
Article

Radial Power Spectral Characterizations of Near Surface Rainfall Rates Using Time-Height Vertically Pointing Doppler Radar Measurements

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Abstract: It has been shown that the Micro-Rain Radar (MRR) can be used to derive rainfall rates every 10 m over a depth of 1.28 km using the mean vertical air velocity corrected Doppler raindrop fall speed spectra. Furthermore, it has been shown that by assuming a reasonable advection velocity for the rain, these data can be analyzed to produce spatial radial power spectra often readily fit using a power function. Previous work has shown, however, that each spectrum applies only to each particular set of data and usually lacks the statistical qualifications necessary to be considered generally applicable. However, this limitation does not preclude the potential existence of other generalizations that can be used to explore the rainfall formation processes. The intent of this study, then, is to perform an initial look for such possible behaviors using time-height profiles of the rainfall rate. It is found that once the rainfall rate, R , exceeds about 20 mm h⁻¹, there is, apparently, an associated flattening of the spectra with increasing R so that the smaller scales play an ever increasingly important role in such rain near the ground perhaps reflecting the increasing importance of such scales in the formation of pockets of more intense convective rain. The true generality of this finding needs additional scrutiny using more data particularly from two spatially separated MRR as is currently under preparation.

Keywords: Raindrop size distributions (DSD) from Doppler radar; Computing radial power spectra using radar Doppler spectra; Vertical pointing Doppler rain observations.

1. Introduction

Gathering high spatial and temporally resolved measurements of the rainfall rate, R , is essentially impossible except by using a vertical pointing Doppler radar. There have been many attempts to collect such measurement, but a primary difficulty in the interpretation of the Doppler radar observations has been the effect of the vertical motion of the air that can significantly affect the determination of the drop sizes (e.g., [1] - [4]). Recent results [5] offer one solution found by shifting a Doppler spectrum until the drop size distribution deduced from this spectrum yields a calculated radar reflectivity factor, Z , equivalent to that observed while also satisfying the definition of the rainfall rate in still air. This shift then corresponds to the vertical air velocity to within the bin resolution of the spectrum itself. Using this technique, the effect of the mean vertical velocity has been removed from the spectrum so that the rainfall rate, R , can be calculated as has been done for the cases illustrated in [5] and reused here in addition to one more set of observations from a Micro-Rain Radar (MRR) operated at the College of Charleston.

The MRR radar is a continuous wave radar that operates at a wavelength of 1.24 cm as described in detail, for example, in [6]. At this wavelength, there is attenuation. However, looking vertically with a spatial resolution of 10 m, the radar observations only extend to 1.28 km. Jameson [7] found a near perfect correlation between the attenuation rate and the rainfall rate for gamma distributions of raindrops of one way attenuation rate A such that of A (dB km⁻¹) = 0.116 R (mm h⁻¹) so that attenuation is the most significant in the more intense rains and at greater distances. Nevertheless, however, this can be a problem, at times, in convective rain. In [5] this difficulty was

addressed by comparing the observed attenuated radar reflectivity Z_a to the reported effective reflectivity factor Z_e in the MRR output over all the observations which also included several measurements not significantly affected by attenuation, as discussed further in the second Appendix in that paper. As a consequence we have the rainfall rates estimated every 10 m up to 1.28 km every 10 seconds as illustrated, for example, for one time period in Figure.1 (from [5]).

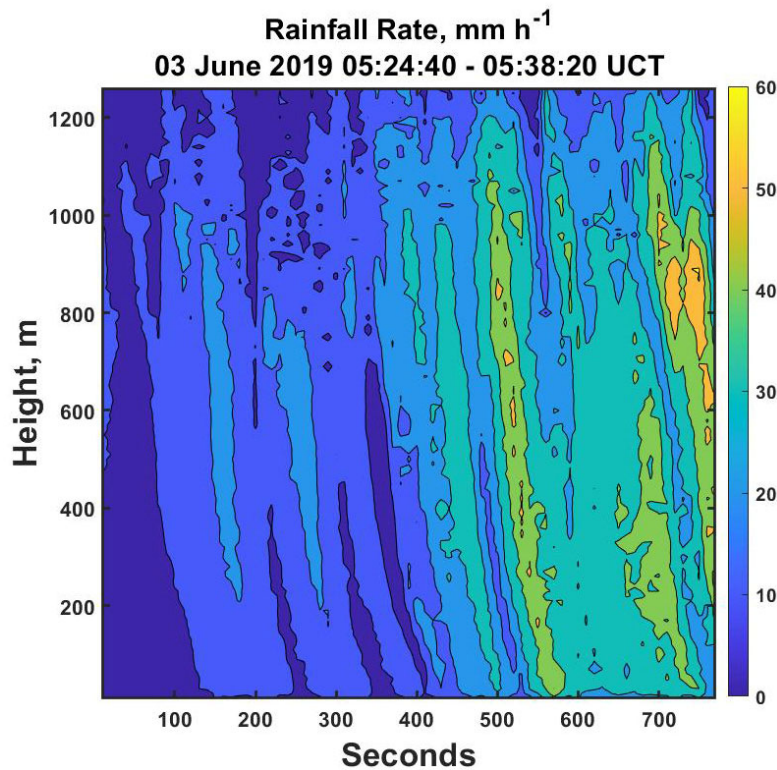


Figure 1. Time-height plot of the base 10 logarithm of the vertical air velocity corrected Doppler spectra rainfall rate using data from a NASA Wallop's Island MRR radar (adapted from [5]).

By assuming that the temporal scale can be associated with a distance scale, say, through a constant advection speed, one can Fourier transform such data into the two dimensional spatial power spectrum using any convenient program such as the `fft2` function in Matlab[®]. In addition, one can, in turn, compute the one dimensional radial power spectrum function by first transforming the height-horizontal distance plane of the FFT (Δx , Δy) into polar coordinates (Δr , $\Delta \theta$) where Δr and $\Delta \theta$ are the measures of the distances and angles between pairs of FFT points. By then integrating over all the $\Delta \theta$ for each Δr , one can derive the radial power spectrum function as was done in [7].

However, if one wishes to explore the changes in the radial power spectrum with distance (or, as in this study, equivalently, the temporal evolution using an advection velocity), additional steps are required. Suppose that the observations between a pair of 10 s observations separated by 70 s represents, instead, a pair of observations by two different MRR separated horizontally by 490 m where we are assuming an advection velocity transformation between time and space of 7 m s⁻¹, well within the wide range of values found in the literature but admittedly arbitrary. The motivation for doing this is that we anticipate simultaneous, spatial measurements by two MRR as part of our current NSF grants so that this study assists the development of the necessary analyses techniques for analyzing such future observations. Nevertheless, it will be shown here that the analyses just using these proxy data reveal some interesting results as discussed toward the end of this paper.

Specifically, we use the Matlab® program interp2 to produce a field of interpolated values at spacings in the horizontal and vertical directions of 7.5 m and 0.16 m, respectively. These fields are then processed to yield the radial power function corresponding to each pair as just described above. The right-most data are then moved to be the left-most and the next 10 s observation is taken to be the right-most data. This, then, is taken to be equivalent to processing a time series of 10 s observations by two spatially separated instruments. These data are then processed as for the previous pair of data and this whole process is repeated until all pairs are used. In the next section, the results are shown for four different sets of measurements using two different MRR.

2. Examples of data processing of proxy measurements by two spatially separated vertically pointing MRR

We begin using the observations reported in Figure 1 by focusing on the segment between 700 and 770 seconds. These data are replotted in Figure 2a with the corresponding interpolated data in Figure 2b. Clearly, the interpolated data do not perfectly match the observations. This is not surprising since the interpolation is only between the two observations, one at 700 and the other at 770 s, so that what actually lies in between is unknown. This will always be true with actual observations using two spatially separated radars as well. However, we are not so much interested in the appearance of the data as we are in its statistical properties as represented by the radial power function. The question, then, is the radial power function still preserved in spite of these effects of interpolation?

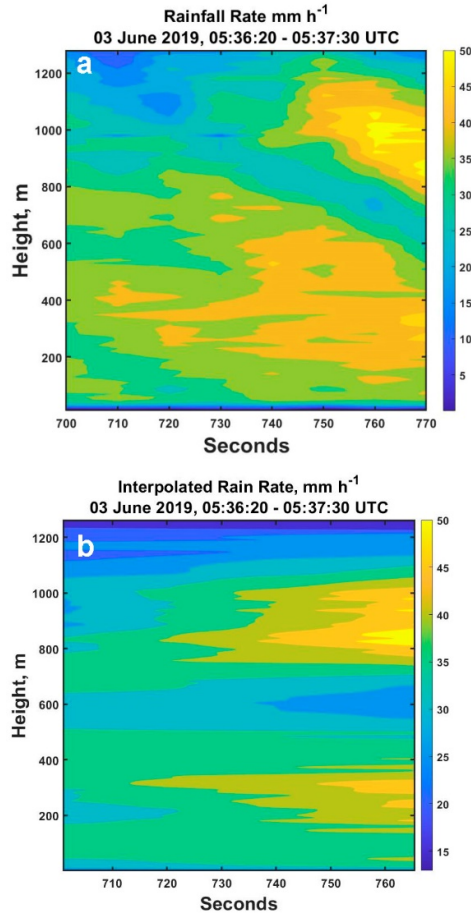


Figure 2. (a) Plot of the actual time height rainfall rates just between 700 and 770 seconds from Figure.1 and (b) the values from the interpolation of the observations at 700 s and 770 seconds alone as discussed in the text. The two fields obviously differ as one would expect.

When we perform the calculation for the power spectrum as just described in the previous section at the resolution of the actual observations, we have the results in Figure 3a showing that by and large the two power spectra are quite similar and seem to exhibit similar shapes. Of course there will be deviations because of the incomplete information within the interpolated field as illustrated in Figure 3b where the power fit of the data is now at the resolution of the interpolated field, namely at spacings in the horizontal and vertical directions of 7.5 m and 0.16 m, respectively. While in reasonable agreement, there is now a small difference of 0.29 between the exponents of the two fits, likely as a consequence of the finer resolution of the interpolated data leading to the introduction of more smaller scale components.

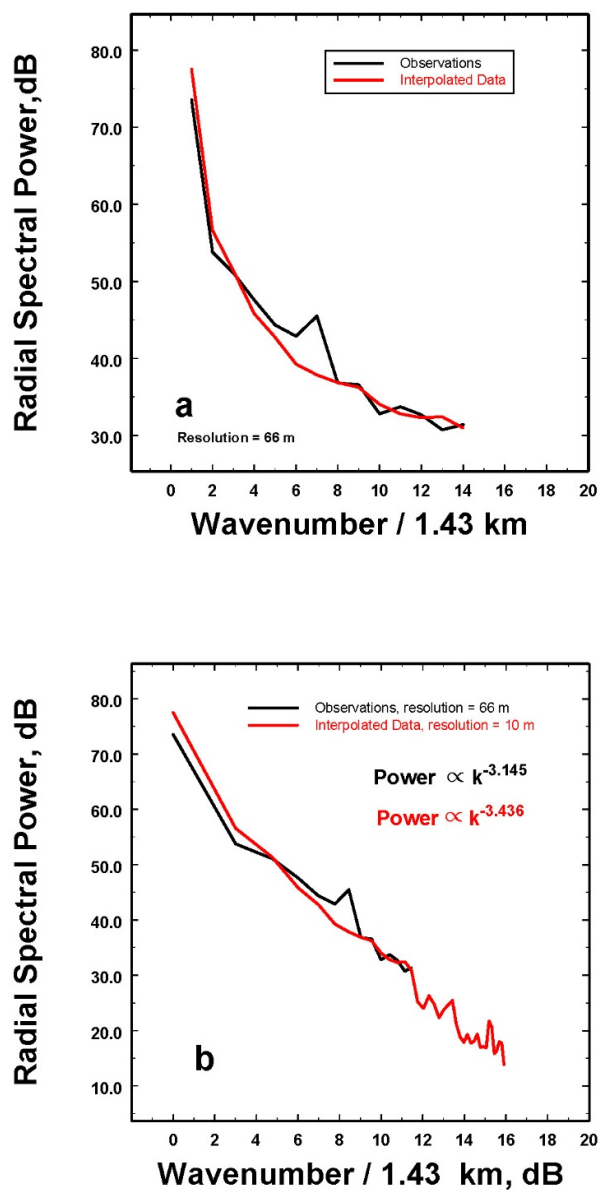


Figure 3. (a) Plots of the radial power spectra for the observed and interpolated data in Figure 1 at the resolution consistent with the observations and for an advection velocity of 7 m s^{-1} . The same as in (a) except for a spatial resolution consistent with the finer scale interpolated values. The two plots show that in spite of the differences in Fig. 1, the power spectra remain quite similar with a larger value of the absolute value of the exponent due to the finer scales in the interpolated field.

Because any two pairs of observations will always miss data in between, it is essential to use the full time series of such pairs of observations so as to reduce the loss of information. The result of doing so is illustrated for these data in Figure 4 where the time series of the logarithm of the radial spectral powers are plotted and contoured in time (distance if you multiply by 7 m s^{-1}) along with a plot of the exponents of the power fits to the radial powers (red line). To our knowledge, this is the first time such data have ever been calculated or displayed.

Systematic changes in the slopes of the exponents of power fits to the radial power spectra are evident. There are then two questions. How are these changes related to the properties of the power spectra and how might they be related to the rainfall rate observations? The first question appears to be readily addressed. In this example, between 200-700 s, the powers in the larger wavenumbers (smaller scales) remains approximately constant while the powers in smaller wavenumbers (longer scales) increases so the slopes of the power fits tend to increase. With regard to the rainfall rate, this, in turn, suggests in this example that higher rainfall rates are associated with ρ having steeper slopes, that is that larger scales were more important in the formation of the heavier rain in this case. These and the results in the next two examples were all from a rain event associated with the passage of a cold front over Wallop's Island Virginia so that, perhaps, it makes sense that longer scales play an important role because of the larger scale dynamics involved. This is, of course, just speculation, and as we shall see, is not always the case.

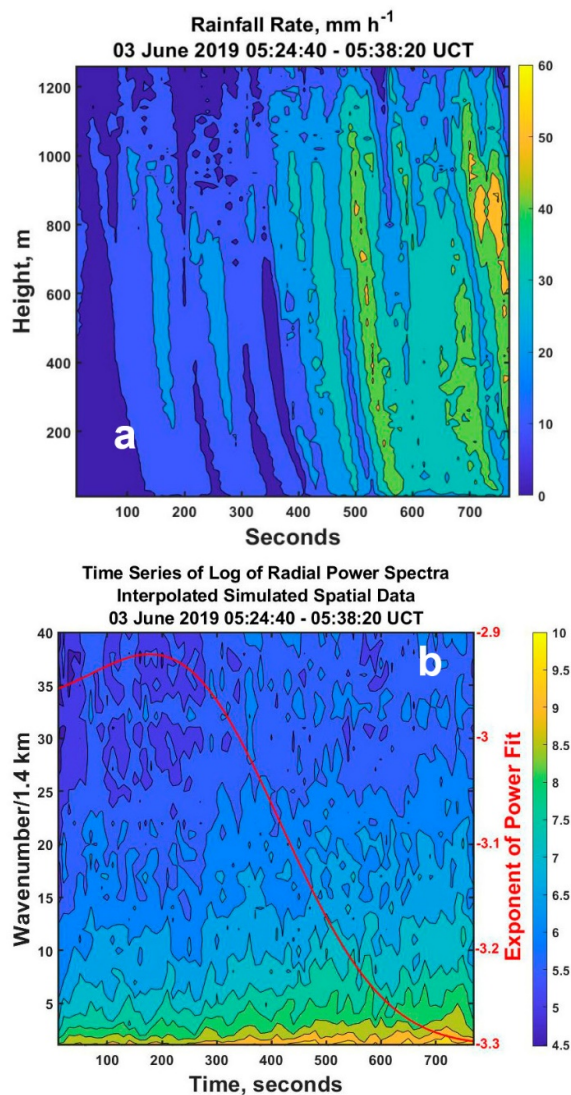


Figure 4. The same as Figure.1, namely, the time-height plot of the base 10 logarithm of the vertical air velocity corrected Doppler spectra rainfall rate. (b) Contour plot of the Log_{10} of the radial spectral powers versus the wavenumber with an overlay of the exponents over all the 83 radial power spectra as discussed further in the text.

Moreover, for some earlier data associated with this meteorological situation, the same trends appear as shown in Figure.5. Again it appears that the exponent of the power fit is determined by the relative contributions at these different wavenumbers. During the more intense rain between 400 s– 720 s, this ratio of power from the lower wavenumbers ($\kappa \sim 4$, longer wavelengths) to that at larger wavenumbers ($\kappa \sim 100$, smaller wavelengths) is relatively steady at about 50 dB while between about 720 s – 850 s this ratio changes substantially by dropping down to 37 dB. Consequently, during the lighter rain, the power at large wavelengths decreases with respect to the values at small wavelengths, and the the slope of the power law decreases.. Therefore, it appears, as before in these meteorological conditions, that the longer wavelengths play a more active role in the formation of the intense precipitation, while the opposite is true during the lighter rain when the smaller scales appear to play a greater role perhaps due to the more spatially granular nature of anvil precipitation from a convective storm.

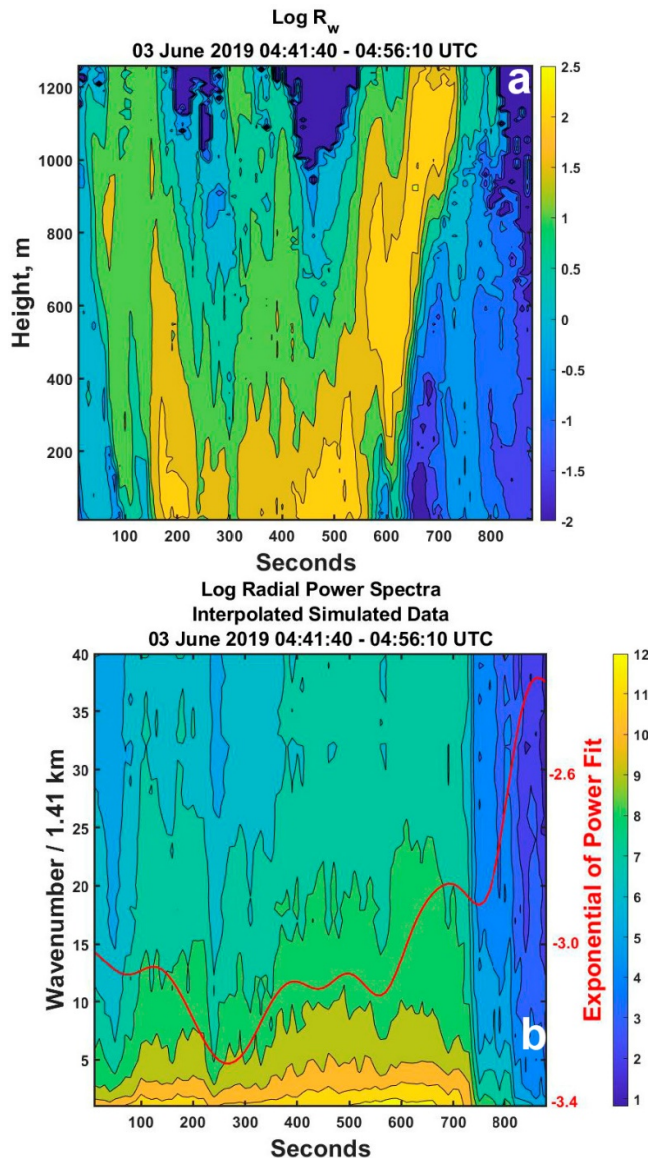


Figure 5. (a) The time-height profile measured by the NASA MRR radar at Wallop’s Island, Virginia (adapted from [5]).(b) Contour plot of the Log_{10} of the radial spectral powers versus the wavenumber with an overlay of the exponents over all the 88 radial power spectra as in the previous figure and as discussed further in the text.

The same trends also appear (Figure 6) between the previous two periods. Here, too, the slopes appear to change systematically in response to the power ratio in a manner similar to the previous two examples even though the rainfall rates are all much lighter. Hence, for this period, it is also reasonable to conclude that the more intense rains, although considerably weaker, are still associated with longer scales, while, again, the opposite appears during the very light rain. Consequently, for this set of observations, the relation the behavior between the slopes and rainfall rate intensity appear to show consistent behavior perhaps because of the dominant role played by the larger scale dynamic system associated with a passing cold front. However, this need not always be the case as we show next.

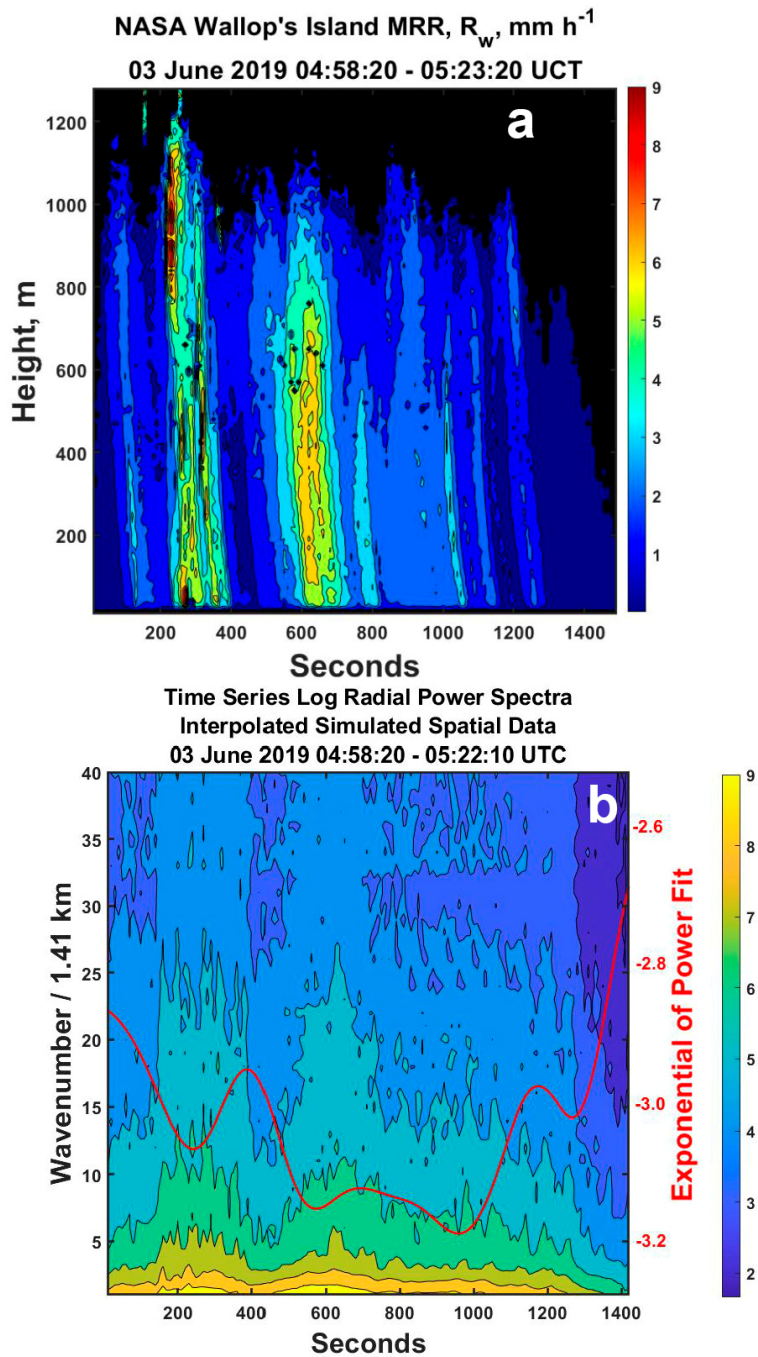


Figure 6. Similar to Figures 4 and 5 except for the middle time period between the two times in those figures. This time there are 142 power spectra represented in (b).

Isolated summertime convective storms are more likely dominated by the smaller scales of local dynamics associated with differential surface heating as illustrated for a storm in Figure 7 near Charleston, S.C. As Figure 8 shows, the slopes of the power fits are still determined by the relative contributions to the power spectra by the longer as compared to the shorter wavelengths. Yet as Figure 7 illustrates, the response of the rainfall rates to the exponent of the power fits is now just the opposite from the previous case with flatter spectra associated with the more intense rain and steeper slopes occurring when the rain is less intense. Perhaps this suggests that in these kinds of more isolated convective storms, it is now the smaller scales that dominate the formation of the more intense rain through localized microphysical processes.

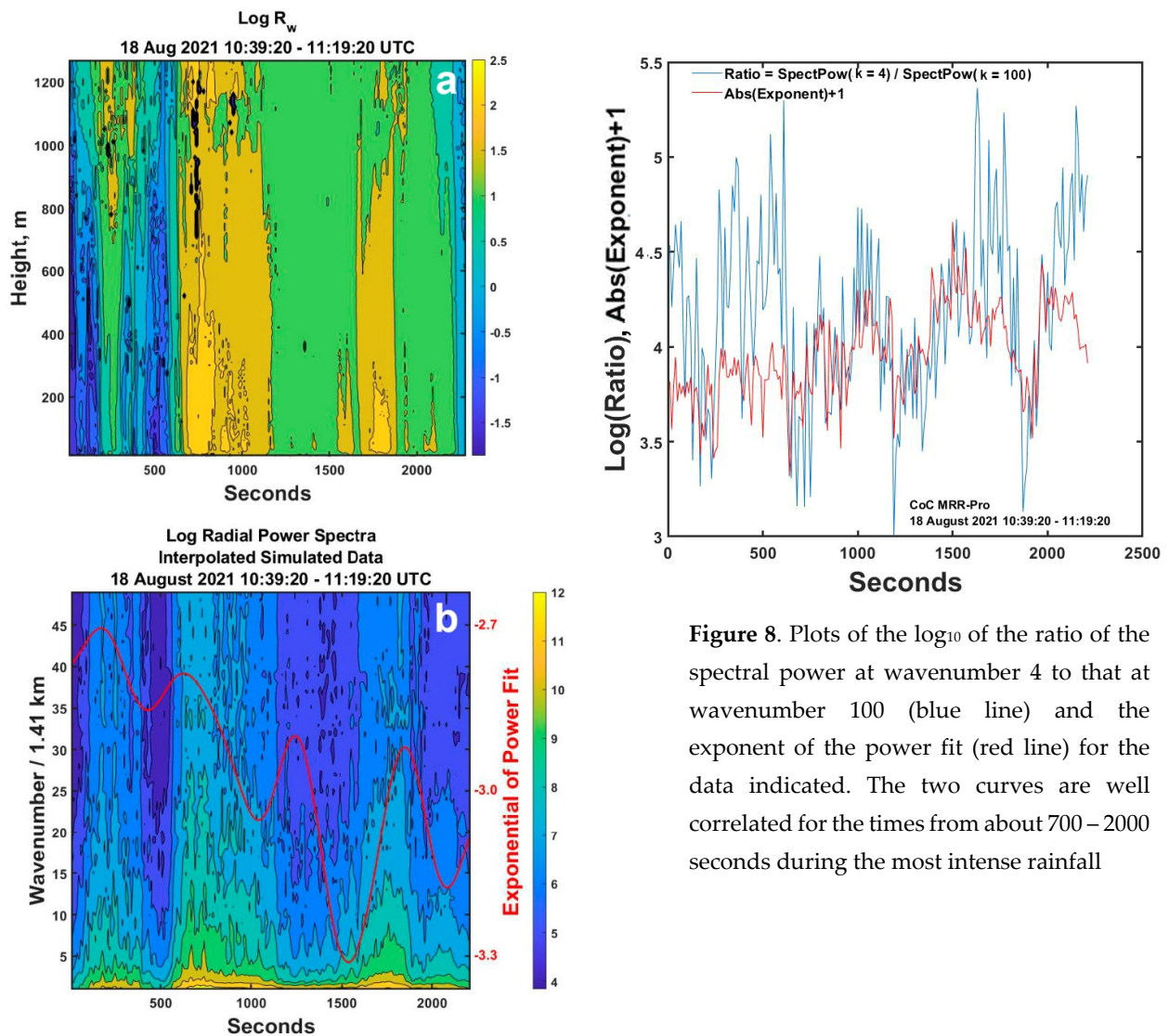


Figure 8. Plots of the \log_{10} of the ratio of the spectral power at wavenumber 4 to that at wavenumber 100 (blue line) and the exponent of the power fit (red line) for the data indicated. The two curves are well correlated for the times from about 700 – 2000 seconds during the most intense rainfall

Figure 7. Similar to the previous figures except the data are from the College of Charleston MRR radar at a different location and date compared to the previous figures. This time 221 radial power spectra are plotted.

In any event, the contrast in these cases is intriguing and suggests the need for much more extensive observations in order to explore further results such as these. Yet even though these results differ, might there still be possible relationships between the 10 s mean values of the spectral power intensities, the mean rainfall rates and the mean slopes? We explore this next by considering simultaneously all of the 535 values from all of these data.

These results are illustrated in Figure 9. These data were interpolated to a uniform grid using the method of [9]. Not surprisingly, the total spectral power increases with the mean rainfall rate in the column. It is also apparent that, at least for these data, once R exceeds about 20 – 25 mm h⁻¹, the exponents begin to decrease systematically in absolute magnitude from around 3.1 toward 2.8 with increasing R . This trend suggests that often for the greater R , the smaller scales become increasingly important to the production of intense rainfall. While one can speculate as to why this might be true, this observation needs further corroboration and research into potential reasons for this intriguing result.

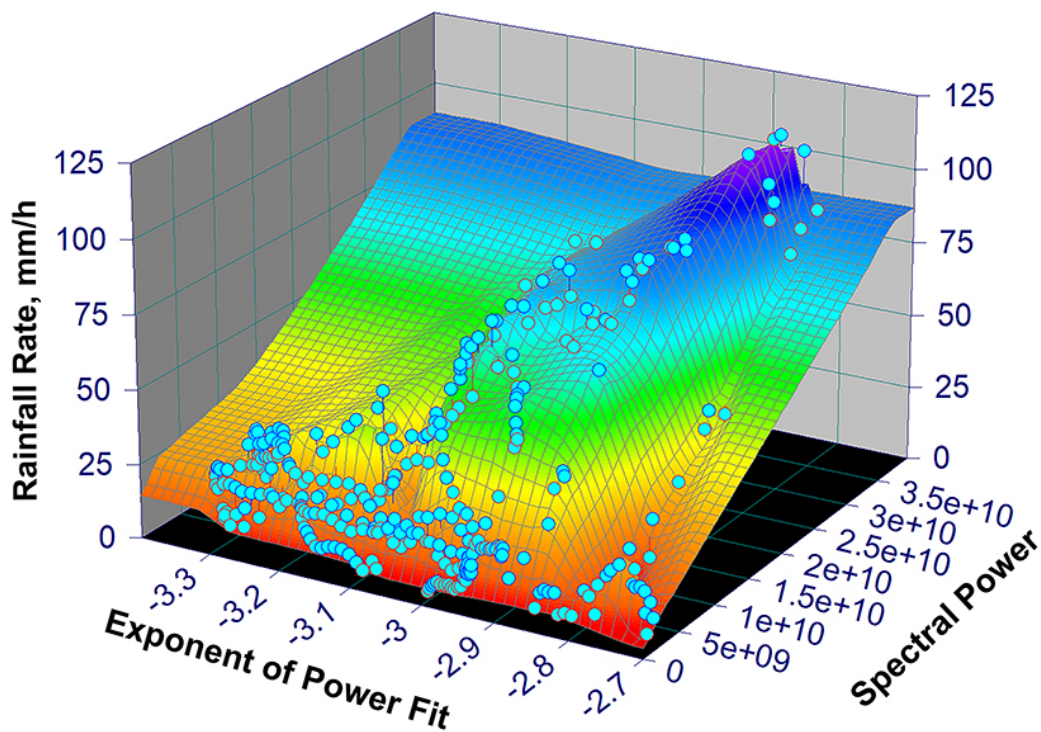


Figure 9. A surface plot of all of the 10 s mean values of the data in Figures 4 – 7. Note that once the mean rainfall rate is larger than around 20 – 25 mm h⁻¹, there is a systematic decrease in the magnitude of the mean exponent suggesting an increasingly important role of smaller scales as the rainfall rate increases as discussed further in the text.

3. Conclusions

Using Micro-Rain Radar (MRR) Doppler radar measurements and the technique reported in [5] to correct for the mean vertical air velocity and attenuation, time height profiles of rainfall rates were calculated which could then be statistically analyzed for four different times using two different MRR every 10 m up to 1.28 km height and at every 10 s. Recent statistical analyses of these data [9] revealed that these data were all essentially statistically heterogeneous so that the only mathematically and physically meaningful quantities were the radial power functions derived from the 2D power distributions as just described in this work as well. These functions were then fit using a power function with a negative exponent lying between 2.5 to 3.3 where the smaller values were sometimes associated with heavier rain and sometimes in the opposite sense with lighter rain so that no clear trend was immediately apparent in the 10 s data.

However, in an attempt to add some clarity, rather than just considering each individual case at a time (Figures 4 – 7), we next considered all of the data simultaneously. This was done by computing the mean R , the total radial spectral powers along and the derived exponents of the power fits every 10 s. The results thus far from this limited set of observations seems to suggest that as the rainfall rate increases, small scale structures appear to play an increasingly important role in generating intense rainfall. Perhaps this is not too surprising given the microphysics that would be required to concentrate large quantities of moisture in a wet atmosphere. Obviously, however, more work is required to investigate these preliminary results using much larger and varied sets of similar observations in a wide variety of meteorological conditions. We intend to pursue this, hopefully in the immediate future, using measurements from two spatially separated MRR instruments.

Author Contributions: There are two authors who contributed to this work, namely A. R. Jameson (AJ), M. L. Larsen (ML). AJ devised and developed this idea, ML provided some of the data associated with the College of Charleston, carefully checked the science and made important suggestions.

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Data Availability Statement The data and Matlab programs are at <https://data.mendeley.com/datasets/rfw8h7h2pk/3>

Conflicts of Interest: We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome. All of the sources of funding for the work described in this publication are acknowledged above.

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