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Dark Energy from Virtual Gravitons (GCDM model vs ΛCDM model)

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Abstract
The dark energy from virtual gravitons is consistent with observational data on supernova with the same accuracy as the ΛCDM model. The fact that virtual gravitons are capable of producing of a de Sitter accelerated expansion of the FLRW universe was established in 2008 (see references). The combination of conformal non–invariance with zero rest mass of gravitons (unique properties of the gravitational field) leads to the appearance of graviton dark energy in a mater-dominated era. This fact explains the relatively recent appearance of the dark energy and answers the question “Why now?”. The transition redshifts (where deceleration is replaced by acceleration) that follow from the graviton theory are consistent with model independent transition redshifts derived from observational data. Prospects for testing the GCDM model (the graviton model of dark energy where G stands for gravitons) and comparison with the ΛCDM model are discussed.

Key wrods: dark energy; supernova observations; virtual gravitons

1.Introduction

As is known, more than 20 years ago, two research groups independently discovered the accelerated expansion of the universe [Riess et al.1998, Perlmutter et al., 1999]. This effect was called the dark energy (DE). The nature of DE is still unknown. A big number of approaches were considered, including quite unusual ones [Kamenshchik et al., 2001, Bento et al., 2002]. The most popular, apparently, is the hypothesis about scalar fields of different kinds, filling the universe. A fairly complete bibliography (reviews) can be found, e.g., at Carroll et al., (2001); Mukhanov, (2005); Copeland et al., (2006); Weinberg, (2008); Chernin, (2008); Yoo and Watanabe, (2012), Heisenberg, (2018); One of the most popular theories based on the idea of scalar field is so called quintessence (Wetterich,1988; Caldwell et al., 1998; Caldwell and Linder, 2005; see also Weinberg, 2008). The quintessence is dark energy in the form of a time varying scalar field which is slowly rolling down toward to the minimum of its potential. Another direction of the search was attempts to generalize Einstein's theory in the classical and quantum way (see e.g., Heisenberg, 2018). Let us also note an interest attempt by Verlinde (2016) to construct an emergent gravity in which “the space-time and gravity emergent together from entanglement structure of an underlining microscopic theory”. Another interesting direction of modern research is associated with attempts to construct a cosmology with a variable cosmological constant (Alexander, Cortes, Liddle et al., 2019).

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For now, despite the large number of DE models, the community settled down on the simplest choice, which is the hypothesis that the cause of acceleration is the cosmological constant, which was introduced into his theory by Einstein himself more than 100 years ago. Later he gave up on it. However, after Zeldovich (1968) showed that the cosmological constant is the energy of a non-gravitational vacuum, it again acquired the "rights of citizenship." In the last hundred years the cosmological constant has appeared every time astrophysics and cosmology faced with problems which, it seemed, could not be solved otherwise. For example, at the end of 60x, when quasars were discovered, and at one time it seemed that they were concentrated near z = 2 (Hoyle and Burbidge, 1966; Longair Scheuer, 1967; Burbidge and Burbidge, 1967). This effect was attempted to be explained by the fact that there is a positive cosmological constant (Petrosian et al. 1967; Shklovsky, 1967, Kardashev, 1967; Rowan-Robinson, 1968; Petrosian and Salpeter, 1970]. Later quasars with large redshifts were discovered, and this hypothesis was dropped. As Petrosian (1974) *remarked: "In the absence of strong* evidence in favor of Lemaître models, we must again send back the Lemaître models and along with them the cosmological constant until their next reappearance".[2] The next reappearance took place in 1998-1999 in connection with the discovery of dark energy [Riess et al., 1998; Perlmutter et al., 1999][3].

The community has now opted for the ΛCDM model despite known problems with it. As is known, two main problems with it are 122 order of magnitude difference between theoretical and observational value of the cosmological constant and the coincidence problem (why dark energy is appear recently, why now?). A very significant argument in favor of ΛCDM is the fact that it describes the observational data very well, despite these theoretical contradictions. From the other hand, all observational data (with no exceptions) based on the attempts to find the equation of state parameter $w = p/\rho$ where $p\, and\, \rho$ are the pressure and density of matter filling the universe. In case of cosmological constant $w_\Lambda = -1$. All observational data shows that $w = -1$, and this fact is considered by the community as confirmation of the validity of the ΛCDM model. However, the virtual gravitons also produce the de Sitter expansion with $w_G = -1$(Appendix B). To find the difference between the models, it is necessary to measure w(z) (Section 5).

As we already mentioned, the goal of this paper is to compare the ΛCDM model with the GCDM model where G stands for gravitons. To do so we are going to calculate distance modules for both models and compare them (Sections 3 and 4). We will see that the GCDM model is consistent with SNe Ia observations with the same accuracy as the ΛCDM model. At the same time, the GCDM model is free from the contradictions that the ΛCDM model encounters.

2. Dark Energy from Virtual Gravitons

A graviton theory of dark energy was presented by Marochnik, Usikov & Vereshkov (2008) (we denote this work as MUV (2008)). A rigorous mathematical foundation of the theory was given by MUV (2013) and Vereshkov & Marochnik (2013). The present paper is dedicated to the comparison of graviton theory with supernova observations only, so we do not touch upon any

[2] Quoted from O'Raifeartaigh et al. (2017).

[3] In fact, the history of the cosmological constant from its inception in 1917 to the present day is an exciting "astronomical adventure novel," detailed in a beautifully written historical and astronomical study (O'Raifeartaigh et al.,2017).

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theoretical problems. Nevertheless, we believe that it is necessary to remind a reader at least a basic information of graviton theory. MUV (2008) showed that quantum fluctuations of the metric (gravitons) and their back reaction on the isotropic and homogeneous (on average) background provide the mechanism for cosmological acceleration (Appendix A). The dark energy effect is a consequence of the vacuum polarization and graviton creation by non–stationary gravitational field of the Universe. The energy density of gravitons is a functional of the background geometry. In the non–empty Universe the background geometry is defined by all contributing cosmological subsystems — by gravitons, matter, and radiation. The combination of conformal non–invariance with zero rest mass of gravitons (unique properties of the gravitational field) leads to a macroscopic quantum effect: condensation of gravitons in a quantum state with wavelength of the order of the distance to the horizon. The same unique properties of gravitational field are the causes for the appearance of dark energy in our era (Appendix D), and it answers the old unanswered question "Why now?".

In the process of the evolution of the Universe, the density of gravitons, because of their condensation, starts dominating over the sum of the energy density of other subsystems of the cosmological media. The self–consistent state of background and gravitons, which evolves asymptotically, represents self–polarized vacuum in the de Sitter space. The regime of the de Sitter-like expansion is beginning to form in the current Universe. MUV (2008) presented three new exact solutions for the one–loop quantum gravity. The de Sitter solution is one of these. All exact solutions can be found when the theory is presented as Bogoliubov–Born–Green–Kirkwood–Yvon (BBGKY) hierarchy equations for moments of the graviton spectral function (Appendix B).

In this paper, we show that a dark energy of graviton origin, with the case of zero cosmological constant, is consistent with SNe Ia observational data with the same accuracy as the ΛCDM model. In terms of observational data, the difference between these models lies beyond the accuracy of observations (Section 4). We show also that the appearance of graviton DE during the modern epoch of the Universe evolution is a direct consequence of the combination of conformal non–invariance with zero rest mass of gravitons, which are unique features of gravitational field (Appendix C). In Sections 5 and 6, we discuss observational tests that might distinguish GCDM and ΛCDM models.

3. Distance Modules

To fit supernova data, we use distance modulus as follows

$$\mu(z,h,\Omega_0) = 25 + 5\log(\frac{3000}{h}(1+z)R(z)) \qquad (1)$$

$$R(z) = \int_0^z \frac{dz'}{\widehat{H}(z')} \qquad (2)$$

$$\widehat{H}(z) = [\Omega_m(1+z)^3 + \Omega(z)]^{1/2} \qquad (3)$$

Where $\mu(z,h)$ is distance modules, the Hubble constant $H = h \cdot 100 \mathrm{km/s} \cdot \mathrm{Mpc}$. The first term in (3) represents energy density of non-relativistic matter, and the second one represents the energy density of DE as a function of z. In case of cosmological constant, $\Omega(z) = \Omega_0 = \Omega_\Lambda = \mathrm{const}$. In case

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of DE, $\Omega(z)$ is the fraction of DE in the total energy balance. Let us denote distance modules for cosmological constant case as $\mu_{\Lambda}(z, h, \Omega_0)$. Our program is to compare runs for $\mu_G(z, h, \Omega(z))$ and $\mu_{\Lambda}(z, h, \Omega_0)$ to show that the difference between them is smaller than observational errors. We do not include CMB input into (21). The input of radiation into the current energy balance is very small, it is of the order of $\Omega_{\gamma} = 5.38 \cdot 10^{-5}$ (Zyla et al, 2020). For example, for the Planck case with $\Omega_{DE} = 0.685$ the input of radiation to the region z=1000 is about 8%. It is reasonable to assume that the effect of radiation on the dynamics of expansion can be neglected when its contribution to the dynamics becomes no more than 8%.

The usual practice is to use the parameter of the equation of state w and determine it from the observational data (see e.g., Riess et al. (2007)). The equation of state of dark energy is unknown, therefore for lack of better the dark energy equation of state can be taken in the $p=w\varepsilon$ form (sometimes in more general forms like $p/\rho=w_0+w_1 \cdot f(a)$). After that, one can look for the w parameter which provides the best fit. Usually, the supernova data are used in combination with other data. All of them show that $w \approx -1$, and this fact, as it is generally accepted, leads to the idea that DE effect is due to the cosmological constant. In case of dynamical DE (and in case of gravitons, particularly), the second term of (3) must follow from the solution of Einstein's equations for the model. In case of cosmological constant, the Friedmannian equations for the flat Universe have a well-known exact analytical solution (see e.g., Frieman, Turner& Huterer, 2008)

$$a(t) = \left(\Omega_m / \Omega_{\Lambda}\right)^{1/3} \left(\sinh\left[3\sqrt{\Omega_{\Lambda}} Ht / 2\right]\right)^{2/3}$$

This allows us to calculate all values of interest analytically. Unlike to the cosmological constant case, there is no exact analytical solution to the system consisting of gravitons and non-relativistic matter. So, we must go for the numerical integration of Einstein's equations (in our case they are the equations of BBGKY chain) for the graviton model.

4. GCDM vs ΛCDM

In this section, we will show that the difference between $\mu_{\Lambda}(z, h, \Omega_0)$ and $\mu_G(z, h, \Omega(z))$ for all cases of interest are less than observational errors in SNe Ia database. In other words, we will show that both ΛCDM and GCDM are statistically indistinguishable. As it was mentioned by the Planck team (Ade et al., 2013), there is a "tension" between WMAP and Planck data. In accordance with WMAP today's value of DE is $\Omega_0=0.721$ (Bennett et al., 2012) meanwhile Plank's data give $\Omega_0 \approx 0.685$ (Ade et al., 2013). Recent data obtained from the analysis of the large-scale structure give $\Omega_m=0.339 \pm 0.032/0.031$ for ΛCDM model (Abbott et al.,2021). The combination of these data with available baryon acoustic oscillation, redshift space distortion, SNe Ia data and with Planck CMB lensing leads to the following content of the nonrelativistic matter and Hubble constant in the universe for ΛCDM model (Abbott et al., 2021) $\Omega_m=0.306 \pm 0.004/0.005; h=0.680 \pm 0.004/0.003$. This leads to the following DE contents which we need for our calculations $\Omega_0=0.694$ .We consider three cases $\Omega_0=0.721$ (Bennett et al., 2012), $\Omega_0=0.685$ (Ade et al., 2013) and $\Omega_0=0.694$ (Abbott et al., 2021). In this work we use Union 2.1

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compilation from the Supernova Cosmology project (Suzuki et al., 2012; Amanullah et al., 2010).

We calculate distance modules $\mu(z, h, \Omega_0)$ using (3). As we already mentioned, in distinction of generally accepted method of calculation of $\mu(z, h, \Omega_0)$ which uses the hypothesis on the equation of state parameter of DE $p=w\varepsilon$,with the following determination of w, we calculate $\Omega(z)$ directly by numerical integration of Einstein equations presented in the form of BBGKY chain (B.1, B.2). In such an approach, the cosmological constant case corresponds to $\Omega(z)=\Omega(0)=\Omega_0=$const in eqn. (3). Thus, we must compare $\mu_\Lambda(z, h, \Omega_0)$ calculated for $\Omega(z)=\Omega_0=$const (cosmological constant) with $\mu_G(z, h, \Omega(z))$ calculated for $\Omega(z)$, i.e. graviton dark energy.

Our computational model has only one free parameter $\Omega^0{}_{DE}$ which is a fraction of the graviton energy in the total energy balance at the start of calculation, run, (Appendix C). Each run started from some value of $\Omega^0{}_{DE}$ and finished when $\Omega(z)$ becomes equal to Ω_0. Attention. In our calculations, the scale factor is increasing from the past to future, i.e., from a=1 to $a = a_{today}$, where a_{today} corresponds to the observed today DE fraction of the full energy, which we denote as $\Omega_0 \equiv \Omega_{DE}(z=0)$. First, we conducted a series of runs to localize the interval of $\Omega^0{}_{DE}$ where the statistical deviation between calculated distance modulus and their values from the data base for SNe Ia (formulas (1-3) above) is minimal. We found that the interval $0.000125<\Omega^0{}_{DE}<$ 0.000162 provide the statistical deviation is about 1 sigma. Second, we found the best fitting $\Omega^0{}_{DE}$ separately for the three cases Ω_0 = (0.721, 0.694, 0.685).

We ran our numerical simulations for the following initial conditions, $\Omega^0{}_{DE}$ =0.000162; 0.000154; 0.000142; 0.000125 . These runs automatically lead to the following initial z_0 for all Ω_0 chosen (see Table).
The SLS is situated at $z \approx 1100$ with the width of this region $\Delta z=90$ (Hadzhiyska and Spergel, 2019). So, we get the initial conditions for such runs in the following regions of interest

$\Omega^0{}_{DE}$	Ω_0	z_0
0.000162	0.721 0.694 0.685	1058 1000 982.6
0.000154	0.721 0.694 0.685	1106-1108 1047-1048 1028-1029
0.000142	0.721 0.694 0.685	1211-1212 1145-1146 1124-1125
0.000125	0.721	>1318

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	0.694	1297-1298
	0.685	1272-1275

The right column in the table shows what the initial values are for the observational values of dark energy (second column). The first column shows four randomly selected EDEs, each markedly less than 0.009. We present here the results of numerical simulation for the $\Omega^0{}_{DE}$=0.000154 for all three cases Ω_0=0.685, Ω_0=0.721 and Ω_0=0.694 from this Table. The results for all three other cases are similar.

To start with, we choose the initial value of the energy density of DE as $\Omega^0{}_{DE}$=0.000154 at the initial point $z_0 \approx 1028$ for Ω_0=0.685. Such a run produces $\Omega(z)$ shown on the Fig.1. For all figures below $\Omega(z)$ is the fraction of DE in the total energy balance

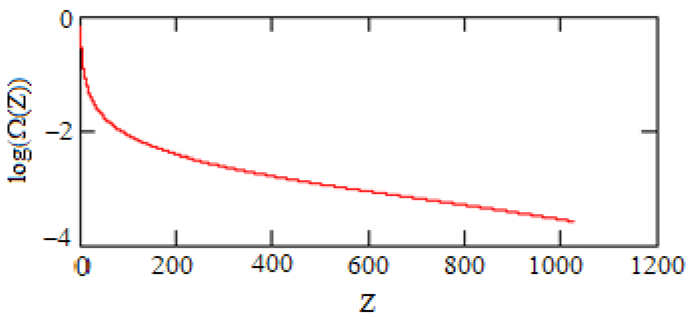

Fig. 1. Graviton DE as a function of z for the Ω_0=0.685 case.

In the region of interest, i.e., from z=0 to z=2, where supernovas observed are situated:

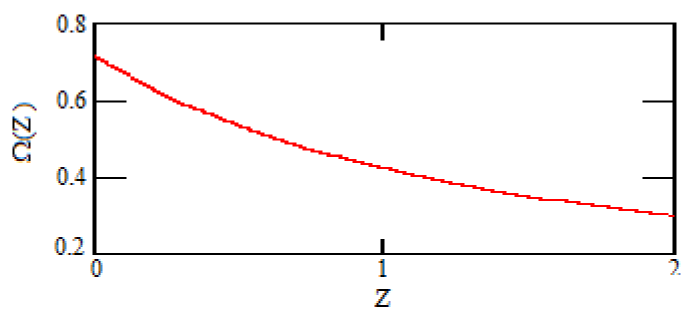

Fig.2 Graviton DE as a function of z for Ω_0=0.685 in the region $0 \le z \le 2$.

To compare $\Omega(z)$, obtained by a numerical integration of BBGKY chain (B.1), (B.2) with the observational data, a high accuracy analytical approximation of it in the whole interval $0 \le z \le 1028$ was obtained in a form of

$$\Omega(Z) := \frac{\alpha + \gamma \cdot Z + \varepsilon \cdot Z^2 + \eta \cdot Z^3 + \iota \cdot Z^4}{1 + \beta \cdot Z + \delta \cdot Z^2 + \zeta \cdot Z^3 + \theta \cdot Z^4} \qquad (4)$$

The coefficients $\alpha, \beta, \gamma, \delta, \varepsilon, \varsigma, \eta, \vartheta, \iota$ are as shown below:

α= 0.685, β=0.62974, γ=-0.046398, δ=0.10038, ε=0.109381, (5)

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ζ=0.115417, η=0.0058062, θ=0.0057869, ι=-4.135292*10^{-6}

Such $\Omega(z)$ generates distance modules which are showed together with the observational errors on Fig.3**.**

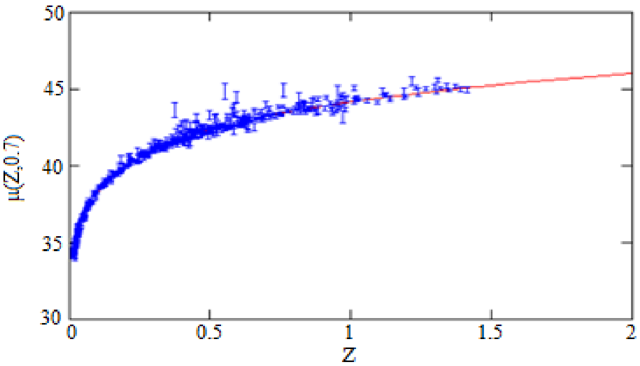

Fig.3. Solid curve is the distance modules μ_G(z,h) for graviton DE run with h=0.7 for the case Ω_0=0.685 . The blue bars show supernova positions together with observational errors

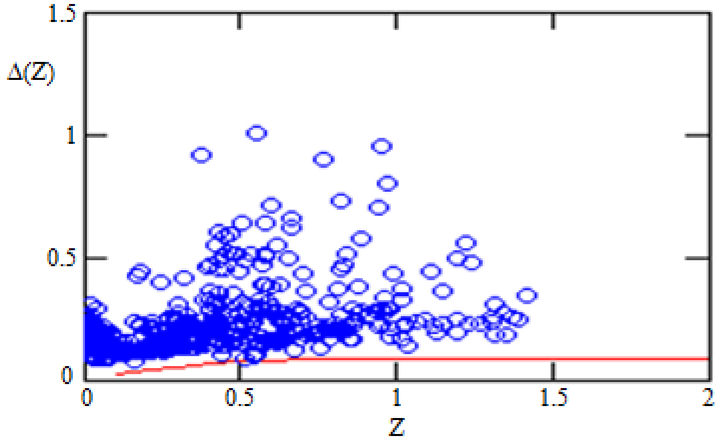

Fig 4. The solid curve $\Delta(z)=\mu_G(z, h)-\mu_\Lambda(z, 0.685, h)$ is difference between distance modules of GCDM and ΛCDM models for Ω_0=0.685 . The circles are the observational errors. One can see that the difference between these models is below all observational errors. The distance modules for the cosmological constant μ_Λ(z, 0.685, h) and for gravitons were calculated by (1-3) for h=0.7 [4].

The Fig.4 visually demonstrates the fact that the difference Δ between ΛCDM and GCDM models is less than observational errors. The same fact follows from the consideration of statistical sums (chi-square criterions). The fit proceeds by minimizing the χ^2 statistical criterion

[4] It worth to note that even big enough $\Omega^0{}_{DE}=0.01\Omega_0$ at the point $z_0=3.6$ leads to a picture similar to Fig. 4

$$\chi_{\mu}^{2}=\sum_{i=1}^{N}\frac{(\mu_i-\mu_i^{obs})^2}{\sigma_{\mu,i}^{2}} \qquad (6)$$

where the sum is over all supernova defined in the Union 2.1 compilation; $\mu^{obs}{}_i$ is the 'observed' distance modulus to the i-th supernova in the Union 2.1 table; $\sigma_{\mu i}$ is the observational error (standard deviation) in the value estimation $\mu^{obs}{}_i$ [51]; μ_i is a theoretical estimation for the same distance as $\mu^{obs}{}_i$. The statistical sum S is defined as

$$S=\chi_{\mu}^{2}/2 \qquad (7)$$

If the theoretical estimation is correct, and experimental errors of observed values $\mu^{obs}{}_i$ are also correct (being independent of each other), and normally distributed, then $\chi^2{}_\mu$ follows the Chi-squared distribution (Seber and Lee, 2003). When the number of members in the sum is equal N, the average (mean) value of $\chi^2{}_\mu$ is equal to N, and the variance is equal to 2N (Faraway, 2003). Practically, if $N \geq 50$, at it's extremum, the Chi-squared distribution becomes very close to the normal distribution. In other words, the probability that the estimated $\chi^2{}_\mu$ will deviate from the average value, N, on more than three-sigma, $\pm\, 3\cdot sqrt(2N)$, is about 100%-99.7%=0.3%. Mostly, with the probability 68%, the deviation should be observed within one-sigma interval about the mean value. To be exact, when one is fitting experimental data with a model having m free parameters, the value of χ^2 in average reduces by the factor $(N-m)/N$ (Andrae et al., 2010). In our case we used models with only one fitting parameter ($m=1$) and many observations (N=580), so we can safely neglect this factor in the chi-squared statistics. For the $\Omega_0=0.685$ case of cosmological constant we get the minimal statistical sum $S^{0.685}{}_{\Lambda,min}=284.521$ for h=0.7, and

$$(\chi^{0.685}{}_{\Lambda})^2=569.042 \qquad (8)$$

In this case the standard deviation is

$$\sigma^{0.685}{}_{\Lambda}=\sqrt{2\chi^{2}{}_{\mu,\Lambda}}=33.735 \qquad (9)$$

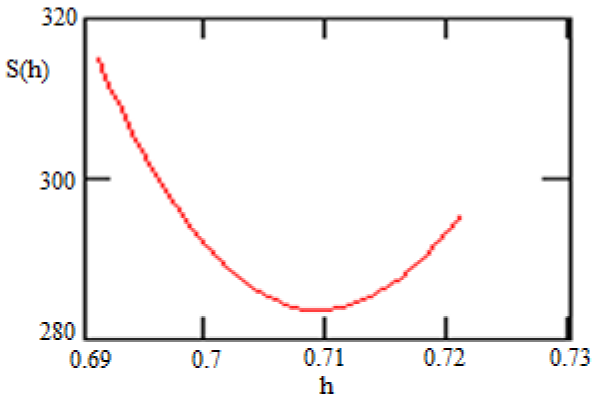

Fig. 5. Statistical sum S(h) as a function of h. The minimal statistical sum for DE in this case is $S_{DE}{}^{0.685}{}_{min}=283.68$ with $h=0.709$. Such h occupies an intermediate position between the typical values of h given in the Hubble tension problem (Ade et al., 2015) and Riess et al., 2019, 2021).

The chi-square and sigma here are

$$(\chi^{0.685}{}_{min\ DE})^2=567.36 \quad \text{and} \quad \sigma^{0.685}{}_{DE}=33.68 \qquad (10)$$

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The difference $(\chi^{0.685}{}_{\min\Lambda})^2-(\chi^{0.685}{}_{\min DE})^2=1.682<<\sigma^{0.685}{}_{DE}$, $\sigma^{0.685}{}_{\Lambda}$. In other words, these two models (ΛCDM and GCDM) are indistinguishable, so that the visual result shown on the Fig.5 is confirmed by direct calculation of statistical sums.

Now let us find the best fit to the cosmological constant for $\Omega_0=0.721$ and $h=0.70$.[5]

The statistical sum $S^{0.721}{}_{\min\ \Lambda}=281.125$ leads to the following chi-square $\chi_{\mu,\Lambda}{}^2$ and standard deviation σ_Λ for the case

$$\chi^{0.721}{}_{\min\Lambda}{}^2=562.250 \qquad (11)$$

$$\sigma^{0.721}{}_{\Lambda}=\sqrt{2\chi^2{}_{\mu,\Lambda}}=33.533 \qquad (12)$$

For the $\Omega_0=0.721$ case, the calculations started at the point $z_0=1106$. Numerical integration leads to the dark energy distribution showed on the Fig. 5

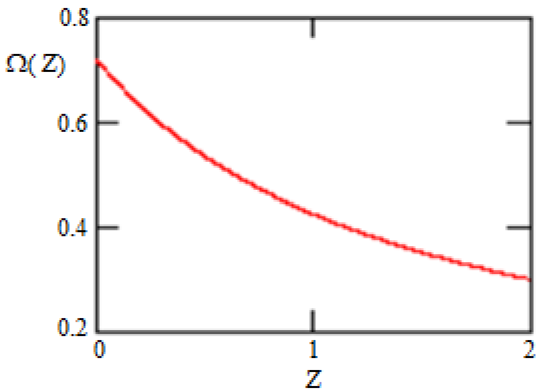

Fig. 6. Graviton DE for $\Omega_0=0.721$.

Graviton DE in the region of interest z in the interval [0, 2]. This Fig.6 is the same as Fig. 4 but for $\Omega_0=0.721$. This dependance obtained by numerical integration of BBGKY chain may be described by the following simple approximation

$$\Omega(z) := \frac{1}{\alpha+\beta\cdot z+c\cdot z^2} \qquad (13)$$

α=1.389213, β=0.9601587, c=0.00370186

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[5] The tension between numerical value of Hubble constant H was a subject for intense discussion between two research groups (Yuan et al., 2019), Freedman et al., 2019). The last publication of Freedman (2021) shows that the last measurement of h gives Ho = 69.8 ± 0.6 (stat) ± 1.6 (sys) km/s/Mpc (Freedman, 2021). Note that we obtained h=0.71 as a number minimizing the statistical sums for the cosmological constant for all three cases, which is closed to Freedman's h=0.698.

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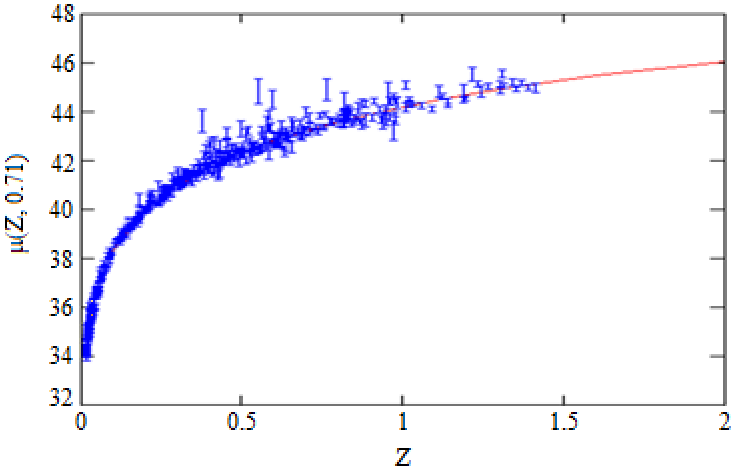

Fig. 7 Same as on the Fig.3 but for the Ω_0=0.721 case.

The statistical sum for the Ω_0=0.721 can be calculated the same way as for Ω_0=0.685 case. The dependance of statistical sum on the Hubble constant can be illustrated with Fig. 8 for the Ω_0=0.721 case.

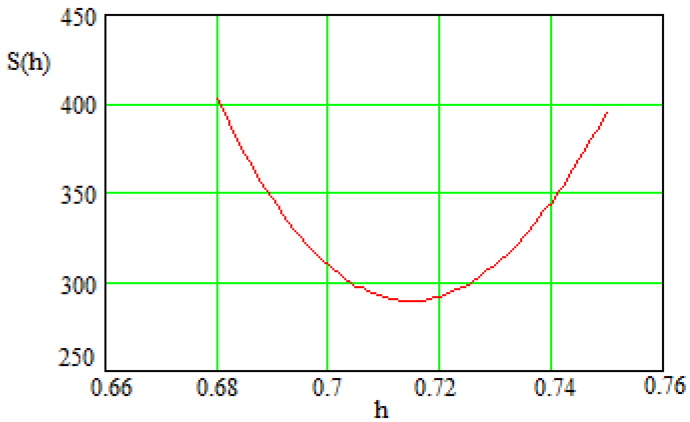

Fig. 8. It shows how statistical sum S(h) depends on Hubble constant for the Ω_0=0.721 case. The minimal statistical sum here is $S^{0.721}{}_{\min DE}$=289.945 with h=0.714.

For this case we get

$$\chi^{0.721}{}_{\min\ DE}{}^2=2S=579.89 \text{ and } \sigma^{0.721}{}_{DE}=34.05. \qquad (14)$$

Comparing (24-25) with (27), we get an expected result

$\Delta\chi^2=(\chi^{0.721}{}_{\min DE})^2-(\chi^{0.721}{}_{\min \Lambda})^2=579.89\text{-}562.250=17.24 < \sigma^{0.721}{}_{DE},\ \sigma^{0.721}{}_{\Lambda}$

Again, the difference between statistical sums less than standard deviations which means that both cases statistically indistinguishable. Visually, this fact can be seen on Fig 9.

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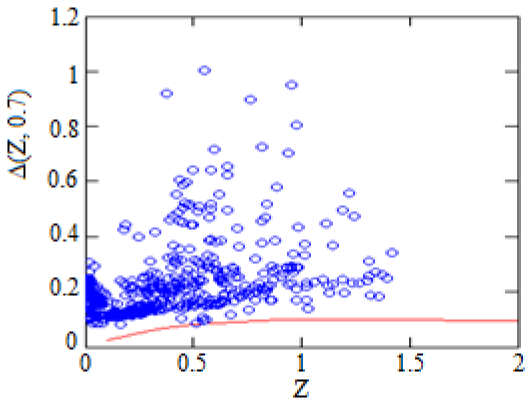

Fig. 9 Same as on the Fig.3 but for the Ω_0=0.721 case. Again, one can see that the difference between graviton and cosmological constant distance modules is less than observational errors.

Similar consideration of the Ω_0=0.694 case from the table, corresponding to z_0=1047-1048, leads to the similar result shown on the Fig. 9

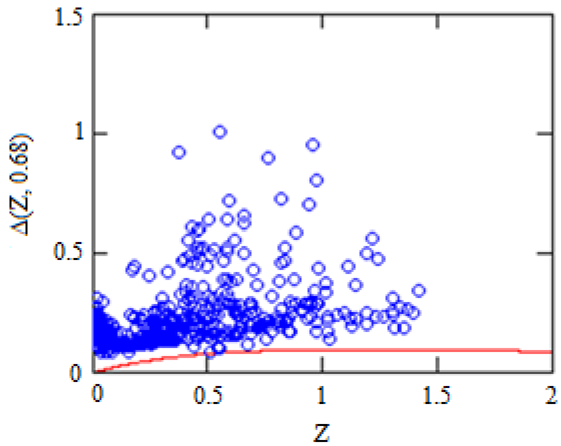

Fig. 10. For this case again Δ is the difference between ΛCDM and GCDM models (solid line) are less than the observational errors shown as circles

Below we show the graviton DE obtained for this case and corresponding stat sums.

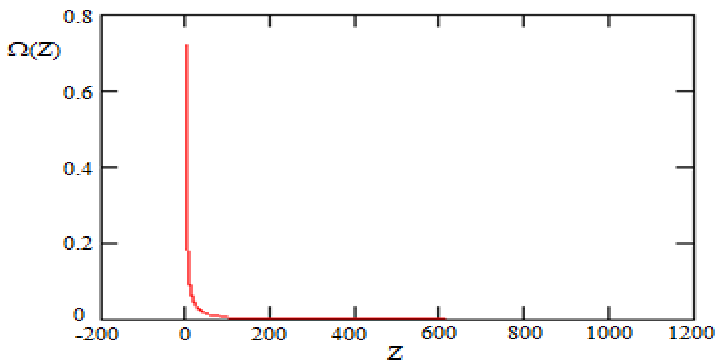

Fig.11 Graviton DE as a function of z on whole interval of redshifts z for Ω_0=0.694 .

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The behavior of $\Omega(z)$ on the interval $0 \le z \le 3$ is shown on Fig.12**.**

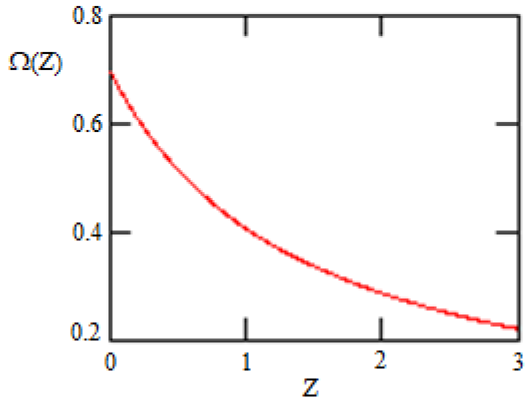

Fig.12. Same as Fig.11 but on the interval of interest 0<z<3 for Ω_0=0.694 .

The minimal statistical sums for this case are $S_{min\ DE}^{0.694}$=284.904 and $S^{0.694}_{min\ \Lambda}$=283.003, respectively. Obviously, $2S^{0.694}_{min\ \Lambda} - 2S^{0.694}_{min\ DE}$=1.901<<1$\sigma$, where 1$\sigma_\Lambda$=23.79 and 1$\sigma_{ED}$=23.87. Thus, in this case too we also have observational errors greater than the difference between models.

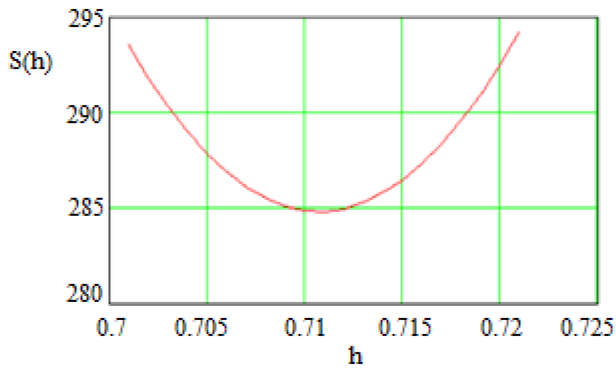

Fig.13 Statistical sum S(h) as function of h for the case of Ω_0=0.694. Here $S_{min\ DE}^{0.694}$=284.862 for h=0.711.

Thus, the results of this section confirm the fact that ΛCDM and GCDM models statistically indistinguishable. Also, they show that the Hubble constants h providing minimums for statistical sums are 0.709, 0.711, 0.714 for the cases considered which are close to the Freedman (2021) result Ho = 69.8 ± 0.6 (stat) ± 1.6 (sys) km/s Mpc.

5. What are the Prospects for the GCDM Testing and Comparison with ΛCDM model?
We believe that the supernova testing is remaining as the best tool for this task. Let us remind a quote from Riess et al. (2007), which is not outdated today. "SNe Ia remain one of our best tools for unraveling the properties of dark energy because their individual measurement precision is unparalleled and they are readily attainable in sample sizes of order 10^2, statistically sufficient to measure dark energy-induced changes to the expansion rate of ~1% ».

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We see at least two possible ways to test the difference between GCDM and Λ CDM models in the frame of supernova data. One of them is comparison of equations of state for GCDM and Λ CDM models. And the second one is comparison of transition redshifts at which the deceleration of expansion changes to acceleration.

Our program computes both energy density and pressure of GCDM from the start z_0 to the end when $\Omega(z)=\Omega_0$. So, we can get the dependence $p(\rho)$ for GCDM and compare it with $p=-\rho$ for Λ CDM model. The Fig. 13 shows this comparison for the Planck case $\Omega_0=0.685$.

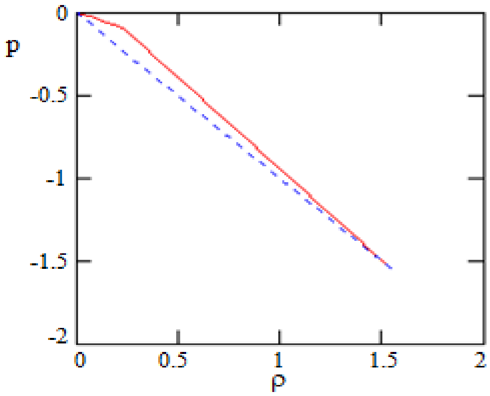

Fig. 14. The solid line is equation of state of the GCDM model and dash-line is that for Λ CDM. Both start from the same point where $p=-\rho$ (in accordance with our choice of initial conditions)

Obviously, to compare these two models we have to have w(z) determined observationally. Usual practice is to suggest that w=const and to look for this constant from observational data. In attempt to consider $w \neq \text{const}$, Planck Collaboration (Ade et al.,2013) tested the model $w=w_0+w_a(1-a)$ where a is a scale factor (Linder, 2003). In z terms it reads

$$w=w_0+w_a(1-a)=w_0+w_a\cdot\frac{z}{1+z} \qquad (15)$$

Ade et al (2013) have found

$$w_0=-1.04_{-0.69}^{+0.72};\quad w_a<1.32 \quad \text{(95\%; Planck +WP+BAO)} \qquad (16)$$

As we see from (16), w(0) can significantly differ from -1. Thus, even light modification of the model (15) can be used for the successful determination w(z) and comparison GCDM and ΛCDM models.

6. Transition Redshifts

Recall that the Friedmannian equation for deceleration/ acceleration in our case reads

$$q=-\frac{\ddot{a}}{aH^2}=\frac{\kappa}{6H^2}[\rho_{de}+3p_{de}+\rho_m(\frac{a_{today}}{a})^3] \qquad (17)$$

In the case of cosmological constant, $\kappa\rho_{de}=\Lambda, \kappa p_{de}=-\Lambda$, the eqn. (17) leads to the well-known result that the redshift $z_{q=0}$ where deceleration is changed for the acceleration, i.e. where $q=0$ reads

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$$1+z_{q=0}=\left(\frac{2\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{1/3} \quad (18)$$

For parameters $\Omega_\Lambda=0.72$ that we use in our simulations one gets $z_{q=0}\approx 0.73$. For the Planck parameters $\Omega_\Lambda=0.685$, we obtain $z_{q=0}\approx 0.63$. So, for the ΛCDM model one gets $z_{q=0}\approx 0.63\text{-}0.73$ considering a tension between WMAP and Planck data. For the GCDM model we have to expect (Marochnik, 2013)

$$1+z_{q=0}\approx\left(\frac{\Omega_0}{1-\Omega_0}\right)^{1/3} \quad (19)$$

Which leads to $z_{q=0}=0.37$ for WMAP data and $z_{q=0}=0.29$ for the Planck data. Now what we get from our numerical simulations. The Fig.15 shows the case $\Omega_0=0.72$

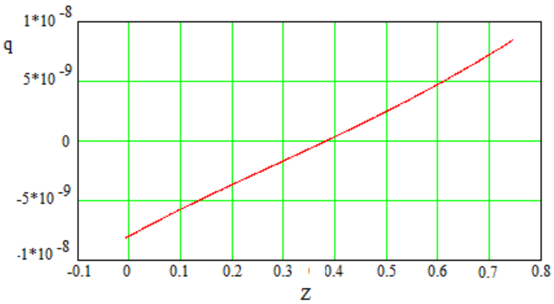

Fig. 15. One can see that the transition redshift is $z_{q=0}=0.38$ for $\Omega_0=0.72$.

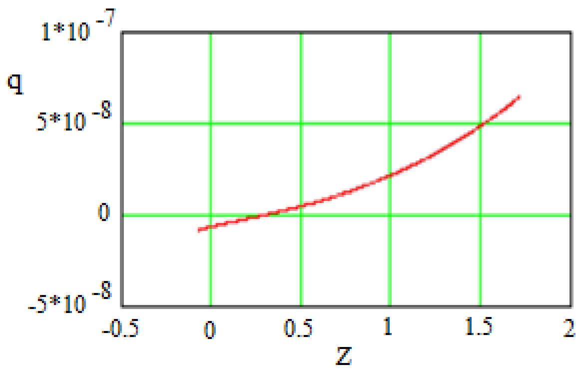

Fig. 16. The same as Fig 15 but for $\Omega_0=0.685$. One can see that $z_{q=0}=0.29$

Direct measurements of the transition point parameter $z_{q=0}$ were made by several authors (see below). All these works must be divided on two categories. The first group of the authors directly used the ΛCDM model in their studies. The second group of the authors considered the model-independent approach. As expected the first group have obtained $z_{q=0}$ close to numbers following from (31) (Blake et al (2012) $z_{q=0}<0.7$; Busca et al (2013) found 0.82 $\pm$0.08; Farooq & Ratra (2013) $z_{q=0}=0.74\pm0.05$; Sutherland and Rothnie (2015) $z_{q=0}=0.7$; Rani et al.(2015) $z_{q=0}=0.7$; Vitenti & Penna-Lima (2015) $z_{q=0}=0.65$). Daly et al. (2008)

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$z_{q=0} \approx 0.78_{-0.27}^{+0.08}$). The second group of authors (model-independent results) found that $z_{q=0}$ is much closer to our numbers. They are Daly & Djorgovski (2007) $z_{q=0} = 0.35 \pm 0.07$; Fa-Yin Wang and Zi-Gao Dai (2006) $z_{q=0} \approx 0.29_{-0.06}^{+0.07}$ Riess et al. (2007) $z_{q=0} \approx 0.43 \pm 0.07$; Shapiro& Turner (2006) $z_{q=0} \approx 0.3$; Moresco et al. (2016) $z_{q=0} = 0.4 \pm 0.1$ (99.9% confidence level.)
The observational determination of transition redshifts is an ill-defined problem due to the necessity to measure the second derivative of scale factor $\ddot{a}$. Such a procedure produces significant errors that can be seen from the above references. Nevertheless, the increase in accuracy with the model-independent approach to measuring transition redshifts can help to make a choice between these models

7. Conclusion

The GCDM model of graviton dark energy is consistent with the supernova observational data with the same accuracy as the ΛCDM model. The graviton DE naturally explains the reason why the DE appears during the matter dominated era. The transition redshifts that follow from graviton theory are consistent with the model independent transition redshifts found from observational data. The way to find out which of the models (GCDM or ΛCDM) better fits the observational data lies in the need to abandon the practically standard assumption that w=const and consider w=w(z) when analyzing observational data. Another prospect for evaluating the GCDM model and comparison with ΛCDM lies in increasing the accuracy of transition redshifts measurements.

Acknowledgment
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Appendix A. Outline of Graviton Theory of Cosmological Acceleration (MUV, 2008)

We operate in the framework of one–loop quantum gravity because the theory cannot be renormalized in higher loops. However, the effect of condensation of gravitons is created by general properties of gravitational field (conformal non–invariance and zero rest mass of gravitons)[6]. The results of one–loop theory in which only a gravitational field is considered, are mathematically robust due to the finiteness of one–loop quantum gravity. In our case, the finiteness is provided by the compensation of diverged contributions of gravitons and ghosts to observable quantities.

[6]Hypotheses on the possibility of graviton condensate formation in the Universe was proposed by Hu (2005) and Antoniadis, Mazur, and Mottola (2007) in a general form. A description of these effects by an adequate mathematical formalism is the problem at the present time

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Our original model of the empty Universe consists of the background and gravitons only[7]. In the self–consistent theory of gravitons, the macroscopic metric is described by regular Einstein equations

$$R_i^k - \frac{1}{2}\delta_i^k R = \kappa < \Psi | \hat{T}_{i(grav)}^k + \hat{T}_{i(ghost)}^k | \Psi > . \qquad \text{(A.1)}$$

The energy momentum tensor (EMT) of gravitons $\hat{T}_{i(grav)}^k$ and ghosts $\hat{T}_{i(ghost)}^k$ should be obtained by solving operator equations of motion and averaging over a quantum ensemble $|\Psi>$. Note the average EMT of nontrivial ghost fields interacting with gravity must appear in the right-hand side of (1) because there are no gauges that eliminate the diffeomorphism group degeneracy in the General Relativity. Our gauge selection was based on two principles. First, both background and gravitons should be considered in the same reference frame. Second, the gauge should provide automatically the one–loop finiteness. We found the only gauge that satisfies both conditions come from the set of synchronic gauges.

Our calculations were done in the frame of one–loop approximation over quantum fields. In the flat isotropic Universe, the equations (1) read

$$3H^2 = \rho_g \equiv \frac{1}{16}D + \frac{1}{4}W_1, \qquad -2\dot{H} - 3H^2 = p_g \equiv \frac{1}{16}D + \frac{1}{12}W_1, \qquad \text{(A.2)}$$

where $H = \dot{a}/a$ is the Hubble function and $a(t)$ is the scale factor. Here D and W_1 are moments of the spectral distribution function of gravitons that is renormalized by ghosts. The moments are:

$$D = \ddot{W}_0 + 3H\dot{W}_0,$$

$$W_m = \sum_{\mathbf{k}} \frac{k^{2m}}{a^{2m}} \left(\sum_{\sigma} < g | \hat{\psi}_{\mathbf{k}\sigma}^{+} \hat{\psi}_{\mathbf{k}\sigma} | g > - < gh | \bar{\theta}_{\mathbf{k}} \theta_{\mathbf{k}} | gh > \right), \qquad m = 0,1,2,...,\infty. \qquad \text{(A.3)}$$

Here and later the dots are time derivatives. Heisenberg's equations for Fourier components of the transverse 3–tensor graviton field and Grassman ghost field are:

$$\ddot{\hat{\psi}}_{\mathbf{k}\sigma} + 3H\dot{\hat{\psi}}_{\mathbf{k}\sigma} + \frac{k^2}{a^2}\hat{\psi}_{\mathbf{k}\sigma} = 0 \qquad (A.4.1)$$

$$\ddot{\theta}_{\mathbf{k}} + 3H\dot{\theta}_{\mathbf{k}} + \frac{k^2}{a^2}\theta_{\mathbf{k}} = 0 \qquad (A.4.2)$$

Considering of normalization of fields in accordance with (2) and (3), canonical commutation relations for gravitons and anticommutation relations for ghosts read

$$\frac{a^3}{4}\left[\dot{\hat{\psi}}_{\mathbf{k}\sigma}^{+}, \hat{\psi}_{\mathbf{k}'\sigma'}\right]_- = -i\hbar\delta_{\mathbf{kk}'}\delta_{\sigma\sigma'},$$

$$\frac{a^3}{8}\left[\dot{\bar{\theta}}_{\mathbf{k}}, \theta_{\mathbf{k}'}\right]_+ = -\frac{a^3}{8}\left[\dot{\theta}_{\mathbf{k}}, \bar{\theta}_{\mathbf{k}'}\right]_+ = -i\hbar\delta_{\mathbf{kk}'}. \qquad \text{(A.5)}$$

Equations (2, (3), (4) and quantization rules (5) have been obtained by the path integral (Faddeev and Popov, 1967; De Witt, 1967). They have been obtained from the class of synchronic gauges that automatically provide one–loop finiteness of observables. One–loop effects of vacuum

[7] In our current paper, we added the non-relativistic matter with the equation of state p=0 (see below)

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polarization and particle creation by background field are contained in equations (A.4) for gravitons and ghosts. These equations are linear in quantum fields, but their coefficients depend on the non–stationary background metric. Correspondingly, in the background equation (A.2) we keep the average values of bilinear forms of quantum fields only. In this model, quantum particles interact through a common self–consistent field only.

In this work, we are examining self–consistent theory of gravitons and ghosts with the wavelengths of the order of distance to the horizon, i.e.,

$$\frac{k^2}{a^2} \sim H^2, |\dot{H}|. \qquad \text{(A.6)}$$

Here $H = \dot{a}/a$ is the Hubble function, $a(t)$ is a scale factor, and upper dotes denote time derivatives. When describing modes (A.6), one should keep in mind two things. First, in the area of the spectrum (A.6), there are no reasonable approximations, which could be used to solve Friedmannian equations, if the law of cosmological expansion $a(t)$, $H(t)$ is not known in advance. Second, the quantum gravity processes of vacuum polarization, spontaneous graviton creation by self–consistent field and graviton–ghost condensation are the most intensive in this region of spectrum. From (A.6) it is also obvious that the threshold for quantum gravitational processes involving zero rest mass gravitons and ghosts is absent. These processes at the scale of horizon occur at any stage of evolution of the Universe, including the modern Universe. The theory that allows quantitatively describe such quantum gravitational effects is constructed by creation of the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy (BBGKY-chain). Now, the BBGKY chain can be created for the moments W_n of spectral function W_k.

Appendix B. Bogoliubov-Born-Green-Kirkwood-Yvon Hierarchy (BBGKY Chain)

In the presence of non-relativistic matter M it reads (MUV, 2008, 2013)

$$\dot{D} + 6HD + 4\dot{W}_1 + 16HW_1 = 0, \qquad (B.1)$$

$$\dddot{W}_n + 3(2n+3)H\ddot{W}_n + [\frac{1}{16}(4n^2+6n+3)D + (n+1)^2 W_1 + (8n^2+18n+9)\frac{M}{2a^3}]\dot{W}_n$$

$$+\frac{n}{3}\left\{\frac{1}{2}\dot{W}_1 + H\left[\frac{n^2}{2}D + (2n^2+3n+3)W_1 + (8n^2+18n+9)\frac{M}{a^3}\right]\right\}W_n$$

$$+4\dot{W}_{n+1} + 8(n+2)HW_{n+1} = 0 \qquad (B.2)$$

$$n = 1, \ldots, \infty. \qquad (B.3)$$

where M is mass of the non-relativistic matter in the Universe. Equations (B.1) and (B.2) form the BBGKY chain. Each equation of this chain connects the neighboring moments. Equations (B.1) and (B.2) must be solved jointly with the Einstein equations (A.2). The set of equations (A.2, B.1, B.2) (to which the definition $\dot{a}/a = H$ is added) can be solved numerically with initial conditions determined by the scale factor, moments of the spectral function and their derivatives

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$$D(0);\ W_n(0);\ \dot{W}_n(0);\ \ddot{W}_n(0) \qquad n=1,\dots,\infty. \qquad \text{(B.4)}$$

The initial condition for the Hubble function is calculated via the equation of the constraint following from (B.1, B.2)

$$H(0) = +\sqrt{\frac{1}{48}D(0) + \frac{1}{12}W_1(0) + \frac{M}{3}} \qquad \text{(B.5)}$$

As it follows from (B.4)), in a general case we must have an infinite array of initial conditions to solve the BBGKY chain. We can however to use one of the exact solutions (see below) which allows us to reduce the problem of initial conditions to just one parameter which is initial value for the energy density of DE (Section 5). For the empty space ($M=0$), the BBGKY chain has three exact solutions, which are the attractors (MUV, 2008). One of these attractors is the exact de Sitter solution to equations (B.1, B.2) for the empty ($M=0$) space. It reads

$$H^2 = \frac{1}{36}W_1, \quad D = -\frac{8}{3}W_1, \quad W_{n+1} = -\frac{n(2n+3)(n+3)}{2(n+2)}\mathrm{H}^2 W_n, \quad a = a_0 e^{\mathrm{Ht}} \quad n \ge 1 \qquad \text{(B.6)}$$

Where $H = const;\ W_n = const;\ D = const$

The substitution of (B.6) to (B.1, B.2)) leads to the equation of state of GCDM model, which reads

$$-\mathrm{p_g} = \rho_\mathrm{g} = \frac{\mathrm{W_1}}{12\kappa}\ \text{, i.e.,}\quad \mathrm{w_G} = \frac{\mathrm{p_g}}{\rho_\mathrm{g}} = -1 \qquad \text{(B.7)}$$

The explicit form of the equation of state reads (MUV, 2008)

$$-\mathrm{p_g} = \rho_\mathrm{g} = \frac{3\hbar \mathrm{N_g} \mathrm{H}^4}{8\pi^2} \qquad \text{(B.8)}$$

Here $\mathrm{N_g}$ is the number of gravitons in the modern Universe[8]. Note also that as was stated above we deal with the horizon size wavelengths. From (B.6) we get (MUV,2008)

$$\lambda \sim \frac{a}{\mathrm{k}} \sim \sqrt{\frac{\mathrm{W_1}}{|\mathrm{W_2}|}} = \frac{\sqrt{0.3}}{\mathrm{H}} \qquad \text{(B.9)}$$

The important consequences follow from (B.8). The equation of state (B.8) is invariant with respect to Wick rotation $\mathrm{t}=\mathrm{i}\tau$. It follows from the fact that RHS of (B.8) depends on the fourth degree of Hubble constant H^4 and the equation of state $-\mathrm{p_g} = \rho_\mathrm{g}$ is the invariant with respect to the transition from Lorentzian space of our space-time to the Euclidian spacetime and vice versa. This invariance underlies the instanton theory of dark energy (Marochnik, 2013).

Appendix C. Integration of BBGKY chain

As was shown by MUV (2008), the virtual gravitons produce the de Sitter expansion of the empty (with no matter) space (see also Appendix A). To investigate the model of the Universe filled with the matter we have to numerically integrate the BBGKY chain (Appendix B).

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[8] This equation of state is superficially similar to what comes from conformal anomalies. As was shown by Starobinsky (1980), quantum corrections to the Einstein equations due to zero oscillations can provide a self-consistent de Sitter solution in the vicinity Planck's value curvature (see also Zeldovich and Starobinsky, 1972).

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When the BBGKY chain is numerically integrated, we face two problems. The first is that the chain is infinite (i.e., in (B.1), (B.2) $n \to \infty$), so we should cut the BBGKY chain at some $n \le N$. The second problem is that the initial conditions needed to integrate equations (B.1) and (B.2) are unknown, so here we inevitably face the challenge of having to make a physical hypothesis about the initial conditions. When the BBGKY chain (B.1, B.2) is numerically integrated we need define the initial conditions at t=0, which comes to 3N+1 free parameters (the set of differential equations (B.1, B.2) requires 3N+1 initial conditions). Even with N=5 (our choice for the cutoff), it is an exceptionally large space to explore to find the best fit with observations. We decided to solve the problem of the large number of initial conditions by utilizing the exact de Sitter solution for the empty space (B.6), for which the BBGKY chain has only one free parameter Ω_{EDE} due to the recurrence in (B.6). As we will see further the EDE number is the only one initial condition that the computational program use for integration of (B.1, B. 2). We modified the BBGKY for the empty space, introducing a matter component as it can be seen from (B.2). We do not know the exact solution of the BBGKY chain with the matter component included, but we can integrate the chain numerically and see the result. The computer calculation shows that the model with the matter included still produces de Sitter-like behavior at small z, regardless of the small variations in initial matter component. So, the de Sitter behavior acts as an attractor. This result is understandable, because the end state is a state where the matter is no longer a significant player.

Regarding the choice of the number of terms in BBGKY chain, we found that the calculation of the N=5 is no longer much different from the calculation for the N=6 (the difference is about 2-3% which is much less than observational errors). That's why we're cutting off the integration chain on the N=5. We start calculations at the moment t=0 and take the initial scale factor $a(t=0) \equiv a(0)=1$. As we mentioned above, we continue the calculation until the calculated value of dimensionless energy density of DE, $\Omega(z)$, becomes equal to the current observational value of the energy density of DE $\Omega(z=0) = \Omega_0$. In what follows we omit the DE index, so that by the letter Ω_0, we denote the dimensionless content of DE at the end point of given calculation, and $\Omega(z)$ denotes the dimensionless content of DE at the current point z. Respectively, the energy density of nonrelativistic matter today is $\Omega_m = 1 - \Omega_0$. In accordance with (B.7), the energy density of DE is defined by the value of $W_1 = 12\kappa\rho_g$. So, variations of $W_1(0)$ are in fact variations of $\Omega^0{}_{DE}$. So that variations in EDE are the variations in the initial conditions for the integration of the set of differential equations (B.1, B.2). As it can be seen from Section 4, the initial value of $W_1(0)$ providing the best fit for the supernova observations for all cases of interest corresponds to such value of the EDE which is consistent with CMB anisotropy and located in the vicinity of SLS. The time in formulas above is defined in the time unit of the Hubble constant. We used time step of 4 years in the numerical integration, comprising to about 3 billion steps.

Appendix D. Why Now? Why the Dark Energy of Graviton Origin Should Appear in the Matter-Dominated Era?

The answer to the question “Why Now?” follows directly from the unique features of gravitational field, conformal non-invariance and zero rest mass of graviton (Marochnik, 2013).

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We first need to go back to the equation (A.4.1), which must be rewritten in terms of proper time $\eta=\int dt/a$. In the terms of proper time, (A.4.1) takes the form

$$\phi''+(k^2-a''/a)\phi=0 \quad \text{where } \phi=\psi/a \qquad \text{(D.1)}$$

prime symbol defines $d/d\eta$ and for simplicity $\kappa\sigma$ indexes operator signs were omitted in $\varphi_{k\sigma}$ and $\hat{\psi}_{k\sigma}$. There exist only two states of substances filling the Universe, the difference between which is not "felt" by the graviton. The first substance is the modern matter, the equation of state of which is $p=0$ and its expansion law is $a=const\cdot\eta^2$. The second substance is the dark energy with the equation of state $p=-\rho$ and expansion law $a=-1/(H\eta)$. Only in these two substances a''/a are the same: in both cases they are $a''/a=2/\eta^2$. This means the graviton equation (D.1) is the same for both $p=0$ and $p=-\rho$ substances. The two solutions form the basis of all solutions for the second order differential equation (D.1). This fact might explain why DE of graviton origin with the equation of state $p=-\rho$ "prefers" to appear in the modern universe filled with a matter with the equation of state $p=0$. Only from the present state of the universe with the equation of state $p=0$ do gravitons "freely pass" into the state of the de Sitter expansion with the equation of state $p=-\rho$, without "feeling" the difference between the regimes[9]. Note that the term a''/a is a consequence of conformal non-invariance of gravitational field.

As it was mentioned by Karwal & Kamionkowski (2016) and Poulin, Smith, Karwal and Kamionkowski (2019), the Hubble tension "might be explained by the presence of an exotic dark-energy density in the early Universe of the type that might arise in some of these ax verse scenarios". Note that the graviton dark energy can appear exactly at the "right time" when "the exotic dark energy" must appear to solve the Hubble tension problem. Therefore, for now, our choice is to limit ourselves to the EDE estimates based on the $\Omega^0{}_{DE}<0.009$ limit.

Appendix E. About Early Dark Energy

The cosmological history of the Universe is well-known. In accordance e.g., to Frieman, Turner and Huterer (2008), «the Universe has gone through three distinct eras: radiation dominated, $z\geq 3000$; matter-dominated $3000\geq z\geq 0.5$ and dark energy dominated $z\leq 0.5$». In terms of z, this means that the appearance of the "pure" matter-dominated era one can expects by $z\leq 1000$. In other words, the birth of DE takes place in the modern era of the evolution of the Universe and probably near $z\approx 1100$. However, there is nothing to prevent the DE from appearing anywhere in between $1100\geq z\geq 3.5$ (at $z_0<3.5$, the change in the sign of deceleration to acceleration ceases to satisfy the observational data, see Section 6). The following idea may serve as some argument

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in favor of the appearance of DE in the region $z \approx 1100$. There is no reason why DE should not appear at the first opportunity, i.e. in the area of $z \approx 1100$. Also note that $z \approx 1100$ is a special place because the surface of the last scattering (SLS) is situated at the same place (Hadzhiyska and Spergel, 2019). Thus, we can expect of the appearance of graviton DE in the vicinity of SLS. It is here the appearance of a noticeable amount of DE should be observable due to appearance of anisotropy in CMB. As it was mentioned by Ade et al. (2013) “The presence or absence of dark energy at the epoch of last scattering is the dominant effect on the CMB anisotropies”. Such early dark energy was named by EDE by Wetterich (2004). The analysis of CMB anisotropies produces the most precise bounds on EDE (Doran et al., 2001; Caldwell et al. 2003; Calabrese et al. 2011; Reichardt et al. 2012; Sievers et al. 2013; Hou et al. 2012; Pettorino et al. 2013). As noted by Ade et al. (2013), the upper limit on the EDE is $\Omega_{EDE} < 0.009$ (95%; Planck+WP+highL). Obviously, the initial value of DE that we use in our calculations $\Omega^0{}_{DE}$ corresponds in its meaning to the concept of early dark energy (EDE) introduced hypothetically in the listed above works. Therefore, we start our calculation assuming that the initial condition for DE at the start is below of 0.009. We start our numerical integration with the initial scale factor $a_0 = 1$. The computer "stops" at the moment when DE is equal to its present value $\Omega_{DE}(0) = \Omega_0$. At this moment, the scale factor is equal to its modern value $a = a_{today}$, and this means that the initial $z = z_0$ is equal to Therefore, for now, our choice is to limit ourselves to the EDE estimates based on the $\Omega^0{}_{DE} < 0.009$ limit.. While taking the estimation of Ω_0 from different groups of observers [Bennett et al, 2012; Ade et al., 2013; Abbot et al, 2021], we found that the initial z_0 are always grouped near the region of SLS (see Table in Section 4).

***The data underlying this article are available in the article and in supplementary material in* Supernova Cosmology Project (lbl.gov)**

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