a half of a nonagon. The motif additionally contains several polygons at the interstices, including distorted fivefold stars, starfish, which serve to combine these various regular multi-armed stars.

There have been several attempts to determine how medieval craftsmen could have constructed such elaborate patterns. This subject has been studied many times by several experts [15, 16]. In principal, any shape, regardless of its complexity, can be constructed either with a compass and straightedge [17, 18, 19], with computer-aided methods [20, 21, 22] or by developing appropriate tiles [23, 24]. The amazing ornament with multiple star patterns [25] shown in Figure 5, right, is analyzed in detail in the following section.

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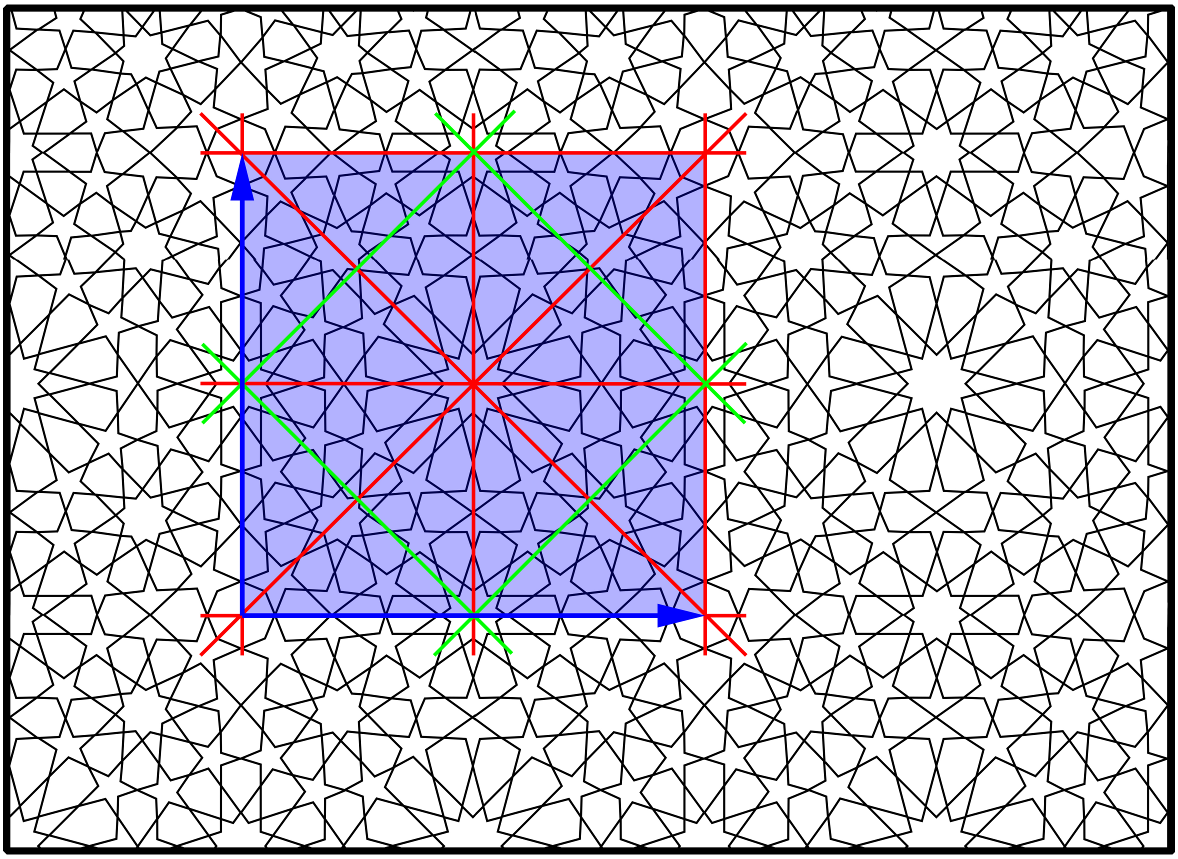

Figure 4: An ornament from Schneider's archive [11], which belongs to the group *p4mm*. It consists of several polygons. Shown in blue are the translation vectors that define the stacking. Highlighted is a square unit cell whose corners and center are axes of fourfold symmetry. Red are the mirror reflection lines, green are the glide reflection lines. The glide lines meet at the axes of twofold symmetry. The axes of rotational symmetry are not shown for simplicity.

## 4 Multi-Star Patterns

Construction of any pattern with suitable tiles is a practical method and is well suited for hobby designers. Tiles are good for reproducing patterns that have a simple star, like a decagon [22]. If the pattern has more than one regular polygon, the number of tiles exceeds the practical limits for proper reproduction. For such cases, computer-aided design promises good results. Brian Wichmann's archive contains a rich collection of ornaments consisting of several polygons, including their digital reproduction [14].

When several multi-pointed stars are incorporated into a unit cell, deviations from an ideal mesh inevitably occur. An example is the complex *p4mm* pattern of the Rum Seljuks, shown in Figure 4. To simplify the analysis, we show the motif of the pattern in Figure 6, which is an equilateral triangle with 90° vertex angle. The motif contains a regular octagon (yellow) at the fourfold-symmetry axis, on the far left of the figure, at an intersection of two mirror-reflection lines (red). The interior angles of the octagon are 135°. On the right, at the other end of the horizontal mirror line where it intersects a vertical mirror line, lies a ten-pointed star {10,3} (blue) on a twofold-symmetry axis of the pattern. The interior angles of the decagon are 72°. Along the mirror line vertically above it we see the twelve-pointed star {12.4} (red) on the other fourfold-symmetry axis of the pattern. The interior angles are 60°. Between the two fourfold-symmetry axes, *i.e.*, between the twelve-pointed star and the octagon, there is a nine-pointed star {18,5} (green) on the mirror-reflection line.

We see that the pattern accommodates eight-, nine-, ten- and twelvefold regular polygons. The harmonious combination of all these shapes places a high standard for the overall design. The combination of polygons with different number of legs is achieved with the help of *mating polygons* placed between each pair of polygons. In the figure, the mating polygons belonging of a particular polygon have the same but lighter color as the polygon. The two petal shapes between the octagon and the

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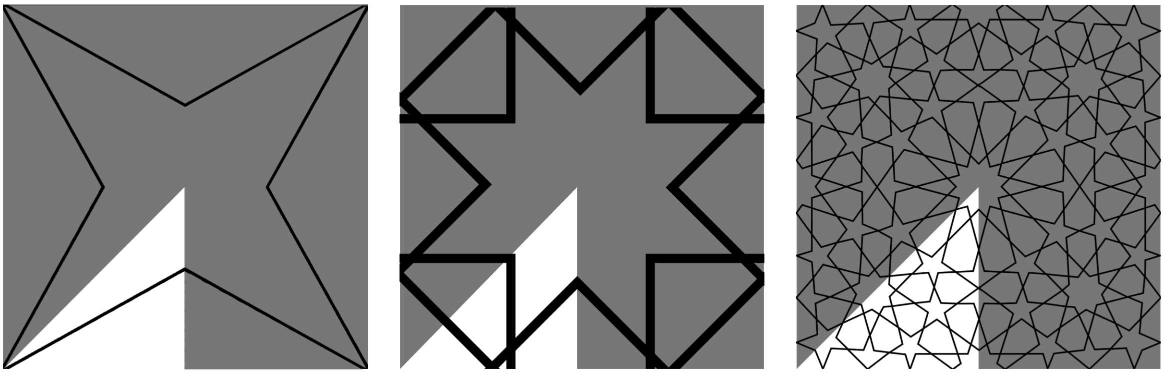

Figure 5: The unit cells of the three examples of the *p4mm* group created by the Rum Seljuks. For each case, the motif is the lighter area of the unit cell.

ten-pointed star have base angles 135° and 140°. The nine-pointed star is also connected to the twelve-pointed star by two petal shapes with vertex angles 80° and 60°. And finally, the connection between the twelve-pointed star and the ten-pointed star is made by two ivy leaves with acute angles of 30° and 36°. We drew an auxiliary line connecting the nine-pointed star and the ten-pointed star. This line connects the polygons with two petal shapes, light blue and light green, with acute angles of 72° and 80°, and divides the motif into two triangles with apex angles 81° and 99°. These angles complete the sum of the interior angles of the triangles to 180°. However, we expected to find 80° because the auxiliary line appears to be a local mirror-reflection line for the polygons it intersects and few other adjacent mating polygons. The line connecting the centers of the octagon and the twelve-pointed star is a mirror-reflection line of the motif, along which the nine-pointed star and the octagon are connected with an arrow and a distorted starfish.

The mating polygons serve not only to fill the space between the main polygons, but also to connect them perfectly by adjusting their side lengths and interior angles. Only the mating polygons, which are around and in close contact with the regular main polygons, are identical. These are twelve arrows and twelve petal shapes around the twelvefold-symmetrical red star, ten arrows and ten petal shapes around the tenfold-symmetrical blue star, nine arrows and nine petal shapes around the ninefold-symmetrical green star, and eight starfish and eight petal shapes around the eightfold-symmetrical yellow octagon. The rest of the area is occupied by some ivy leaves and starfish. There are at least four different petal shapes, three different arrows, three different ivy leaves, and three different starfish. Therefore, this particular pattern would require many different pieces if reproduced with tiles, while other construction techniques promise more success [14].

## 5 Conclusions

We have presented three examples of Seljuk ornaments in the same symmetry group *p4mm* with different complexity of design. They were implemented in Sivas in 1271 (Fig. 2), in Niğde in 1223 (Fig. 3), and in Aksaray in 1242 (Fig. 4). Aksaray and Niğde are about 90 km apart, while Sivas is further away (300 km). This observation shows that the Seljuk dynasty of Rum created these patterns in the same time and place in completely different artistic nature, which speaks for their creativity. There are few star patterns found in the monuments of the Great Seljuk Sultanate in present-day Iran, Afghanistan and India [23, 24].

The Seljuks came to Anatolia in great masses after 1071. After settling in the Konya area, they spread throughout Anatolia and founded important settlements that are still large Turkish cities today. It is worth mentioning that no decorations have been found on mosques, Koran schools or caravanserais built before the late 12th century [10]. The Seljuks brought with them some knowledge of monument construction and decoration when they came from regions south of the Caspian Sea and were influenced mainly by the classical Persian school. They penetrated into Asia Minor, where there were still some remains of Greco-Roman art culture [4]. They came into close contact with Eastern Romans (Byzantine Empire) and Armenians. There is evidence that the Seljuks in Anatolia were significantly influenced by these cultures [11].

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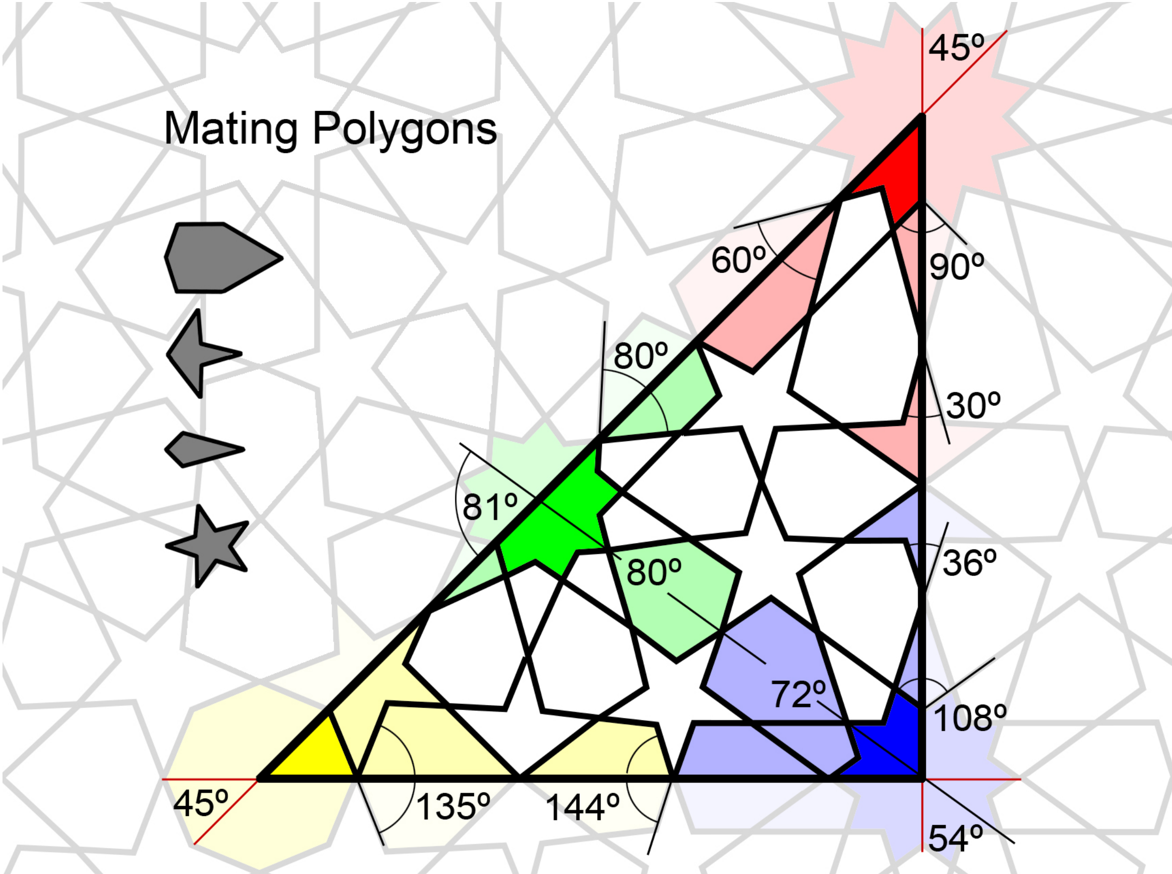


Figure 6: The motif of the multi-star ornament of the group *p4mm*. The motif contains parts of an octagon (yellow), a nonagon (green), a decagon (blue) and a dodecagon (red). The shapes in lighter shades of each star polygon serve as matching units to the neighboring star. They are displayed separately as *mating polygons* and are called *petal shape*, *ivy leave*, *arrow* and *starfish*, from top to bottom.

There are several Seljuk artifacts that testify to Persian, Byzantine, Armenian, and even Greco-Roman influence (*cf.* Figs. 2 and 3). There seems to be a consensus that star ornaments and especially multi-star ornaments belong to Islamic art [14, 16, 15]. Since the majority of Seljuk designs contain such star ornaments, we must conclude that the interaction of Seljuks of Rum during the 12th century must have been very intense with Arabic-Islamic culture.

The cultural interaction between Arabs and Seljuks led to the present situation where most Seljuk designs are labeled *Islamic*. In the Middle Ages, groups of people were identified mainly by their denomination. National identities were not developed until the late 18th century. This practice certainly leads to the loss of natural identities, and we cannot readily infer the driving force of art creation, whether it was inspired by the Word of God or folkloric talent. However, there is another way to interpret these cultural interactions. We note that German scholars [9, 10, 11] prefer the term *Seljuk art*, while others insist on *Islamic art*. Considering the fact that the Seljuks created about 200 different masterpieces with star patterns [11] found on monuments in Asia Minor before the middle of the 15th century, which clearly surpass other medieval contributions, I believe that it is more appropriate to speak of *Seljuk geometric art* than of *Islamic design*. After all, the Seljuks, although very moderate, were also Moslems, and their work, largely influenced by Hellenistic, Roman, Armenian, and Arab creations, is therefore in many ways a multi-national art product.

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[25] The combination of the two techniques, the straightedge and the trigonometric construction, demonstrates that $(1 + \sqrt{2} - \sqrt{3}) = 2\,sin\,20°$, $1/2(\sqrt{5} - 1) = 2\,sin\,18°$ or $\sqrt{2 - \sqrt{3}} = 2\,sin\,15°$.