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Compatibility of drought magnitude based method with SPA for assessing reservoir volumes: analysis using Canadian river flows

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Abstract: The traditional sequent peak algorithm (SPA) was used to assess the reservoir volume (V_R) for comparison with deficit volume, D_T, (subscript T representing the return period) obtained from the drought magnitude (DM) based method with draft level set at the mean annual flow on 15 rivers across Canada. At an annual scale, the SPA based estimates were found to be larger with an average of nearly 70% compared to DM based estimates. To ramp up DM based estimates to be in parity with SPA based values, the analysis was carried out through the counting and the analytical procedures involving only the annual SHI (standardized hydrological index, i.e. standardized values of annual flows) sequences. It was found that MA2 or MA3 (moving average of 2 or 3 consecutive values) of SHI sequences were required to match the counted values of D_T to V_R. Further, the inclusion of mean, as well as the variance of the drought intensity in the analytical procedure, with aforesaid smoothing led D_T comparable to V_R. The distinctive point in the DM based method is that no assumption is necessary such as the reservoir being full at the beginning of the analysis - as is the case with SPA.

Keywords: Deficit volume; drought intensity; drought magnitude; extreme number theorem; Markov chain; moving average smoothing; standardized hydrological index; sequent peak algorithm; reservoir volume.

1. Introduction

A considerable amount of research can be traced to the hydrologic drought models that focus on the estimation of drought duration and magnitude (previously termed as severity) using the river flow data. Two major elements of the hydrologic drought studies have been the truncation level approach and the analysis by simulation and/or analytical methods. The analytical methods are pursued by the use of the frequency analyses of drought events in terms of duration and deficit volumes. The noteworthy contributions in this area of frequency analyses are that of [1], [2], [3], [4], among others. The other route in the domain of analytical methods is the use of the theory of runs, which is well documented in [5 - 13]. Several hydrologic drought indices have been suggested such as standardized runoff index (SSI) [14], streamflow drought index (SDI) [15], and standardized hydrologic index (SHI) [12, 13]. These indices are essentially standardized (statistically) values of historical stream flows or in some transformed version (normalization in a probabilistic sense) at the desired time scale. The standardized hydrological index (SHI) is the standardized value (statistical) of river flows with the mean 0 and the standard deviation equal to 1, unlike the standardized precipitation index (SPI), which is normalized after standardization [16]. On the monthly time scale, it is the month-by-month standardization and so on at the weekly time scale.

The major application of the SPI refers to drought monitoring which is an essential element in the process of drought early warning and preparedness. Applications of SPI are amenable because of the widespread availability of precipitation data. Though some attempts have been made to classify the hydrological drought [15] on the lines of SPI, yet such uses of hydrological drought indices are limited. However, there have been investigations on the use of SPI to relate the propagation of meteorological droughts to hydrological droughts in Spanish catchments [17] and for the U.K. catchments [18], among others. Despite such limitations, hydrological drought indices have potential in the estimation of drought magnitude that plays an important role in the assessment of shortage of water in rivers and consequently in reservoirs. Even with the aforesaid studies, few investigations other than Sharma and Panu [19, 20] have been made to link the deficit volume to reservoir volume, and also how and at what time scale of analysis would be aptly meaningful in this regard.

The term drought magnitude has been variously defined in the earlier literature such as the drought severity [5, 6] and the deficit volume [2]. In this paper, the term deficit volume (denoted by D) represents the deficiency or the shortage of water below the truncation level in a river flow sequence, and the drought magnitude (M) refers to the deficiency in terms of the SHI (standardized flow) sequences. The deficit volume and drought magnitude are related by the linkage relationship: $D = \sigma \times M$ [5], in which σ is the standard deviation of the flow sequence. The analyses are usually conducted in the standardized domain to assess deficit volume, D through the above linkage relationship.

To the best of the authors' knowledge, no research investigations other than those of authors [19, 20] have been reported in the literature on the application of drought indices and magnitude based analyses, and models for sizing reservoirs. This paper represents one of the pioneering attempts to address and bridge the above gap and demonstrate the utility of such analyses of drought magnitude in assessing the size of reservoirs. The standardized hydrological index (SHI) has been used in this analysis utilizing streamflow data from Canadian rivers. The data on annual, monthly and weekly flow sequences were analyzed using the draft at the mean annual flow for sizing

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the reservoirs. However, the authors' preliminary investigations indicate that the detailed analysis related to the sizing of reservoirs be conducted at an annual scale in view of ease and simplicity in handling annual streamflow data.

2. Preliminaries on Methods for Sizing the Reservoirs

The two textbook based methods [21], [22] for sizing the reservoirs are the Rippl graphical procedure and the sequent peak algorithm (SPA). In the Rippl method, the graphical plot of cumulative inflows as well as outflows is used to derive an estimate of the reservoir size. In the SPA, the calculations are conducted numerically using the cumulative or residual mass curve methods to obtain the estimate of reservoir volume, V_R. In the drought magnitude based method, SHI sequences are obtained after standardization. It corresponds to truncating the annual river flow sequences at the mean level or SHI value = 0. Since the drought lengths and corresponding magnitudes yield conservative values for the design of reservoirs, therefore SHI = 0 as a truncation level is preferred and has been used in the analysis. The SHIs below the truncation level are referred to as the deficit (dubbed as d), whereas above the level are referred to as the surplus (dubbed as s). In a historical record of N (= T) years, there shall emerge several spells of deficit and surplus, and the longest spell length of deficits (representing L_T) is recorded. Likewise, the corresponding deficits are added to represent the largest magnitude (M_T). These deficits are being referred to as drought intensities and represent truncated values of SHIs below the truncation level. The foregoing approach of calculation of L_T and M_T is dubbed as the counting procedure in the ensuing sections. The largest deficit volume (D_T) during the drought period is computed as $D_T = \sigma \times M_T$ [5]. It is noted that the unit of D_T is the same as that of σ because M_T is a dimensionless entity. It is stated that the quantity D_T obtained using either the DM based counting or analytical procedure is perceived equivalent to V_R calculated by the SPA method. In the counting procedure, the entities M_T and D_T are respectively obtained from the historical or observed data and hence are denoted as $M_{T\text{-}o}$ and $D_{T\text{-}o}$, where subscript "$_o$" stands for the observed.

2.1. Estimation of deficit volumes by DM model

The majority of models for estimation of drought magnitude are built on the frequency distribution of drought events [2], [4]. The moving average (MA) and sequent peak algorithm (SPA) form the important tools for analysis [9], [2]. In the other approach, probability based relationships are hypothesized for estimating drought magnitude (M) using the relationship: drought magnitude = drought intensity × drought length [23]. As mentioned above in the text, drought intensities essentially are deficit spikes and are derived by truncating a SHI sequence. The deficit spikes have a negative sign because each spike lay on the downside (negative side) of the truncation level, with the lower bound as - ∞ and an upper bound as truncation level such as z_0, which is also a negative number with maximum value as 0. It is tacitly assumed that the SHI sequences obey standard normal probability density function (pdf) which after truncating at the desired level (z_0) shall result in a truncated normal pdf, whose mean and variance would be different from 0 and 1. One can develop a probabilistic relationship for M_T expressed as follows, through the use of the extreme number theorem [7], [24] that implicitly involves drought intensity and drought length (L_T).

$$P(M_T \leq Y) = \exp\left[-Tq(1-q_q)(1-P(M \ \leq Y)\right] \qquad (1)$$

In which, q represents the simple probability of drought and q_q represents the conditional probability that the present period is drought given the past period is also drought and T is equivalent to return period; M stands for the drought magnitude which takes on non-integer values represented by Y, and P (.) represents the notation of cumulative probability. Since Y's (such as Y_1, Y_2, Y_3, Y_4, ...) correspond to values of M, thus the largest of them will represent M_T. In the above expression, M is construed to follow a normal pdf with mean and variance related to the mean and variance of drought intensity and a characteristic drought length. The characteristic drought length is related to mean drought length and the extreme drought length, L_T.

At the annual level, the flow sequences in Canadian rivers have been found to follow the normal pdf [13], leading SHI sequences to obey the standard normal pdf. Therefore, the assumption of deficit spikes to obey truncated normal distribution is reasonably justified. Based on the above premises, a detailed derivation has been tracked by Sharma and Panu [19, 20] and are not reproduced here for the sake of brevity. The final expressions for the purpose of the present paper are described as follows.

$$E(M_T) = \sum_{j=0}^{n_1} \frac{(Y_{j+1} + Y_j)}{2}\left[P\left(M_T \leq Y_{j+1}\right) - P\left(M_T \leq Y_j\right)\right] = M_{T-e} \qquad (2)$$

To compute [P ($M_T \leq Y_{j+1}$) – *P* ($M_T \leq Y_j$)] in Equation (2), the integration of the normal probability function is numerically performed as described in Sharma and Panu [12]. Theoretically, the upper limit of summation (n_1) in Equation (2) is ∞, but for numerical integration purposes, a finite value is chosen. For drought magnitude analysis based on annual flows, a value of n_1 = 30 (with an increment in j = 0.05 was found to be large enough to ensure sufficient accuracy in the process of numerical integration. For brevity, henceforth E (M_T) shall be written as M_{T-e}, i.e. an estimated value of M_T. It may be noted that Equation (2) involves both the mean and variance of drought intensity to arrive at a value of M_{T-e}. Likewise, the estimated value of D_T is designated as D_{T-e} (= $\sigma \times M_{T-e}$).

A particular version of M_{T-e} involving the mean of drought intensity only can be written as follows:

$$M_{T-e} = \text{abs}\left[-\frac{\exp(-0.5\, z_0^2)}{q\sqrt{2\pi}} - z_0\right].\, L_T \qquad \text{("abs" means absolute)} \qquad (3)$$

Where, L_T is the largest drought length obtained using Markov chain based algorithm, q is drought probability at the truncation level z_0. For example, a standard normal pdf respectively can be truncated at $z_0 = 0.0$ (mean level) and $z_0 = -0.30$ (which is 70 % of a mean level) and the corresponding drought probability q can be found from standard normal probability tables to be 0.5 and 0.38.

3. Data Acquisition and Calculations of Reservoir Volumes

Fifteen rivers from prairies to Atlantic Canada (Figure 1, Table 1) were involved in the analysis. The rivers encompassed drainage areas ranging from 97 to 56369 km^2 with the data bank spanning from 38 to 108 years. The flow data for these 15 rivers were extracted from the Canadian hydrological database [25]. To increase the number of samples, some of the rivers with large data sizes such as the Bow, English, Lepreau, Bevearbank, and North Margaree were also analyzed by forming 2-4 subsamples with the data size of 40 years or more. This type of analysis created around 30 samples from 15 rivers to obtain a robust and reliable estimate of the performance statistics. Based on the above premises, the results of various analyses are described in the sections to follow.

Figure 1 Location of the river gauging stations used in the analysis [Source: Environment Canada]

Table 1 Summary of statistical properties of annual flows of the rivers under consideration

Name, location, and the numeric identifier in Figure 1 of the River	Period of Record (Year)	Area (km^2)	Mean (m^3/s)	cv	γ	ϱ
[1] Bow River at Banff, AB05BB001, (51°10’ 30’’N, 115°34’10’’W)	108 (1911-18)	2210	39.12	0.13	0.05	0.06
[2] South Saskatchewan River at Medicine Hat AB05JA001, (50°03’ 00’’N, 110°40’ 00’’ W)	59 (1960-18)	56369	167.08	0.35	0.20	0.12
[3] English River at Umfreville, ON05QA002, (49°52’ 30’’N, 91°27’30’’W)	97 (1922-18)	6230	58.75	0.32	0.30	0.21
[4] Pic River near Marathon, ON02BB003, (48°46’ 26’’N, 86°17’49’’W)	48 (1971-18)	4270	50.21	0.24	-0.06	0.13
[5] Pagwachaun River at highway#11, ON04JD005, (49°46’ 00’’N, 85°14’ 00’’W)	50 (1968-18)	2020	23.07	0.25	0.18	0.06
[6] Nagagami River at highway#11, ON04JC002, (49°46’ 44’’N, 84°31’ 48’’W)	38 (1981-18)	2410	24.59	0.22	-0.14	0.08
[7] Batchwana River near Batchwana, ON02FB001, (46°59’ 36’’N, 84°31’ 31’’W)	51 (1968-18)	1190	22.20	0.20	0.21	0.03
[8] Goulis River near Searchmont, ON02FB002, (46°51’ 37’’N, 83°38’ 18’’W)	51 (1968-18)	1160	18.17	0.21	0.21	0.08
[9] North French near Mouth, ON04MF001, (51°05’ 00’’N, 80°46’ 00’’W)	52 (1967-18)	6680	95.48	0.21	0.004	-0.04
[10] Beaurivage A. Sainte Entiene, QC02PJ007, (46°39’ 33’’N, 71°17’ 19’’W)	75 (1926-00)	709	14.19	0.26	1.15	0.19
[11] Lepreau River at Lepreau, NB01AQ001, (45°10’ 11’’N, 66°28’ 05’’W)	100 (1919-18)	239	7.41	0.22	0.53	0.10
[12] Bevearbank River at Kinsac, NS01DG003, (44°51’ 04’’N, 63°39’ 50’’W)	97 (1922-18)	97	3.04	0.19	0.15	-0.19
[13] N. Margaree at Margaree valley, NS01FB001, (46°22’ 08’’N, 60°58’ 31’’W)	90 (1929-18)	368	17.03	0.14	0.49	0.17
[14] Upper Humber R. at Reidville, NF02YL001, (49°14’ 34’’N, 57°21’ 36’’W)	66 (1953-18)	2110	80.05	0.13	0.42	0.15
[15] Torrent River at Bristol pool, NF02YC001, (50°36’ 26’’N, 57°09’ 05’’W)	59 (1960-18)	624	24.81	0.15	0.72	0.18

Note: cv, γ, & ϱ respectively represent the coefficient of variation, skewness, and lag-1 autocorrelation of annual flows.

The first step in the analysis was to discern the role of time scale in influencing the reservoir size. Therefore, reservoir volumes (V_R) were assessed using the SPA at the demand level equivalent to the mean flows at the annual, monthly, and weekly scales. The procedure advanced in Linsley et al. (21) was used to calculate the V_R. In turn, the V_R values were compared with the deficit volumes, D_{T-o}. The calculations were done by writing Macros in Visual Basic and coupling them with associated data in the Microsoft Excel framework. Therefore, flows were standardized at the above three time scales to obtain SHI sequences. In all three time scales, the values of the drought probability, q, were obtained by the counting procedure in which an SHI sequence was chopped at level 0 (mean level). In general, the annual flows tend to follow a normal pdf in the Canadian settings so, at the mean level, q values cluster around 0.50 (Table 2, column 2). In view of the gamma pdf of the flow sequences, at the monthly and weekly scales [13], the q values are significantly larger than 0.50 (Table 2, columns 3 and 4).

Table 2 Calculations of storage volumes at the mean level of flows for varying time scales.

River name & data size	Computations of Storage Volumes (m^3)					
	Sequent Peak Algorithm (SPA)			Drought magnitude (DM)		
	Annual	Month	Week	Annual	Month	Week
1	2	3	4	5	6	7
S. Saskatchewan 1960-2011, N=52	q=0.50 2.19×10^{10}	q=0.58 2.24×10^{10}	q=0.59 2.28×10^{10}	q=0.50 1.63×10^{10}	q=0.58 $7.91(\mathit{8.52})\times10^{9}$	q=0.59 $6.54(\mathit{7.30})\times10^{9}$
S. Saskatchewan 1970-2011, N=42	q=0.50 1.59×10^{10}	q=0.58 1.63×10^{10}	q=0.58 1.65×10^{10}	q=0.50 8.41×10^{9}	q=0.58 $7.11(\mathit{7.79})\times10^{9}$	q=0.58 $6.13(\mathit{6.80})\times10^{9}$
Bow River 1940-2011, N=72	q=0.50 1.61×10^{9}	q=0.54 1.62×10^{9}	q=0.55 1.68×10^{9}	q=0.50 7.79×10^{8}	q=0.54 $5.65(\mathit{4.27})\times10^{8}$	q=0.55 $4.84(\mathit{4.10})\times10^{8}$
Bow River 1911-1960, N=50	q=0.48 1.67×10^{9}	q=0.54 1.77×10^{9}	q=0.55 1.85×10^{9}	q=0.48 1.08×10^{9}	q=0.54 $5.31(\mathit{5.05})\times10^{8}$	q=0.55 $3.51(\mathit{3.89})\times10^{8}$
Bow River 1960-2003, N=44	q=0.50 1.21×10^{9}	q=0.54 1.24×10^{9}	q=0.55 1.28×10^{9}	q=0.50 6.21×10^{8}	q=0.54 $5.55(\mathit{4.14})\times10^{8}$	q=0.55 $4.79(\mathit{4.24})\times10^{8}$
English River 1922-2009, N=88	q=0.52 7.86×10^{9}	q=0.57 7.97×10^{9}	q=0.58 8.06×10^{9}	q=0.52 3.73×10^{9}	q=0.57 $3.67(\mathit{3.20})\times10^{9}$	q=0.58 $3.80(\mathit{3.24})\times10^{9}$
English River 1922-66, N=45	q=0.49 5.60×10^{9}	q=0.57 5.92×10^{9}	q=0.58 6.00×10^{9}	***q***=0.49 3.19×10^{9}	q=0.57 $3.41(\mathit{2.88})\times10^{9}$	q=0.58 $3.55(\mathit{2.92})\times10^{9}$
English River 1975-2011, N=37	q=0.50 4.49×10^{9}	q=0.55 4.51×10^{9}	q=0.55 4.59×10^{9}	q=0.50 2.85×10^{9}	q=0.55 $2.18(\mathit{2.10})\times10^{9}$	q=0.55 $2.27(\mathit{2.17})\times10^{9}$
Beaverbank River 1961-2000, N=40	q=0.50 8.00×10^{7}	q=0.59 8.73×10^{7}	q=0.65 9.10×10^{7}	q=0.50 6.37×10^{7}	q=0.59 $4.63(\mathit{4.78})\times10^{7}$	q=0.65 $2.96(\mathit{2.72}\ )\times10^{7}$
Pic River 1971-05, N=35	q=0.51 1.48×10^{9}	q=0.58 1.77×10^{9}	q=0.60 1.82×10^{9}	q=0.51 9.68×10^{8}	q=0.58 $1.17(\mathit{1.13})\times10^{9}$	q=*0.60* $9.94\ (\mathit{8.09})\times10^{8}$
Goulish River 1968-10, N=43	q=0.49 1.10×10^{9}	q=0.57 1.21×10^{9}	q=0.63 1.25×10^{9}	q=0.49 8.11×10^{9}	q=0.57 $3.35(\mathit{2.48})\times10^{8}$	q=*0.63* $3.70(\mathit{2.49})\times10^{9}$

Note: The italicized values in parentheses are calculated at the mean levels (variable means) of the respective months and weeks without standardization of the flow sequences.

The drought magnitudes, $M_{T\text{-}o}$ were computed for each time scale and accordingly converted to $D_{T\text{-}o}$. On the annual scale, there is only one set of μ and σ, whereas there are respectively 12 and 52 such sets of μ and σ at the monthly and weekly scales. Therefore, in the calculations of $D_{T\text{-}o}$ at the monthly and weekly scales, an averaged out (arithmetic average) value, denoted by σ_{av} was used in the analysis. Various versions of σ such as σ_{min} (subscript min for minimum), σ_{max} (max for maximum) and the geometric mean of 12 monthly and 52 weekly values were tried, and the arithmetic mean turned out to be the best estimator [12]. The $D_{T\text{-}o}$ values were also estimated without standardization and truncating the flow series at the variable mean levels corresponding to the respective monthly and weekly time scales. In the standardized domain, the variable means are homogenized with a common mean = 0.

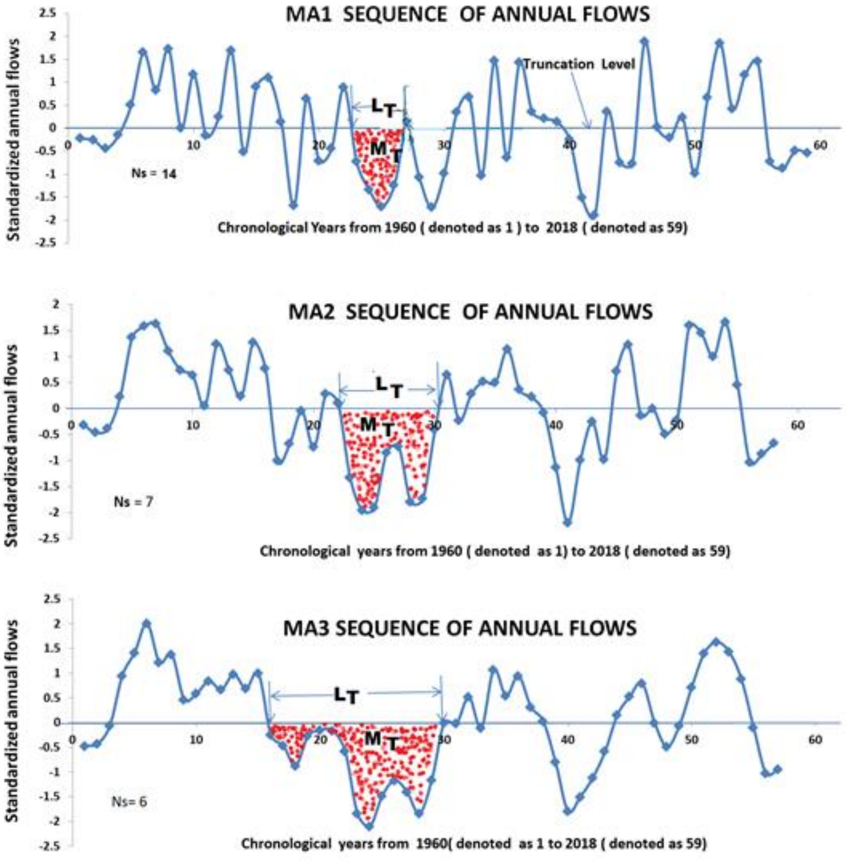

Figure 2 Redistribution of drought lengths and magnitudes with varying MA smoothing.

The MA sequences can be formed from flows or the SHI sequences, alike. However, it is convenient to apply flow sequences to compute the V_R using SPA, whereas DM based method explicitly requires SHI sequences. When the annual SHI (or flow) sequence is used without involving any moving average operations then such a sequence is designated as moving average 1 (MA1) sequence. In other words, a non-averaged value of SHI (or flow) is essentially the annual SHI (or flow). When consecutive 2 or 3 or annual SHIs (or flows) are averaged out then such a running sequence is termed as MA2 or MA3 sequence. Figure 2 displays MA1, MA2 and MA3 annual SHI sequences with the drought parameters for the South Saskatchewan River. The

MA1 sequence (flows) was subjected to analysis to compute the mean (μ), standard deviation (σ) and lag-1 autocorrelation (ϱ). The aforesaid statistics were also evaluated for the MA2 and MA3 (flows) sequences and are shown in Table 3. Using the above values of mean and standard deviation, the MA1, MA2 and MA3 flow sequences were converted to respective SHI sequences. In the process of analysis, the number of drought spells (Ns) dropped from the MA1 through MA3 sequences and are presented in column 5 of Table 3. After a few MA smoothing, Ns attained nearly an equilibrium state and thus suggesting no further MA smoothing were warranted. For example, in Table 3, Ns values for MA3 smoothing marginally deviate from MA2 but significantly drop from MA1.

Table 3 Summarized V_R (SPA) and D_T (DM method -counting) on the MA smoothed annual SHI Sequences.

River details	MA number	Mean (m³/s)	q	Ns	σ (m³/s-yr.), ϱ	Computation of Reservoir Volume (m³/s-yr.)				
						DM Method				SPA*
						M_{T-o}	M_{T-e}	‘D_{T-o}	D_{T-o}	V_R **
1	2	3	4	5	6	7	8	9	10	11
Bow river N=108(1911-18)	MA1	39.12	0.48	26	5.18, 0.06	5.84	7.02	30.27	30.27	78.11
	MA2	39.10	0.49	15	3.79, 0.57	13.02	13.22	49.35	67.46	--
	MA3	39.08	0.51	15	3.23, 0.68	14.52	15.15	47.48	**75.23**	**--**
Bow River N=62(1955-16)	MA1	38.44	0.52	17	5.12, 0.11	4.98	6.72	25.45	25.45	54.73
	MA2	38.48	0.46	10	3.84, 0.57	11.08	10.52	42.53	**56.67**	**--**
	MA3	38.51	0.53	9	3.25, 0.65	12.20	11.39	39.65	62.44	--
Saskatchewan River N=59(1960-18)	MA1	167.1	0.51	14	58.83, 0.20	8.86	6.62	521.09	508.59	708.07
	MA2	167.5	0.48	7	45.95, 0.66	10.72	11.09	509. 58	**630.68**	--
	MA3	167.9	0.54	6	40.63, 0.81	13.71	13.07	557.04	806.41	--
Saskatchewan River N=42(1970-11)	MA1	159.8	0.50	12	60.62, 0.09	4.40	5.88	266.72	266.72	504.28
	MA2	158.4	0.46	6	44.40, 0.53	9.46	8.41	420.20	**573.67**	--
	MA3	157.3	0.43	5	37.11, 0.74	10.39	9.85	385.37	629.53	--

* SPA denotes sequent peak algorithm, ** V_R reservoir volume by the closest SPA. The bold letters signify values .corresponding to V_R.

For a comparative analysis on V_R, the counting procedure was applied to the MA1 sequences. The V_R (Table 3, column 11 and in the subsequent text) were computed using SPA for comparison with D_{T-o}. The counting for D_{T-o} was done in terms of M_{T-o} (SHI sequences, Table 3), which were truncated at the level of 0 ($z_0 = 0$), then converted to D_{T-o} ($D_{T-o} = \sigma \times M_{T-o}$). In the MA1 smoothing, there is only one standard deviation, so after calculating the value of M_{T-o1}, the value of D_{T-o1} was obtained using the above relationship by replacing σ with σ_1, i.e. the standard deviation of the MA1 sequence.

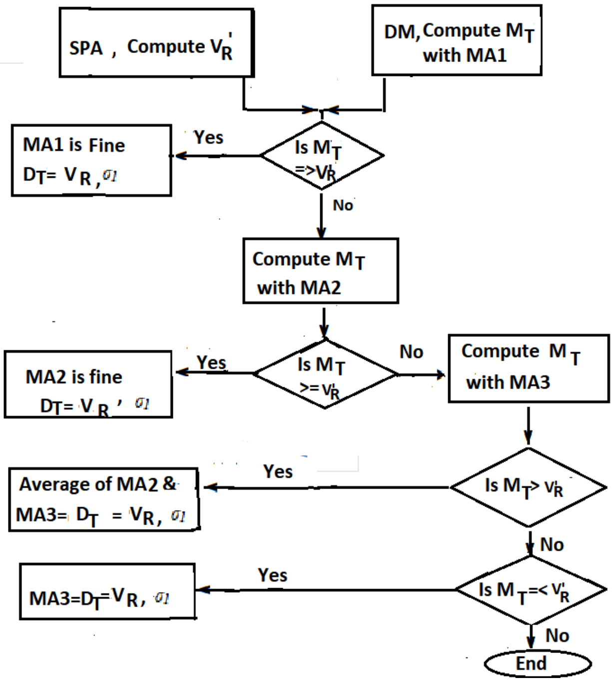

Figure 3 Flow-diagram for computing deficit volumes under various options in the DM method

When the $D_{T\text{-}o}$ based on the MA1 analysis did not match or were far less than the V_R, then the analysis was extended to MA2 and at times to MA3 sequences. For the MA2 smoothing, the value of $D_{T\text{-}o}$ (denoted as $D_{T\text{-}o2}$) was obtained using the corresponding value of the standard deviation (denoted by σ_2) and $M_{T\text{-}o}$ (say $M_{T\text{-}o2}$), i.e. $D_{T\text{-}o2} = \sigma_2 \times M_{T\text{-}o2}$. For further comparison, another value of $D_{T\text{-}o2}$ was also computed using σ_1 (based on MA1 or the original flow sequence), i.e. $D_{T\text{-}o2} = \sigma_1 \times M_{T\text{-}o2}$ (Table 3). Likewise, for the MA3 smoothing, two values of $D_{T\text{-}o3}$ were obtained; one based on σ_3 ($D_{T\text{-}o3} = \sigma_3 \times M_{T\text{-}o3}$; σ_3 being the standard deviation obtained after MA3 smoothing) and another value as $D_{T\text{-}o3} = \sigma_1 \times M_{T\text{-}o3}$. The aim is to choose the MA smoothing that will provide the best equivalence of $D_{T\text{-}o}$ to V_R. The above sequence of computations is portrayed in a flow diagram (Figure 3). In the flow diagram, the symbols $D_{T\text{-}o}$ and $M_{T\text{-}o}$ are denoted by D_T and M_T for the sake of brevity and ease of writing and they can also represent $D_{T\text{-}e}$ and $M_{T\text{-}e}$ with the common multiplier σ_1. In the flow diagram V_R' stands for the standardized value of V_R, i.e. = V_R/σ_1, which corresponds to M_T.

Parallel to the counting procedure, the M_T values (denoted as $M_{T\text{-}e}$) were obtained by using the analytical procedure (Equations 2 and 3). The corresponding values of $D_{T\text{-}e}$ were computed ($D_{T\text{-}e} = \sigma_1 \times M_{T\text{-}e}$). The V_R values were compared with the values of $D_{T\text{-}e}$. Two values of $M_{T\text{-}e}$ were computed for each situation, i.e. one value using the mean as well as the variance in Equation (2) and the other value by simply using the mean based on Equation (3), thus yielding two values of D_{T-e}. Both the values were compared with V_R for arriving at an appropriate value of D_{T-e} for further analysis and use.

4. Results

4.1. Role of the time scale on the size of reservoirs

To discern the role of the annual, monthly and weekly time scales, V_R (SPA) and D_{T-o} (counting procedure in the DM method) from the respective flows and SHI sequences were computed and are summarized for 6 rivers in Table 2. It was found that the values of V_R at the aforesaid time scales were fairly close to each other with a tendency to slightly increase (0 to 20% with an average value of 10%) at the monthly and weekly scales compared to the annual scale. The monthly and weekly D_{T-o} values tended to decrease compared to annual values of D_{T-o} with an average reduction of 25% (range from 21 to 59%). The other point that emerged from the calculations was that the V_R was greater than D_{T-o} at all-time scales. For example, at the annual scale, V_R values were found to be larger (ranging from 0 to 200%), with an average of nearly 70%. The reduction in the storage requirement in terms of deficit volumes makes sense because at monthly and weekly scales the actual drought periods are estimated more accurately due to time scale effects and are usually shortened and thus requiring less amount of water to meet the demand. On the contrary, in the SPA based calculations for V_R, the fluctuations at a shorter time scale would be larger requiring greater reservoir volume to damp out such fluctuations to meet the constant demand.

The above numbers displaying large discrepancies between the SPA and DM based (MA1) estimates highlight that either the SPA yields excessive values of reservoir volume or the DM method yields too small estimates at the draft level of mean annual flow (MAF, 1μ). At the annual scale, however, when the draft was lowered to 0.90μ or less, the estimates by the SPA and DM method converged to the same value [20]. In other words, a region with a draft level between 0.90μ and 1μ requires special consideration for the estimation of reservoir volumes by the DM method. The SPA based estimates can be construed as fixed with little scope to lower them in view of the inherent algorithm imbued in it. But the DM based estimates can be boosted by utilizing the MA procedure to attain parity with SPA based estimates. The SPA has been in vogue since the 1960s [26] and is universally accepted to design the reservoir capacity, therefore, the focus in this study is to arrive at a suitable MA smoothing that should yield D_T comparable to V_R at the draft level of 1μ.

The DM based estimates (*italicized*, Table 2) at the monthly and weekly scales without standardization were found to be slightly different (mostly smaller) in comparison to the standardization based values (SHI sequences).

On average, the standardization based estimates were found about 12% larger than those based on the non-standardized values. This discrepancy can be perceived to arise because of σ_{av}, which has been taken as the representative value of the standard deviation to convert the magnitude in deficit volume ($D_T = \sigma_{av} \times M_T$). Needless to mention that σ_{av} is an estimator representing 12 values of monthly σ's and is unlikely to be the best for all situations, but is construed to be a better option compared to other options mentioned earlier. Since standardization is purely statistical in this operation, the pdf of monthly sequences is less likely to play the role in explaining the aforesaid discrepancy. At the annual scale, however, it should be noted that the standard deviation has only one value, thus estimates by both routes turned out to be identical. Therefore, only one value of $D_{T\text{-}o}$ is reported in Table 2. Since these estimates (i.e. without standardization) do not require the use of standard deviation in the calculations and thus can be deemed more accurate. However, the standardization procedure is better amenable to statistical analysis, and estimates based on this approach are more conservative (i.e. higher compared to the non-standardization, Table 2). Based on the foregoing reasoning, the route involving standardization (i.e., the use of SHI sequences) for the estimation of $D_{T\text{-}o}$ has been preferred in subsequent analyses and the annual scale is considered as a first choice.

4.2. *Comparison of reservoir sizes using the SPA and the DM based counting procedure*

In computing the $D_{T\text{-}o}$, the first step is to choose the right value of σ at each MA smoothing. For example, in the MA2 smoothing, there are two $D_{T\text{-}o}$: one based on σ_2 (i.e. $'D_{T\text{-}02} = \sigma_2 \times M_{T\text{-}o2}$) and another based on σ_1 (i.e. $D_{T\text{-}02} = \sigma_1 \times M_{T\text{-}o2}$). For the MA1 smoothing, there is only one standard deviation and thus $D_{T\text{-}o1} = {'D_{T\text{-}o1}}$ (Table 3). It is apparent from Table 3 that $'D_{T\text{-}o}$ values either inconsistently decrease or increase in MA2 and MA3 smoothing; whereas $D_{T\text{-}o}$ values are consistently increasing and hence σ_1 is the crucial parameter to be used for matching to the V_R to arrive at an appropriate MA smoothing. In other words, σ_1 must be used as a multiplier with $M_{T\text{-}o}$ in every MA smoothing for estimation of $D_{T\text{-}o}$ and the role of σ_2 and σ_3 is confined to the standardization of the smoothed MA2 and MA3 flow sequences. It should be noted that for consistency and ease, only 1.0 σ_1 is being used as a multiplier. Other fractions of σ_1 (such as 1.2, 1.1, 0.90 or 0.80) were neither considered nor tested for their efficacy in this study.

In assessing the efficacy of various smoothing, the values of $D_{T\text{-}o}$ and V_R were compared on a 1:1 basis and the performance statistics, viz. the Nash-Sutcliffe efficiency (NSE), and the mean error (MER) were used [27]. To arrive at the above estimates of performance statistics, values of V_R and $D_{T\text{-}o}$ were standardized dividing them by σ_1 (MA1). In other words, a 1:1 comparison was made between $D_{T\text{-}o}/\sigma_1 = M_{T\text{-}o}$ and V_R/σ_1 (denoted by V_R'). In doing so, the wild variation in these entities (i.e. $D_{T\text{-}o}$ and V_R) from small to large rivers were homogenized while rendering them non-dimensional, and thus resulting in sensible estimates of NSE and MER. The efficacy is being tested using NSE and MER [27] as these statistics have been widely used during the past 50 years and are time tested measures in hydrologic investigations.

Based on aforesaid calculations, it was found that $M_{T\text{-}o}$ for MA1 sequences turned out to be significantly less than V_R' with a caveat that in a few cases, the values of $M_{T\text{-}o}$ were found to be equal to V_R'. In other words, the values of the $M_{T\text{-}o}$ compared poorly with V_R' which is also apparent from an utterly low value of NSE ≈ 24% and MER ≈ -39% (Table 4). In brief, the V_R' tended to be very conservative (meaning larger), whereas DM based estimates, i.e. $M_{T\text{-}o}$ appeared to be significantly smaller. Since the discrepancies in values of $M_{T\text{-}o}$ and the V_R' were excessively large, therefore, the MA2 and MA3 smoothing were considered.

Table 4 Performance statistics for comparison of SPA based V_R with DM based D_T.

Type of model	Performance statistic	Drought magnitude based analysis			
		MA1	MA2	MA3	Average of MA2 & MA3
Calculations by the counting method from the observed flow data	NSE (%) MER (%)	24.33 -38.83	71.93 -13.04	69.36 17.50	**86.73** **2.23**
DM model with consideration of the mean of drought intensity only, Equation (3)	NSE (%) MER (%)	25.90 -49.80	49.08 -36.30	56.70 -26.83	53.60 -31.56
DM model with consideration of the mean and variance of drought intensity, Equation (2)	NSE (%) MER (%)	46.23 -23.71	**72.01** **0.75**	65.08 13.85	**71.02** **7.30**

Firstly, the MA2 based $M_{T\text{-}o2}$ values were compared to the V_R' on a 1:1 basis. It was discovered that with the MA2 smoothing, the matching to V_R' significantly improved resulting in NSE ≈ 72% and MER ≈ -13% (Table 4). In other words, the underestimation was ameliorated significantly as the value of MER ascended from -39% to -13%. Although the NSE values have improved remarkably, there still existed a scope for improvement in the estimates of the $D_{T\text{-}o}$ because underestimation was endemic as revealed by MER of -13%.

Thus, MA3 smoothing was undertaken (flow chart-Figure 3) and values of $M_{T\text{-}o3}$ were obtained. The MA3 sequences resulted in the over-estimation of $D_{T\text{-}o}$ values with MER = 17.50% although the value of NSE dropped marginally to 69.36% (Table 4). In short, the MA2 smoothing led to the under-counting whereas the MA3 smoothing led to the over-counting of the $D_{T\text{-}o}$ values with NSE being nearly the same. For comparison with values of V_R, therefore, it was considered reasonable to average out the $D_{T\text{-}o}$ values based on the MA2 and the MA3 smoothing. Such a comparison was made by plotting the average values of the $M_{T\text{-}o2}$ and $M_{T\text{-}o3}$ against the values of V_R' and resulted in a remarkably improved match with NSE ≈ 87% and MER ≈ 2% (Figure 4A, Table 4). The important point to be noted is that at every smoothing, a new value of $M_{T\text{-}o}$ will emerge, which is multiplied by the MA1 smoothing based value of σ (= σ_1) to arrive at the new estimate of $D_{T\text{-}o}$.

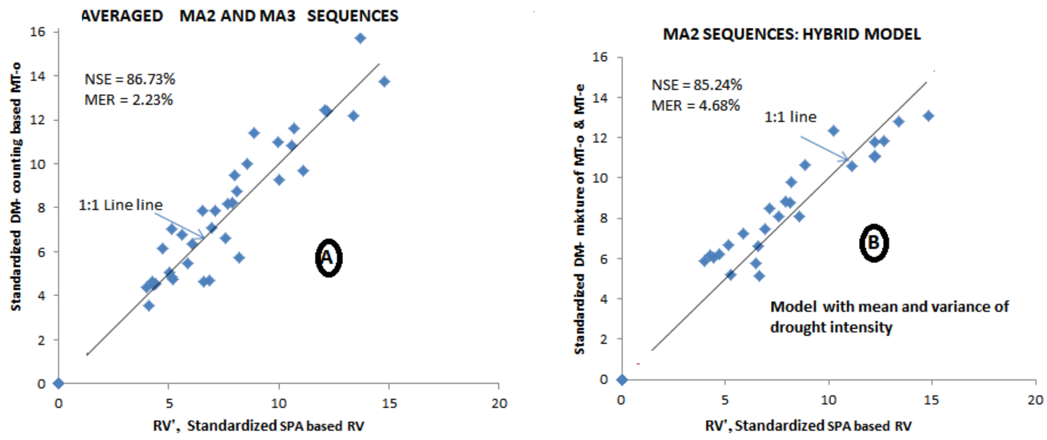

Figure 4 Comparison of SPA based V_R with (A) $M_{T\text{-}o}$ by counting procedure (B) M_T by a hybrid procedure i.e. M_T = a bigger value between $M_{T\text{-}o}$ and $M_{T\text{-}e}$

In the process of moving from the MA1 smoothing to the MA2 smoothing, there has been a considerable reduction in the number of drought spells (column 5, Table 3). Such a reduction suggests that there is a significant increase in the drought length (Figure 2) and in turn, there is also a significant increase in the drought magnitude. In other words, the smoothing procedure led to the amalgamation of smaller drought episodes with the larger ones which resulted in enhanced values of the $D_{T\text{-}o}$ (or $M_{T\text{-}o}$). Such enhanced values have been found to compare well with V_R (or V_R').

4.3. Comparison of reservoir sizes using the SPA and DM based Model

The drought magnitudes ($M_{T\text{-}e}$) in the standardized domain were estimated using Equations (2) and (3). At the annual scale, the characteristic drought length was found equivalent to extreme drought length, L_T [12], obtained from the Markov chain based relationship. In the first version, the $M_{T\text{-}e}$ was estimated by involving only the mean of the drought intensity i.e. Equation (3) while in the second version, both the mean and variance of drought intensity were considered i.e. Equation (2) to arrive at estimates of the $M_{T\text{-}e}$ and hence estimates of the $D_{T\text{-}e}$. The calculation for $D_{T\text{-}e}$ was done using σ_1, i.e. $D_{T\text{-}e} = \sigma_1 \times M_{T\text{-}e}$, which is similar to the case of $D_{T\text{-}o}$. Because of similarity, the best multiplier was σ_1 for all MA smoothing in the estimation of $D_{T\text{-}e}$. For example, there are two estimates of D_{T-e} (viz. $D_{T-e2} = \sigma_2 \times M_{T-e2}$ and $D_{T-e2} = \sigma_1 \times M_{T-e2}$) if the MA2 smoothing was conducted, then the appropriate value will be D_{T-e2} (= $\sigma_1 \times M_{T-e2}$), which, in turn, should be comparable to V_R or the counting based D_{T-o2}. One can advance similar arguments to the MA3 smoothing. Succinctly, σ_1 is the multiplier for all MA smoothing chosen for estimating D_{T-e} in the analytical approach as was the case for D_{T-o}. It was found that the estimation by a simple version involving only the mean of the drought intensity proved too inadequate both in terms of NSE and MER. The calculations showed that values of M_{T-e} are nearly 50% of V_R with the MA1 smoothing and such an underestimation persisted even with the MA3 smoothing leading to the value of MER = - 27% (Table 4). The NSE for the MA3 smoothing was also low with the highest value nearly equal to 57%. Similar was the case for the MA2 smoothing with NSE = 49% and MER = -36% (Table 4).

In view of the abysmal values of the performance statistics by Equation (3), Equation (2) was used to estimate M_{T-e} and its corresponding D_{T-e}. The performance statistics turned out to be encouraging. Although, there was a significant underestimation (≈ -24%) for the MA1 smoothing, however, the underestimation improved remarkably (MER= 0.75%) with the corresponding NSE =72% for the MA2 smoothing. A consideration of MA3 smoothing resulted in a significant overestimation of nearly 14% and a slight reduction of NSE to 65%, which suggested that the MA3 smoothing is less meaningful. However, the estimates of M_{T-e}, based on the MA2 smoothing and the MA3 smoothing were averaged out and the resultant performance statistics improved compared to those of the MA2 smoothing with an acceptable overestimation (7.30%). Likewise, the NSE of 71% was almost equal to 72% that was obtained for the MA2 smoothing. In nutshell, the analytical (model) approach also yielded estimates of M_{T-e} (or D_{T-e}) which are in agreement with those of the counting method. However, the analytical procedure proved a bit rigorous as it involved the numerical integration of relevant equations reported by Sharma and Panu [12] and the resultant output from which, in turn, became input into Equation (2).

It was observed that the MA2 smoothing resulted in similar values of NSE for both the counted D_{T-o} as well as the estimated D_{T-e} (Equation 2). The counted values of D_{T-o} were ameliorated by averaging the values obtained from the MA2 and the MA3 smoothing. Such an averaging by the analytical estimates involving the MA2 and MA3 smoothing resulted in little improvement over the counted values. At this point, it was mooted that the MA2 smoothing is preserved and the larger value between D_{T-o} (M_{T-o}) and D_{T-e} (M_{T-e}) (Table 5) be used as the final estimate of the reservoir volume. For an evaluation of the performance statistics, viz. NSE and MER, the V_R' and M_T

(larger between M_{T-e} and M_{T-o}) were compared on a 1:1 basis (Table 5, Figure 4B).

Table 5 Summary of the V_R', M_{T-o} and M_{T-e} based on the hybrid procedure on MA2 smoothed annual SHI sequences for the selected rivers.

River name and Flow data size	ϱ	σ_1 (m^3/s-yr)	Counting Method M_{T-o} = (D_{T-o}/σ_1)	Analytical Method M_{T-e} = D_{T-e}/σ_1 (Eq. 3)	D_{T-e}/σ_1 (Eq. 2)	Best M_T	V_R (m^3/s-yr)	V_R'
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Bow River (1911-18) N=108	0.57	5.18	13.02	7.55	13.22	**13.22**	78.11	15.08
Bow River (1955-18)N=64	0.57	5.05	11.29	6.37	10.66	**10.66**	56.73	11.23
S. Saskatchewan R. (1960-18) N=59	0.66	58.83	10.72	6.73	11.25	**11.25**	708.07	12.04
S. Saskatchewan R. (1970-11)N=42	0.53	60.62	9.46	5.30	8.41	**9.46**	257.52	8.32
English River (1922-18) N=97	0.58	19.06	9.00	7.39	12.87	**12.87**	254.72	13.36
English River (1965-16) N=52	0.51	18.73	11.55	5.64	9.19	**11.55**	30.44	10.64
Pagwachaun River (1968-18) N=51	0.40	5.79	4.61	5.17	8.33	**5.17***	2.69	5.26
Bevearbank River (1922-1997) N=76	0.30	0.56	4.17	5.45	8.95	**5.45***	2.63	4.69

Note: *(asterisk) means the value is based by comparing M_{T-o} and M_{T-e} (Equation 3) because of ϱ being < 0.42.

The foregoing selection criteria of the larger estimate between the M_{T-e} and the M_{T-o} (named as hybrid procedure) resulted in the value of NSE about 85% with an acceptable level of overestimation (MER = 4.7%). The criteria performed well in a majority of rivers except in cases where ϱ was found to be less than 0.42 for the MA2 sequences. In such cases, the value of the M_{T-e} based on Equation (3) was compared with the estimate of the M_{T-o} and the bigger value was chosen to represent the reservoir volume. In nutshell, the MA2 smoothing is satisfactory for the evaluation of reservoir volumes at the annual scale and conversely, a little gain is achieved by invoking higher MA smoothing, with the caveat that the bigger one between D_{T-e} (= $\sigma_1 \times M_{T-e}$) and D_{T-o} (= $\sigma_1 \times M_{T-o}$) should be chosen as the estimator of deficit volume to correspond with reservoir volume.

5. Discussion

From the foregoing analysis, it was observed, that the discrepancy between the V_R (SPA) and D_T (DM) is significant at the draft level of mean

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annual flow when only MA1 smoothing is applied on SHI sequences. The large discrepancies between the SPA and DM based (MA1) estimates are an eye-opener in that either the SPA yields too large values of reservoir volume or the DM method yields too small estimates. No such estimates can be considered to be absolute as each method has its own logistics and limitations. In the case of SPA, the difference between the full reservoir level (reservoir is assumed to be full at the beginning) and the lowest level reached during the sampling period, is taken as the reservoir volume. During this intervening period, several droughts including the longest one may occur and the recovery in the water levels in the reservoir may succeed. Conversely in the DM based method, no such assumption is invoked and thus the total shortfall of water below the long term mean flow in a river during the longest drought period is regarded as the required reservoir volume. The reservoir should be designed to store the above volume of water during the period of excess flow in the river.

In a bid to attain the same D_T values as V_R using the DM based method, the drought lengths and in turn, the magnitudes were amplified by a moving average procedure that resulted in the MA2 and MA3 sequences. The D_T values based on the MA2 sequences tend to undercount whereas those based on the MA3 sequences tend to over count compared to the V_R. On an annual basis, either MA2 or MA3 smoothing can only be conducted because there is no smoothing operation between these two, i.e. there is no integer number between MA2 and MA3 smoothing. Therefore, any further refinement of results stresses that the analysis should be conducted at a shorter time scale (i.e. the monthly scale). There exists an opportunity for a suitable match between the V_R and D_T values, provided MA smoothing such as 3-, 4-, 5-, 6- or higher monthly SHI sequences are utilized [28]. This is an area for further research justifying the use of monthly based analysis in the design of reservoirs.

It turned out that at the demand level at the mean annual flow of a river, the V_R values tend to be not significantly different from each other irrespective of what time scale is chosen. In contrast, the drought magnitude based method resulted in significant discrepancy among these estimates (D_T) with the annual time scale yielding much higher values compared to those at the monthly and weekly scales. In such a scenario, one can even be tempted to limit the analysis to the annual flow sequences only as it is trivial and the annual flow data can easily be synthesized or generated at any desired location on a river. The aim is to store enough water to meet any exigency or unaccounted episodes. There is also a need to examine other methods of estimating the reservoir capacity and compare them with the DM based estimates. Finally, all the estimates may be averaged out to arrive at the final design value of the reservoir capacity.

6. Conclusions

The analysis was carried out to compare the V_R and D_T respectively, using the SPA and the DM based method (counting and analytical procedures) on the flow data from 15 rivers across Canada. The estimates of V_R at the annual, monthly and weekly scales with the draft set at the mean annual flow level were observed close to each other with a tendency to slightly increase at the monthly and weekly scales. On the contrary, estimates of D_T at monthly and weekly scales tended to decrease compared to the annual scale. At all three time scales, the D_T estimates turned out to be smaller than V_R. To ameliorate the D_T to the level of V_R, the counting and analytical procedures (in the DM based Method) were applied to flow and SHI sequences on an annual scale. In the counting procedure, the best parity was found with the averaged out values of $D_{T\text{-}o}$ that were obtained by MA2 and MA3 smoothing of SHI sequences. Likewise, the analytical procedure yielded similar results (in terms of $D_{T\text{-}e}$) when applied on MA2 and MA3 smoothed SHI sequences. In the analytical procedure, the relationships were built on the extreme number theorem, the truncated normal probability distribution of the drought intensity, normal distribution of the drought magnitude and a Markov chain based value of extreme drought length. The estimation of $D_{T\text{-}e}$ was found inadequate when only the mean of the drought intensity was used. The consideration of the mean and the variance of the drought intensity in the analytical procedure turned out to be satisfactory and corroborated the results obtained from the counting procedure. Another finding of the study was that the MA2 smoothing of SHI sequences is sufficiently provided that the larger value between $D_{T\text{-}o}$ and $D_{T\text{-}e}$ is taken as a counterpart value of the reservoir volume for design purposes. The novel feature of the DM method lies in its ability to assess the reservoir volume without assuming the reservoir being full at the beginning of the analysis as is the case with SPA. Further, the DM based method is capable of considering the return period and associated risk in the design process of reservoirs. However, the DM method requires flow sequences to be stationary unlike the SPA, which applies to stationary and nonstationary flow sequences alike. It is recommended that the study be extended to the monthly scale at varying draft levels such as 80%, 70%, 60%, 50% etc. of the mean annual flow, which are largely used where environmental concerns are the overriding factors in the design of reservoirs across the globe.

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