

# Chromatic Number and Neutrosophic Chromatic Number

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## Abstract

New setting is introduced to study chromatic number. Neutrosophic chromatic number and chromatic number are proposed in this way, some results are obtained. Classes of neutrosophic graphs are used to obtain these numbers and the representatives of the colors. Using colors to assign to the vertices of neutrosophic graphs is applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using strong edge to define the relation amid vertices which implies having different colors amid them and as consequences, choosing one vertex as a representative of each color to use them in a set of representatives and finally, using neutrosophic cardinality of this set to compute neutrosophic chromatic number. This specific relation amid edges is necessary to compute both chromatic number concerning the number of representative in the set of representatives and neutrosophic chromatic number concerning neutrosophic cardinality of set of representatives. If two vertices have no strong edge, then they can be assigned to same color even they've common edge. Basic familiarities with neutrosophic graph theory and graph theory are proposed for this article.

**Keywords:** Neutrosophic Strong, Neutrosophic Graphs, Chromatic Number

**AMS Subject Classification:** 05C17, 05C22, 05E45

## 1 Background

Fuzzy set in **Ref. [15]**, neutrosophic set in **Ref. [2]**, related definitions of other sets in **Refs. [2, 13, 14]**, graphs and new notions on them in **Refs. [5–11]**, neutrosophic graphs in **Ref. [3]**, studies on neutrosophic graphs in **Ref. [1]**, relevant definitions of other graphs based on fuzzy graphs in **Ref. [12]**, related definitions of other graphs based on neutrosophic graphs in **Ref. [4]**, are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

### 1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

**Question 1.1.** *Is it possible to use mixed versions of ideas concerning “neutrosophic strong edges”, “neutrosophic graphs” and “neutrosophic coloring” to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors. Thus they're used to define new ideas which conclude to the structure of coloring. The concept of having strong edge inspire to study the behavior of strong edge in the way that, both neutrosophic chromatic number and chromatic number are the cases of study.

The framework of this study is as follows. In section (1.2), I introduce basic definitions to clarify about preliminaries. In section (2), new notion of coloring is applied to the vertices of neutrosophic graphs. Neutrosophic strong edge has the key role in this way. Classes of neutrosophic graphs are studied when the edges are neutrosophic strong. In section (3), one application is posed for neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section (4), some problems and questions for further studies are proposed. In section (5), gentle discussion about results and applications are featured. In section (5), a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

## 1.2 Preliminaries

**Definition 1.2.**  $G : (V, E)$  is called a **crisp graph** where  $V$  is a set of objects and  $E$  is a subset of  $V \times V$  such that this subset is symmetric.

**Definition 1.3.** A crisp graph  $G : (V, E)$  is called a **neutrosophic graph**  $G : (\sigma, \mu)$  where  $\sigma = (\sigma_1, \sigma_2, \sigma_3) : V \rightarrow [0, 1]$  and  $\mu = (\mu_1, \mu_2, \mu_3) : E \rightarrow [0, 1]$  such that  $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$  for all  $xy \in E$ .

**Definition 1.4.** A neutrosophic graph is called **neutrosophic empty** if it has no edge. It's also called **neutrosophic trivial**. A neutrosophic graph which isn't neutrosophic empty, is called **neutrosophic nontrivial**.

**Definition 1.5.** A neutrosophic graph  $G : (\sigma, \mu)$  is called a **neutrosophic complete** where it's complete and  $\mu(xy) = \sigma(x) \wedge \sigma(y)$  for all  $xy \in E$ .

**Definition 1.6.** A neutrosophic graph  $G : (\sigma, \mu)$  is called a **neutrosophic strong** where  $\mu(xy) = \sigma(x) \wedge \sigma(y)$  for all  $xy \in E$ .

**Definition 1.7.** A path  $v_0, v_1, \dots, v_n$  is called **neutrosophic path** where  $\mu(v_i v_{i+1}) > 0$ ,  $i = 0, 1, \dots, n - 1$ .  $i$ -path is a path with  $i$  edges, it's also called **length** of path.

**Definition 1.8.** A crisp cycle  $v_0, v_1, \dots, v_n, v_0$  is called **neutrosophic cycle** where there are two edges  $xy$  and  $uv$  such that  $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\dots,n-1} \mu(v_i v_{i+1})$ .

**Definition 1.9.** A neutrosophic graph is called **neutrosophic t-partite** if  $V$  is partitioned to  $t$  parts,  $V_1, V_2, \dots, V_t$  and the edge  $xy$  implies  $x \in V_i$  and  $y \in V_j$  where  $i \neq j$ . If it's neutrosophic complete, then it's denoted by  $K_{\sigma_1, \sigma_2, \dots, \sigma_t}$  where  $\sigma_i$  is  $\sigma$  on  $V_i$  instead  $V$  which mean  $x \notin V_i$  induces  $\sigma_i(x) = 0$ . If  $t = 2$ , then it's called **neutrosophic complete bipartite** and it's denoted by  $K_{\sigma_1, \sigma_2}$  especially, if  $|V_1| = 1$ , then it's called **neutrosophic star** and it's denoted by  $S_{1, \sigma_2}$ . In this case, the vertex in  $V_1$  is called **center** and if a vertex joins to all vertices of neutrosophic cycle, it's called **neutrosophic wheel** and it's denoted by  $W_{1, \sigma_2}$ .

**Definition 1.10.** Let  $G : (\sigma, \mu)$  be a neutrosophic graph. For any given subset  $N$  of  $V$ ,  $\sum_{n \in N} \sigma(n)$  is called **neutrosophic cardinality** of  $N$  and it's denoted by  $|N|_n$ .

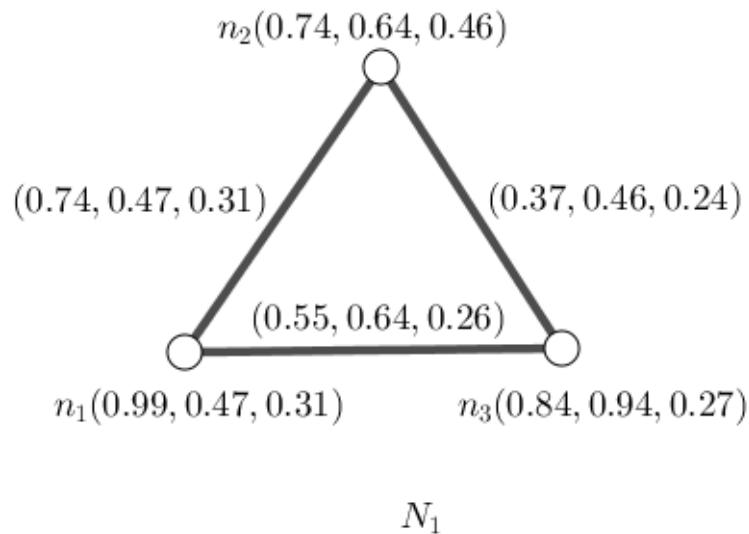
**Definition 1.11.** Let  $G : (\sigma, \mu)$  be a neutrosophic graph. Neutrosophic cardinality of  $V$  is called **neutrosophic order** of  $G$  and it's denoted by  $O_n(G)$ .

**Definition 1.12.** Let  $G : (\sigma, \mu)$  be a neutrosophic graph. The number of vertices is denoted by  $n$  and the number of edges is denoted by  $m$ .

**Definition 1.13.** Let  $N = (\sigma, \mu)$  be a neutrosophic graph. It's called **neutrosophic connected** if for every given couple of vertices, there's at least one neutrosophic path amid them.

**Definition 1.14.** Let  $N = (\sigma, \mu)$  be a neutrosophic graph. Suppose a path  $P : v_0, v_1, \dots, v_{n-1}, v_n$  from  $v_0$  to  $v_n$ .  $\min_{i=0,1,2,\dots,n-1} \mu(v_i v_{i+1})$  is called **neutrosophic strength** of  $P$  and it's denoted by  $\mathcal{S}_n(P)$ .

**Definition 1.15.** Let  $N = (\sigma, \mu)$  be a neutrosophic graph. The number of maximum edges for a vertex, amid all vertices, is denoted by  $\Delta(N)$ .



**Figure 1.** Neutrosophic Graph,  $N_1$

## 2 Chromatic Number and Neutrosophic Chromatic Number

**Definition 2.1.** Let  $N = (\sigma, \mu)$  be a neutrosophic graph. **Chromatic number** is minimum number of distinct colors which are used to color the vertices which have neutrosophic strong edge. Neutrosophic cardinality of the set of these distinct colors when it's minimum amid all of these sets, is called **neutrosophic chromatic number** with respect with first order.

**Example 2.2.** Consider Figure (1). The chromatic number is three and neutrosophic chromatic number is 2.57 with respect to first order.

**Proposition 2.3.** Let  $N = (\sigma, \mu)$  be a neutrosophic complete. Then chromatic number is  $n$  and neutrosophic chromatic number is neutrosophic order.

*Proof.* All edges are neutrosophic strong. Every vertex has edge with  $n - 1$  vertices. Thus  $n$  is chromatic number. Since any given vertex has different color in comparison to another vertex, neutrosophic cardinality of  $V$  is neutrosophic chromatic number. Therefore, neutrosophic chromatic number is neutrosophic order.  $\square$

**Proposition 2.4.** Let  $N = (\sigma, \mu)$  be a neutrosophic strong path. Then chromatic number is two and neutrosophic chromatic number is

$$\min_{x \text{ and } y \text{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

*Proof.* With alternative colors, neutrosophic strong path has distinct color for every vertices which have one edge in common. Thus if  $x$  and  $y$  are two vertices which have one edge in common, then  $x$  and  $y$  have different color. Therefore, chromatic number is two. The representative of colors are a vertex with minimum value amid all vertices which have same color with it. Thus,

$$\min_{x \text{ and } y \text{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

□ 85

**Proposition 2.5.** Let  $N = (\sigma, \mu)$  be an even neutrosophic strong cycle. Then chromatic number is two and neutrosophic chromatic number is

$$\min_{x \text{ and } y \text{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

*Proof.* All edges are neutrosophic strong. Since the cycle has even vertices, with alternative coloring of vertices, the vertices which have common edge, have different colors. So chromatic number is two. With every color, the vertex which has minimum value amid vertices with same color with it, is representative of that color. Thus,

$$\min_{x \text{ and } y \text{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

□ 86

**Proposition 2.6.** Let  $N = (\sigma, \mu)$  be an odd neutrosophic strong cycle. Then chromatic number is three and neutrosophic chromatic number is

$$\min_{x, y \text{ and } z \text{ have different colors}} \{\sigma(x) + \sigma(y) + \sigma(z)\}.$$

*Proof.* With alternative coloring on vertices, at end, two vertices have same color, and they've same edge. So, chromatic number is three. Since the colors are three, the vertices with minimum values in every color, are representatives. Hence,

$$\min_{x, y \text{ and } z \text{ have different colors}} \{\sigma(x) + \sigma(y) + \sigma(z)\}.$$

□ 87

**Proposition 2.7.** Let  $N = (\sigma, \mu)$  be a neutrosophic strong star with  $c$  as center. Then chromatic number is two and neutrosophic chromatic number is

$$\min_{x \text{ is non-center vertex}} \{\sigma(c) + \sigma(x)\}.$$

*Proof.* All edges are neutrosophic strong. Center vertex has common edge with every given vertex. So it has different color in comparison to other vertices. So one color has only one vertex which has that color. All non-center vertices have no common edge amid each other. Then they've same color. The representative of this color is a non-center vertex which has minimum value amid all non-center vertices. Hence,

$$\min_{x \text{ is non-center vertex}} \{\sigma(c) + \sigma(x)\}.$$

□ 88

**Proposition 2.8.** Let  $N = (\sigma, \mu)$  be a neutrosophic strong wheel with  $c$  as center. Then chromatic number is three where neutrosophic cycle has even number as its length and neutrosophic chromatic number is

$$\min_{x,y \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y)\}.$$

*Proof.* Center vertex has unique color. So it's only representative of this color. Non-center vertices form a neutrosophic cycle which have distinct colors for the vertices which have common edge with each other when the number of colors is two. So a color for center vertex and two colors for non-center vertices, make neutrosophic strong wheel has distinct colors for vertices which have common edge. Hence, chromatic number is three when the non-center vertices form odd cycle. Therefore,

$$\min_{x,y \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y)\}.$$

□ 89

**Proposition 2.9.** Let  $N = (\sigma, \mu)$  be a neutrosophic strong wheel with  $c$  as center. Then chromatic number is four where neutrosophic cycle has odd number as its length and neutrosophic chromatic number is

$$\min_{x,y,z \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y) + \sigma(z)\}.$$

*Proof.* All edges are neutrosophic strong and non-center vertices form odd neutrosophic strong cycles. Odd neutrosophic strong cycle have chromatic number which is three. Non-center vertex has same edges with all non-center vertices. Thus non-center vertex has different colors with non-center vertices. Therefore, chromatic number is four. Four representatives of colors form neutrosophic chromatic number where one representative is center vertex and other three representatives are non-center vertices. So,

$$\min_{x,y,z \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y) + \sigma(z)\}.$$

□ 90

**Proposition 2.10.** Let  $N = (\sigma, \mu)$  be a neutrosophic complete bipartite. Then chromatic number is two and neutrosophic chromatic number is

$$\min_{x \text{ and } y \text{ are in different parts}} \{\sigma(x) + \sigma(y)\}.$$

*Proof.* Every given vertex has neutrosophic strong edge with all vertices from another part. So the color of every vertex which is in a same part is same. Hence, two parts implies two different colors. It induces chromatic number is two. The minimum value of a vertex amid all vertices in every part, identify the representative of every color. Therefore,

$$\min_{x \text{ and } y \text{ are in different parts}} \{\sigma(x) + \sigma(y)\}.$$

□ 91

**Proposition 2.11.** Let  $N = (\sigma, \mu)$  be a neutrosophic complete  $t$ -partite. Then chromatic number is  $t$  and neutrosophic chromatic number is

$$\min_{x_1, x_2, \dots, x_t \text{ are in different parts}} \{\sigma(x_1) + \sigma(x_2) + \dots + \sigma(x_t)\}.$$

*Proof.* Every part has same color for its vertices. So chromatic number is  $t$ . Every part introduces one vertex as a representative of its color. Thus, neutrosophic chromatic number is

$$\min_{x_1, x_2, \dots, x_t \text{ are in different parts}} \{\sigma(x_1) + \sigma(x_2) + \dots + \sigma(x_t)\}.$$

□ 92

**Proposition 2.12.** *Let  $N = (\sigma, \mu)$  be a neutrosophic strong. Then chromatic number is 1 if and only if  $N = (\sigma, \mu)$  is neutrosophic empty.*

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*Proof.* ( $\Rightarrow$ ). Let chromatic number be 1. It implies there's no vertex which has same edge with a vertex. So there's no neutrosophic strong edge. Since  $N = (\sigma, \mu)$  is a neutrosophic strong,  $N = (\sigma, \mu)$  is a neutrosophic empty.

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( $\Leftarrow$ ). Let  $N = (\sigma, \mu)$  be neutrosophic empty and neutrosophic strong. Hence there's no edge. It implies for every given vertex, there's no common neutrosophic strong edge. It induces there's only one color for vertices. Hence the representative of this color is chosen from  $n$  vertices. Thus chromatic number is 1. □

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**Proposition 2.13.** *Let  $N = (\sigma, \mu)$  be a neutrosophic strong. Then chromatic number is 2 if and only if  $N = (\sigma, \mu)$  is neutrosophic complete bipartite.*

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*Proof.* ( $\Rightarrow$ ). Let chromatic number be two. So every vertex has either one vertex or two vertices with a common edge. The number of colors are two so there are two sets which each set has the vertices which same color. If two vertices have same color, then they don't have a common edge. So every set is a part in that, no vertex has common edge. The number of these sets is two. Hence there are two parts in each of them, every vertex has no common edge with other vertices. Since  $N = (\sigma, \mu)$  is a neutrosophic strong,  $N = (\sigma, \mu)$  is neutrosophic complete bipartite.

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( $\Leftarrow$ ). Assume  $N = (\sigma, \mu)$  is neutrosophic complete bipartite. Then all edges are neutrosophic strong. Every part has the vertices which have no edge in common. So they're assigned to have same color. There are two parts. Thus there are two colors to assign to the vertices in that, the vertices with common edge, have different colors. It induces chromatic number is 2. □

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**Proposition 2.14.** *Let  $N = (\sigma, \mu)$  be a neutrosophic strong. Then chromatic number is  $n$  if and only if  $N = (\sigma, \mu)$  is neutrosophic complete.*

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*Proof.* ( $\Rightarrow$ ). Let chromatic number be  $n$ . So any given vertex has  $n$  vertices which have common edge with them and every of them have common edge with each other. It implies every vertex has  $n$  vertices which have common edge with them. Since  $N = (\sigma, \mu)$  is a neutrosophic strong,  $N = (\sigma, \mu)$  is neutrosophic complete.

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( $\Leftarrow$ ). Suppose  $N = (\sigma, \mu)$  is neutrosophic complete. Every vertex has  $n$  vertices which have common edge with them. Since all edges are neutrosophic strong, the minimum number of colors are  $n$ . Thus chromatic number is  $n$ . □

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General bounds for neutrosophic chromatic number are computed.

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**Proposition 2.15.** *Let  $N = (\sigma, \mu)$  be a neutrosophic graph. Then chromatic number is at most the number of vertices and neutrosophic chromatic number is at most neutrosophic order.*

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*Proof.* When every vertex is a representative of each color, chromatic number is the number of vertices and it happens in chromatic number of neutrosophic complete which is  $n$ . When all vertices have distinct colors, neutrosophic chromatic number is neutrosophic order and it's sharp for neutrosophic complete. □

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The relation amid neutrosophic chromatic number and main parameters of neutrosophic graphs is computed.

**Proposition 2.16.** *Let  $N = (\sigma, \mu)$  be a neutrosophic strong. Then chromatic number is at most  $\Delta + 1$  and at least 2.*

*Proof.* Neutrosophic strong is neutrosophic nontrivial. So it isn't neutrosophic empty which induces there's no edge. It implies chromatic number is two. Since chromatic number is one if and only if  $N = (\sigma, \mu)$  is neutrosophic empty if and only if  $N = (\sigma, \mu)$  is neutrosophic trivial. A vertex with degree  $\Delta$ , has  $\Delta$  vertices which have common edges with them. If these vertices have no edge amid each other, then chromatic number is two especially, neutrosophic star. If not, then in the case, all vertices have edge amid each other, chromatic number is  $\Delta + 1$ , especially, neutrosophic complete.  $\square$

**Proposition 2.17.** *Let  $N = (\sigma, \mu)$  be a neutrosophic  $r$ -regular. Then chromatic number is at most  $r + 1$ .*

*Proof.*  $N = (\sigma, \mu)$  is a neutrosophic  $r$ -regular. So any of vertex has  $r$  vertices which have common edge with it. If these vertices have no common edge with each other, for instance neutrosophic star, chromatic number is two. But since the vertices have common edge with each other, chromatic number is  $r + 1$ , for instance, neutrosophic complete.  $\square$

### 3 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

**Step 1. (Definition)** Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

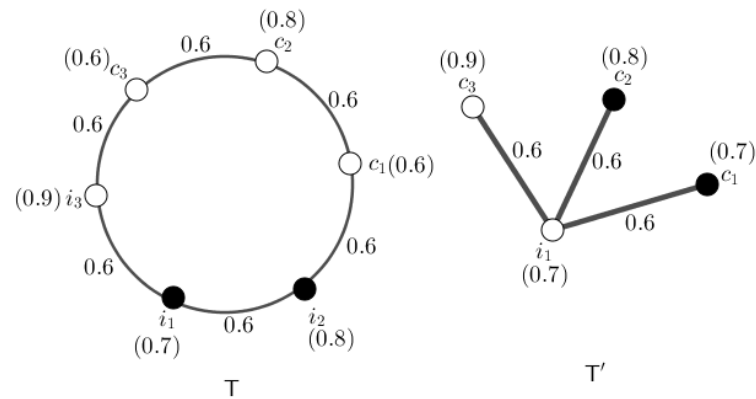
**Step 2. (Issue)** scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

**Step 3. (Model)** As Figure (2), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two section is at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

**Table 1.** Scheduling concerns its Subjects and its Connections as a Neutrosophic Graph in a Model.

Sections of $T$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	$s_7$	$s_8$	$s_9, s_{10}$
Values	0.1	0.8	0.7	0.8	0.1	0.3	0.6	0.5	0.2
Connections of $T$	$s_1s_2$	$s_2s_3$	$s_3s_4$	$s_4s_5$	$s_5s_6$	$s_6s_7$	$s_7s_8$	$s_8s_9$	$s_9s_{10}$
Values	0.1	0.6	0.4	0.1	0.1	0.2	0.4	0.2	0.1

**Step 4. (Solution)** As Figure (2) shows, neutrosophic model, propose to use chromatic number 2 in the case with is titled  $T'$ . In this case,  $i_1$  and  $c_1$  are representative of these two colors and neutrosophic chromatic number is 1.4. The



**Figure 2.** Black vertices are suspicions about choosing them.

set  $\{i_1, c_1\}$  contains representatives of colors which pose chromatic number and neutrosophic chromatic number. Thus the decision amid choosing the subject  $c_1$  an  $c_2$  is concluded to choose  $c_1$ . To get brief overview, neutrosophic model uses one number for every array so 0.9 means  $(0.9, 0.9, 0.9)$ . In Figure (2), the neutrosophic model  $T$  introduce the common situation. The representatives of colors are  $i_2$  and  $c_1$ . Thus chromatic number is two and neutrosophic chromatic number is 1.4. Thus suspicion about choosing  $i_1$  and  $i_2$  is determined to be  $i_2$ . The sets of representative for colors are  $\{i_2, c_1\}$ .

## 4 Open Problems

The two notions of coloring of vertices concerning neutrosophic chromatic number and chromatic number are defined on neutrosophic graphs when neutrosophic strong edges have key role to have these notions. Thus

**Question 4.1.** *Is it possible to use other types edges to define chromatic number and neutrosophic chromatic number?*

**Question 4.2.** *Is it possible to use other types of ways to make number to define chromatic number and neutrosophic chromatic number?*

**Question 4.3.** *Which classes of neutrosophic graphs have the eligibility to pursue independent study in this way?*

**Question 4.4.** *Which applications do make an independent study to define chromatic number and neutrosophic chromatic number?*

**Problem 4.5.** *Which approaches do work to construct classes of neutrosophic graphs to continue this study?*

**Problem 4.6.** *Which approaches do work to construct applications to create independent study?*

**Problem 4.7.** *Which approaches do work to construct definitions which use all three arrays and the relations amid them instead of one array of three arrays to create independent study?*

## 5 Conclusion and Closing Remarks

This study uses mixed combinations of neutrosophic chromatic number and chromatic number to study on neutrosophic graphs. The connections of vertices which are clarified by neutrosophic strong edges, differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of graph theory. One way is finding some relations amid array of vertices to make sensible defintions. In Table (2), some limitations and advantages of this study is pointed out.

**Table 2.** A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Using neutrosophic strong edges	1. Using only one array of three arrays
2. Using neutrosophic cardinality	
3. Using cardinality	2. Study on a few classes
4. Characterizing smallest number	
5. Characterizing biggest number	3. Quality of Results

## References

1. M. Akram, and G. Shahzadi, *Operations on Single-Valued Neutrosophic Graphs*, Journal of uncertain systems 11 (1) (2017) 1-26.
2. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets Syst. 20 (1986) 87-96.
3. S. Broumi, M. Talea, A. Bakali and F. Smarandache, *Single-valued neutrosophic graphs*, Journal of New Theory 10 (2016) 86-101.
4. N. Shah, and A. Hussain, *Neutrosophic soft graphs*, Neutrosophic Set and Systems 11 (2016) 31-44.
5. Henry Garrett, *Big Sets Of Vertices*, Preprints 2021, 2021060189 (doi: 10.20944/preprints202106.0189.v1).
6. Henry Garrett, *Locating And Location Number*, Preprints 2021, 2021060206 (doi: 10.20944/preprints202106.0206.v1).
7. Henry Garrett, *Metric Dimensions Of Graphs*, Preprints 2021, 2021060392 (doi: 10.20944/preprints202106.0392.v1).
8. Henry Garrett, *New Graph Of Graph*, Preprints 2021, 2021060323 (doi: 10.20944/preprints202106.0323.v1).
9. Henry Garrett, *Numbers Based On Edges*, Preprints 2021, 2021060315 (doi: 10.20944/preprints202106.0315.v1).
10. Henry Garrett, *Matroid And Its Outlines*, Preprints 2021, 2021060146 (doi: 10.20944/preprints202106.0146.v1).
11. Henry Garrett, *Matroid And Its Relations*, Preprints 2021, 2021060080 (doi: 10.20944/preprints202106.0080.v1).

- 
12. A. Shannon and K.T. Atanassov, *A first step to a theory of the intuitionistic fuzzy graphs*, Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61. 227  
228
  13. F. Smarandache, *A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic, Rehoboth: American Research Press (1998).* 229  
230
  14. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single-valued neutrosophic sets*, Multispace and Multistructure 4 (2010) 410-413. 231  
232
  15. L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353. 233