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Article

Relativistic Cosmology with an Introduction to Inflation

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Abstract: In this review article the study of the development of relativistic cosmology and introduction of inflation in it is carried out. We study the properties of standard cosmological model developed in the framework of relativistic cosmology and the geometric structure of spacetime connected coherently with it. We examine the geometric properties of space and spacetime ingrained into the standard model of cosmology. The big bang model of the beginning of the universe is based on the standard model which succumbed to failure in explaining the flatness and the large-scale homogeneity of the universe as demonstrated by observational evidence. These cosmological problems were resolved by introducing a brief acceleratedly expanding phase in the very early universe known as inflation. Cosmic inflation by setting the initial conditions of the standard big bang model resolves these problems of the theory. We discuss how the inflationary paradigm solves these problems by proposing the fast expansion period in the early universe.

Keywords: spacetime; relativistic cosmology; big bang model; inflation

1. Introduction

With the advent of general relativity in 1916, spacetime transformed itself into one of the most fundamental interactions of the universe because the geometrical structure of it was taken to demonstrate gravity in a dynamical way [1]. The force of gravity was replaced by the curvature of spacetime mirrored through the structure of metric tensor. Spacetime became an integral part of the universe and a dynamical medium where the whole phenomenal universe exists. Any solution of the field equations of general relativity entails a certain structural geometry of spacetime or just a spacetime that represents a universe itself, therefore determining a solution of the field equations is like to coming across a specific model of the universe.

Cosmology studies the universe as a whole encompassing its beginning in spacetime or as spacetime itself, its evolution, and its eventual ultimate fate. The history of cosmology dates back to ancient Greeks, Indians and Iranians with its roots at that time in philosophy and religion. Before the modern scientific cosmology emerges, it has been nurtured in the womb of Ibrahamic religions especially Judaism, Christianity and Islam. Cosmology as modern science begins with the surfacing of general relativity when Einstein first himself put to use it to formulate a cosmological model of the universe mathematically. The model brought about a dynamic universe but was rendered to be static as there was no cosmological evidence of its contraction or expansion at that time [2]. Einstein's static model was afterward proved to be inconsistent with cosmological observations and was discarded but

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its formulation as the first mathematical model based on the field equations of general relativity laid the foundational stone for the inception of modern relativistic cosmology as science.

Cosmology takes into account the largest scale of spacetime that is the causally connected maximal patch of the cosmos from the perspective of the origin, evolution, and futuristic eventual fate. It gives the universe a mathematical description as large as the cosmological observational parameters reveal. Modern relativistic cosmology was established on general relativity which brings forth the big bang model of the universe. The big bang model was marred with some inward problems in it which were removed by introducing an exponentially expanding phase in the early universe known as inflation. de Sitter presented a model of the empty universe with the cosmological constant term retained. The geometry of the model was proved to be accelerating [3]. The de Sitter universe corresponds to the specific case related to one of the very early solutions of Einstein's Field Equations. As the actual universe must be considered as a local set of perturbations on the geometry of de Sitter having validity in the large. de Sitter geometry represents Euclidean space with a metric that depends on time. It was found that the inflation could be the de Sitter geometry or quasi de Sitter geometry which has an innate impact on the evolution of the geometry of FLRW spacetimes. It further bears its relation with the late-time accelerated expansion of the universe and the dynamic geometry of the spacetime innately cohered with it. Inflation as it was propounded, has a profound impact on the evolution of the universe as the geometry of spacetime. de Sitter universe represents the inflationary phase of the universe with slightly broken time translational symmetry.

Alexander Friedmann predicted theoretically the universe to be dynamic, the one which can expand, contract, or even be born out of a singularity [4]. George Lemaitre unaware of Friedmann's work at that time independently reached the same conclusion. In 1931 he also proposed a theory of the primeval atom which later on was known as the big bang theory by Hoyle etc. [5]. Edwin Hubble first proved the existence of other galaxies besides our's Milky Way and afterward in 1929 discovered that the universe is expanding based on observational evidence [6]. In the late 1940s George Gamow (1904-1968) and his collaborators, Ralph Alpher $(1921-2007)$ and Robert Herman $(1914-1997)$ independently worked on Lemaître's hypothesis and transformed it into a model of the early universe. They supposed the initial state of the universe comprising of a very hot, compressed mixture of nucleons and photons, thereby introducing the big bang model. They did not associate a particular name with the early state of the universe. Based on this model they were successful in calculating the amount of helium in the universe but unfortunately, there was no authentic observational evidence with which their calculations could be compared [7]. The standard relativistic model of cosmology underpinning big bang theory could not explain the global structure of the universe and the origin of matter in it. The distribution of matter homogeneously on large scales and spatial flatness also remained enigmatic. The big bang model just made an assumption about these but could not solve them. In the frame work of effective field theory, the aspects of nonsingular cosmology were explored by Yong Cai1 et al. It is shown that the effective field theory assists in having the clarification about the origin of no-go-theorem and helps to resolve the this theorem [8].

The inflationary era was proposed in the standard model of cosmology which propounds the big bang theory of the creation of the universe. Inflation solves the problems encountered in the big bang cosmology. Gliner in 1965 hypothesized an era of exponential expansion for the universe earlier than any significant inflationary model surfaced [9]. It was found that the scalar fields are dynamic and it was considered in 1972 that during phase transitions the energy density of the universe as scalar field changes [10]. Linde in 1974 realized that scalar fields can play an important role in describing the phases of the very early universe. He speculated that the energy density of a scalar field can play the role of vacuum energy dubbed as a cosmological constant [10]. In 1978, Englert, Brout and Gunzig [12], forwarded a proposal of 'fireball' hypothesis attempting to resolve the primordial singularity problem. They basing their investigations on the entropy contained in the universe and with introducing particle production approached the issue of early evolution of the universe. They inquired into it based on their hypothesis that the universe undergoing a quantum mechanical effect would itself appear in a state

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of negative pressure subject to a phase of exponential expansion. A work was mentioned by Linde in his review article [13] where he sought, in collaboration with Chibisov, to develop a cosmological model based upon the facts known about the scalar fields. They considering the supercooled vacuum as a self-contained source for entropy tried to cause the exponential expansion of the universe to be concerned with it, however they found out instantly that the universe becomes very inhomogeneous after the bubble wall collisions.

Slightly before Alan Guth's original proposal of inflation surfaced, Alexei Starobinsky in 1980 proposed a model of inflation on the base of a conformal anomaly in quantum gravity. His proposal was presented in the framework of general relativity where slight modification of the equations of general relativity was made and quantum corrections were employed to it in order to have a phase of the early universe. Starobinsky's model can be considered as the first model of inflation which is of semi-realistic nature and evades from the graceful exit problem [14]. It was hardly concerned with the problems of homogeneity and isotropy which occur in the relativistic cosmological model of the big bang. Starobinsky's model, as he accentuated, can be considered the extreme opposite of chaos in Misner's model. Tensor perturbations that represent gravitational waves have also been predicted in Starobinsky's model with a spectrum that is flat.

Alan Guth employed the dynamics of a scalar field and with a clear physical motivation presented an inflationary model [15] in 1981 on the base of supercooling theory during the cosmic phase transitions where the universe expands in a supercooled false vacuum state. A false vacuum is a metastable state containing a huge energy density without any field or particle so that when the universe expands from this heavy nothingness state its energy density does not change and empty space remains empty so that the inflation occurs in false vacuum [16]. The inflationary phase in Guth's original scenario is too short to resolve any problem and the universe becomes very inhomogeneous which leads to the graceful exit problem [17,18]. The problem prevents the universe from evolving to later stages and is inherently existing in the originally proposed version of Guth.

The graceful exit problem was addressed independently by Linde, Steinhard and Albrecht [19–24] where they introduced the phase of slow roll inflation inclusively known as new inflation. The resolution of the problem was sought by constructing a new inflationary paradigm where the inflation can has its inception either in an unstable state at the top of the effective potential or in the state of false vacuum. In this scenario the dynamics of the scalar field is such that it rolls gradually down to the lowest of its effective potential. It is of great importance to note that the shifting away of the scalar field from the false vacuum state to other states has remarkable consequences. When the scalar field rolls slowly towards its lowest-so called slow roll inflation, the density perturbations are generated which seed the structure of the universe. The production of density perturbations during the phase of slow roll inflation is inversely proportional to the motion of the scalar field [25–31]. The basic difference between the new inflationary scenario and that of the old one is that the advantageous portion of the inflation in the new scenario, which is responsible for the large scale homogeneity of the universe, does not take place in the false vacuum state, where the scalar field vanishes. This means that new inflation could explain why our universe was so large only if it was very large and contained many particles from the very beginning. In this article we study the standard model of cosmology by investigating the geometric structure of spacetime. We begin with introducing the structure of Euclidean space and that of spacetime in special and general theory of relativity with a quick review of their basics. We discuss problems encountered in the standard big bang cosmology and the inflationary solutions introduced into it by proposing a phase of accelerated expansion in the early universe.

The layout of the paper follows as: Section 2 discusses the structure of Euclidean space beginning with the axioms of Euclidean geometry and the significant role played by the Pythagoras theorem to locate the distance between points of this space. Pythagoras theorem acts fundamentally as a metric formula or line element for Euclidean space. This section contains 4 subsections that investigate the structure of spacetime in Newtonian physics, special relativity, and general relativity with a discussion of the basics of general relativity. Section 3 begins with relativistic cosmology investigating

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cosmological principle, Weyl's principle, and general relativity in its 3 subsections. In Section 4 we investigate the standard model in cosmology represented by the FLRW metric. The section comprises 9 subsections that discuss three possible geometries of its spatial part namely spherical, hyperbolic and Euclidean. We solve the metric using Einstein Field Equations. Section 5 gives the derivation of Friedmann's equations and further derive these equations in the presence of the cosmological term in subsection 5.1. In section 6 we describe how to embed a geometrical object in a space of higher dimensions. This section has 4 subsections discussing intrinsic geometry, extrinsic geometry, the geometry of 2-sphere, and geometry of 3-sphere as they get embedded in higher-dimensional space. Section 7 presents Einstein's static model and has 2 subsections that discuss the instability of Einstein's universe and de Sitter's empty universe model respectively. In Section 8 we describe conformal FLRW line elements and discuss vacuum, radiation, and matter-dominated eras. Critical density, density parameter, deceleration parameter, Friedmann's equations in terms of density parameter, cosmological redshift, luminosity distance, and angular distance formula in its 12 subsections. section 9 is devoted to the discussion of cosmological problems faced by the standard model in its 4 subsections. In Section 10 we embark on inflation and discuss the following topics in its subsections: Starobinsky inflation, scalar field dynamics, slow-roll inflation, slow-roll parameters, and the number of e-folds that quantify inflationary period. Section 11 describes how the proposal of exponential expansion in the early universe solves the cosmological problems. In the last section, we give a summary of the paper. There are four appendices at the end after refernces given in the paper.

2. Euclidean Space

Euclidean geometry is established on a set of simple axioms and definitions derived from these axioms. A space at the level of mathematical abstraction is the set of points where each point represents a specific position in it. When the abstract space is mapped onto physical space each point of it represents a physical location in physical space. Euclidean space is what entails on the base of axioms of Euclidean geometry. Geometrically a space can be described by reducing it to a certain specification of the distance between each pair of its neighboring points. In order to reduce all of the geometry of a space to a certain specification of the distance between each pair of neighboring points we use the metric or line element which measures the space and describes its nature. A line element specifies a certain geometry and its form varies corresponding to different coordinate systems. Five basic postulate lie at the core of Euclidean space and are the basis of standard laws of geometry.

1. Any two points can be joined by a straight line *i.e*. the shortest distance between two points is a straight line. 2. A straight line can be extended to any length. 3. A circle can be drawn with a given a line segment as radius and one end as centre of the circle. 4. All right angles are congruent. 5. Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line *i.e*. parallel lines remain a constant distance apart. Pythagoras theorem was known before Euclid and can also be derived from the five postulates and is used to find distance between any two points in Euclidean space. A mathematical space is an abstraction used to model the physical space of the universe.The Euclidean space consists of geometric points and has three dimensions. Now the Pythagorean theorem for a right triangle describes how to calculate the length of hypotenuse when the lengths of other two sides namely base and altitude are given.

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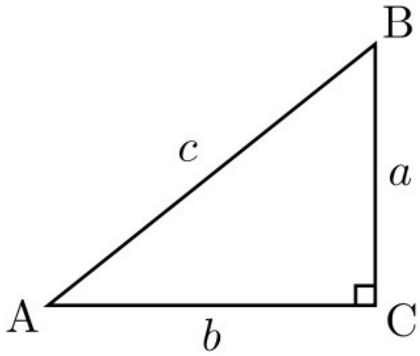

Figure 1. Pythagoras theorem: $c^2 = a^2 + b^2$

$$d^2 = x^2 + y^2 \tag{1}$$

Now since space can be expressed everywhere consisting of geometric points. We can define for every infinitesimally close three points of space forming a right triangle so that we can find distance between two points with the help of Pythagoras theorem. Using rectangular Cartesian coordinate system we can express distance between two points in differential form

$$dl^2 = dx^2 + dy^2 \tag{2}$$

The distance-measure by pythagoras theorem in Eq. (2) will be known as metric or line element in two dimensions and defines Euclidean metric for two dimensional space. The distance measured between two points by the metric in Eq. (2) does not change on rotating the coordinate system in which these two points are specified.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{3}$$

or

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned} \tag{4}$$

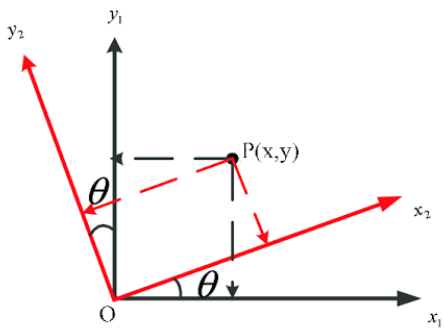

Figure 2. The rotation of two dimensional rectangular coordinate system through angle *theta*

The distance between two points remains invariant which means that

$$dx'^2 + dy'^2 = dx^2 + dy^2 \tag{5}$$

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The Pythagorean theorem in three dimensions can be described as

$$d^2 = x^2 + y^2 + z^2 \tag{6}$$

Three mutually perpendicular planes along three dimensions of the Cartesian coordinate system divide it in 3-planes.

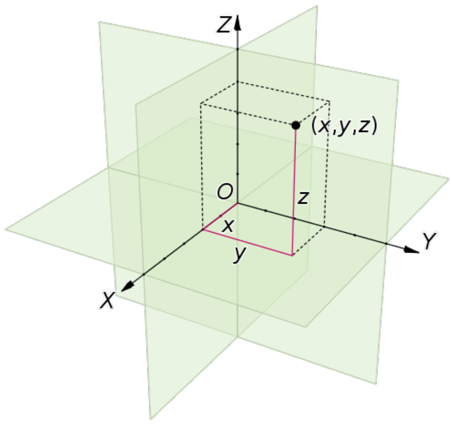

Figure 3. Three dimensional rectangular Cartesian plane representing Euclidean space-three mutual perpendicular planes

Now in reference to a coordinate system each point of this space will have three coordinates (x, y, z) if we approach its structure through Cartesian scheme, i.e. in Cartesian coordinates each point of it is represented by three coordinates which are the distances measured starting from the origin of the coordinate axes along the corresponding axes *i.e* x-axis, y-axis and z-axis respectively. These three axes stand for three dimensions of space. we find the distance between two points with Cartesian coordinate for three points separated infinitesimally

$$dl^2 = dx^2 + dy^2 + dz^2 \tag{7}$$

which gives the metric of three dimensional space The distance between two points with Cartesian coordinates (x, y, z) and (p, q, r) will be

$$ds^2 = (x - p)^2 + (y - q)^2 + (z - r)^2 \tag{8}$$

The infinitesimal distance between any two points (x, y, z) and $(x + dx, y + dy, z + dz)$ can be had using the metric written above in Eq. (7) in three dimensional Euclidean space.

$$ds^2 = [x - (x + dx)]^2 + [y - (y + dy)]^2 + [z - (z + dz)]^2 \tag{9}$$

$$ds^2 = (-dx)^2 + (-dy)^2 + (-dz)^2 = dx^2 + dy^2 + dz^2 \tag{10}$$

Or in tensor form

$$ds^2 = \delta_{\mu\nu} dx^\mu dx^\nu \tag{11}$$

Where $\delta_{\mu\nu}$ is the Kronecker delta function representing a symmetric tensor of rank two and can be expressed as a 3×3 matrix form

$$\delta_{\mu\nu} = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

diagonal of

$$\delta_{\mu\nu} = diag\left[\delta_{\mu\nu}\right] = [+1, +1, +1] \tag{13}$$

and trace of

$$\delta_{\mu\nu} = \sum_{\mu=\nu}\left[\delta_{\mu\nu}\right] = 1 + 1 + 1 = 3 \tag{14}$$

Thus $\delta_{\mu\nu} = diag\,(+1, +1............ + 1)$ in Eq. (13) defines the n-dimensional Euclidean space.

Now Eq. (11) can be expanded using Einstein summation convention

$$ds^2 = \delta_{1\nu}dx^1dx^\nu + \delta_{2\nu}dx^2dx^\nu + \delta_{3\nu}dx^3dx^\nu \tag{15}$$

$$\begin{aligned} ds^2 = &\left(\delta_{11}dx^1dx^1 + \delta_{12}dx^1dx^2 + \delta_{13}dx^1dx^3\right) \\ &+ \left(\delta_{21}dx^2dx^1 + \delta_{22}dx^2dx^2 + \delta_{23}dx^2dx^3\right) \\ &+ \left(\delta_{31}dx^3dx^1 + \delta_{32}dx^3dx^2 + \delta_{33}dx^3dx^3\right) \end{aligned} \tag{16}$$

$$\begin{aligned} ds^2 = &\left((1)\,dx^1dx^1 + (0)\,dx^1dx^2 + (0)\,dx^1dx^3\right) \\ &+ \left((0)\,dx^2dx^1 + (1)\,dx^2dx^2 + (0)\,dx^2dx^3\right) \\ &+ \left((0)\,dx^3dx^1 + (0)\,dx^3dx^2 + (1)\,dx^3dx^3\right) \end{aligned} \tag{17}$$

$$ds^2 = \left(dx^1dx^1 + 0 + 0\right) + \left(0 + dx^2dx^2 + 0\right) + \left(0 + 0 + dx^3dx^3\right) \tag{18}$$

$$ds^2 = \left(dx^1dx^1\right) + \left(dx^2dx^2\right) + \left(dx^3dx^3\right) \tag{19}$$

$$ds^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 \tag{20}$$

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{21}$$

Eq. (21) can also be expressed in the form

$$dsds = dxdx + dydy + dzdz \tag{22}$$

From Eq. (22) we can see that the inner product in three dimensional Euclidean space can be perfectly described that's why three dimensional Euclidean space is an example of a complete inner product space.

2.1. Newtonian Mechanics: The Structure of Space and Time

Space and time are absolute structures in classical physics and can be distinguished from one another in an independent way. Newton's Mechanics is based specifically on three laws of motion, law of gravitation and Galilean principle of relativity which are inherently related with the properties of space and time. Newtonian space is three dimensional extension around us which constitutes absolute space. Absolute space in Newton's own words is described as "Absolute space, in its own nature, without relation to anything external remains always similar and immovable", therefore space is rigid, motionless and can be viewed as colossally empty three dimensional cubic or cuboidal box where material objects reside and all physical phenomena take place. Newtonian space has the properties of Euclidean pace where infinitesimal distance between any two points is a straight line and if three points constitute a right angled triangle, then three sides are related by Pythagoras theorem which ascribes to it the properties of a flat space. Sum of angles in a triangle in such space is 180^0. Newtonian space is homogeneous and isotropic which entails Newtonian Mechanics. Homogeneity implies translational invariance of the properties of space which means that it has similar properties at every point contained in it. The property of being homogeneous is called homogeneity that leads to the invariance of physical laws performed in two or more coordinate systems. Newton's 3rd law, law of conservation of momentum and energy etc. come out as a consequence of homogeneity. It is also isotropic that implies rotational invariance of the properties of space which means it has similar

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properties in all directions therefore it is direction-independent. Thus isotropy implies homogeneity but the converse is not true. The absolute time has been enunciated as follows "Absolute time, and mathematical time of itself and from its own nature flows equably without relation to anything external, and is otherwise called duration" such time exists independent of space and whatever dynamically happens in it and flows uniformly in one direction. An interval of time possesses always unchanging meaning for all times.

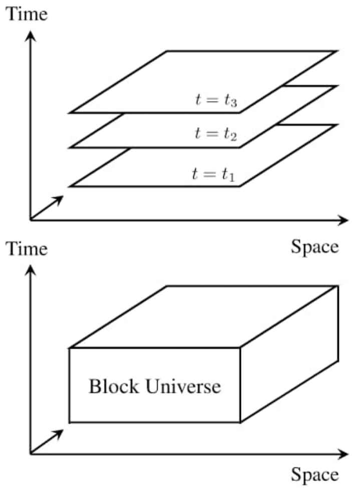

Figure 4. Newtonian space

According to Newtonian Mechanics gravitation and relative motion do not affect the rate at which time flows. From Newton's 2nd law $F = ma$, the isotropy of time can be viewed in case the dynamic system does not change from perpetrating transition from $+t$ to $-t$ since it does not incorporate the element of time explicitly which implies that past and future are indistinguishable but this is paradoxical because time is unidirectional and flows always from past to future. Two observers in two inertial frames of reference in relative motion and equipped with standard measuring clocks record the spacetime coordinates of an event written as (t, x, y, z) and (t', x', y', z') respectively. According to Galilean principle of relativity, the coordinate transformations are

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned} \tag{23}$$

We can calculate how velocities are added according to these transformations by differentiating the spatial parts of Eq. (23) with respect to time t, we have

$$\begin{aligned} \frac{dx'}{dt} &= \frac{dx}{dt} - v \\ \frac{dy'}{dt} &= \frac{dy}{dt} \\ \frac{dz'}{dt} &= \frac{dz}{dt} \end{aligned} \tag{24}$$

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Since $t = t'$, we infer that $\frac{dx'}{dt} = \frac{dx'}{dt'}$ Likewise, acceleration can also be had by differentiating once again Eq. (24), we hve

$$\begin{aligned} \frac{d^2x'}{dt^2} &= \frac{d^2x}{dt^2} \\ \frac{d^2y'}{dt^2} &= \frac{d^2y}{dt^2} \\ \frac{d^2z'}{dt^2} &= \frac{d^2z}{dt^2} \end{aligned} \tag{25}$$

We can observe from Eq. (25) that the accelerations in both frames are same. The time-coordinate t' of one inertial frame remains unaffected during transformation to another inertial frame of reference in classical physics and does not depend on spatial coordinates x, y and z. The set of equations in Eq. (23) is known as Galilean transformations. The motion along y and z spatial dimensions remains unaffected and the time coordinates in the two frames are equivalent which implies that time is absolute as Newton believed meaning that for all the inertial observers the time interval between any two events would be invariant. We notice that the two events having coordinates (t, x, y, z) and (t', x', y', z') respectively with differential of the distance as Euclidean spatial interval described in Eq. (21) as $ds^2 = dx^2 + dy^2 + dz^2$ and the time interval $\Delta t = t' - t$ both remain separately invariant under the Galilean transformations in Eq. (23). This fact makes us consider the nature of space and time as absolute entities in Newtonian Mechanics. We identify the quantity ds^2 as square of the distance between points of three dimensional Euclidean space and invariance of this differential of distance alludes to the fact that it is geometrical structural property of the space itself in its own right. This describes the geometry of space and time according to Newton's views.

2.2. Special Theory of Relativity: The Structure of Spacetime

The theory of special relativity is a theory of the structure of spacetime and in this way constitutes a geometric theory [32]. The fields and particles grow over this spacetime structure and relativistic mechanics is developed according to this structure which corresponds to the postulates of special relativity. According to the Lorentz transformations implied by it space and time are not distinguishable quantities but constitute innately a single continuum to be known as spacetime. One of the Einstein's 1905 papers brought forward this theory founded upon two postulates [33]

(1) The principle of special covariance

Since the laws of physics remain form-invariant *i.e* covariant according to a privileged class of observers known as inertial frames. This is also called principle of relativity.

(2) The principle of invariance of the velocity of light (c)

These two principles overthrew the pre-relativity notions of absolute space and absolute time proposing instead relative concepts. In classical physics as we saw earlier the coordinates of two observers are related by Galilean transformations whereas according to the special relativity the coordinates in two frames are related using Lorentz transformations.

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned} \tag{26}$$

Lorentz transformations contain all the geometric information about space and time and describe the structure of spacetime. Further, we can see that space and time coordinates are absolute according to the

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Galilean transformations for two inertial observers which move relative to each other and are connected through space and time coordinates. Time coordinate has the same magnitude in pre-relativity physics however according to the special relativity which obeys Lorentz transformations time coordinate in one coordinate system is connected to the time coordinate of the second coordinate system through both time and space coordinates which alludes to the fact that space and time coordinates are now to be dealt on equal footings. It is obvious from the Lorentz transformations that the time coordinates are not equivalent in two frames *i.e.* $t \neq t'$ rather t' is innately cohered with both of the coordinates of time and space t and x respectively. It means that time t' of one coordinate frame converts partially in space and partially in time coordinates. Therefore t' does not remain independent but has partially coalesced with space coordinates losing its absolute nature and the principle of relativity forbade us to locate a preferred frame of reference ensuing that absolute notion of time disappears logically. This fact was first perceived by Minkowski when he was recasting the special relativity in the language of geometry. He has presented a very profound and significant geometrical structure underlying special relativity. While delivering a lecture at the meeting of the Göttingen Mathematical Society on November 5, 1907, he introduced the concept of spacetime continuum whereby he asserted that independent space and time have to doom away into mere shadows and only a union of the two can preserve an independent reality. Minkowski viewed that the principle of special relativity can be described by the metric $-dt^2 + dx^2 + dy^2 + dz^2$ on the four-dimensional space R^4 which familiarized the concept of spacetime continuum and paved the way for the formulation of general relativity. A Minkowski metric g on the linear space R^4 is a symmetric non-degenerate bilinear form with signature $(-,+,+,+)$. It means that there exists a basis $\{e_0, e_1, e_2, e_3\}$ such that $g\left(e_\mu, e_\nu\right) = g_{\mu\nu}$ where $\mu, \nu \in \{0, 1, 2, 3\}$ and $g_{\mu\nu}$ is expressed in the form

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{27}$$

so that an we have orthonormal basis and can construct a system of coordinates of R^4 as $\left(x^0, x^1, x^2, x^3\right)$ such that at each point we can have $e_0 = \partial_t$ and ∂_{x^j} where $j = 1, 2, 3$. Now with respect to this coordinate system we can write the metric tensor $(0, 2)$ in the form $g = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \sum_1^3 dx^j$ or $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$ The negative sign with one time component term in the metric indicates that it is not Euclidean space but represents a pseud-Euclidean known as Minkowski space and also guarantees that the speed of light is same in all inertial frames. An expanding Minkowskian spacetime can be expressed in form written below which represents the simplest of all dynamic spacetimes $ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$. It was considered convenient on the dimensional grounds to introduce the coordinates in the form $\left(x^0, x^1, x^2, x^3\right) = (ct, x, y, z)$. Pythagoras theorem applied in Euclidean space R^3 of three spatial dimensions gives the distance of two points as an invariant as we observed in previous section.

$$ds^2 = +dx^2 + dy^2 + dz^2 \tag{28}$$

here ds the length element is a scalar quantity which means that in certain frame of references all the observers will agree upon the length of the measured object. In 1905, Einstein speculated that the measurement of the spacetime interval

$$ds^2 = -dx^2 - dy^2 - dz^2 + (cdt)^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{29}$$

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where

$$\eta_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{30}$$

would not result in identical either in space or in time [34] for the observers in relative uniform motion. However Minkowski noted that the four dimensional entity in Eq. (29) would remain invariant for all such observers. The basic significant idea which Minkowski took notice of was that the spacetime interval remains invariant for all the observers in uniform relative motion meaning that it is also a scalar upon which they all will agree. The metric of Minkowski space which is homogenous and isotropic is given by

$$g_{\mu\nu} = \eta_{\mu\nu} = diag\,(-1, +1, +1, +1) \tag{31}$$

thus the geometry of spacetime is flat in special relativity. If the Minkowskian geometry of spacetime has to expand it can be expressed as, however, in special relativity realm it is not expanding.

$$g_{\mu\nu} = \eta_{\mu\nu} = diag\left(-1, a^2(t), a^2(t), a^2(t)\right) \tag{32}$$

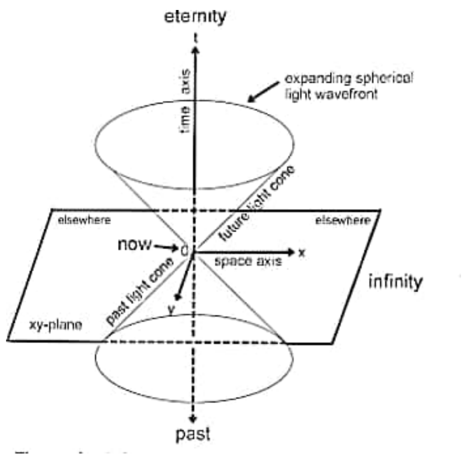

Figure 5. a spacetime frame as null cone structure

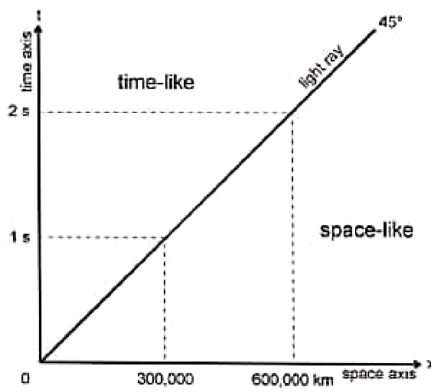

Figure 6. Structure of spacetime where one second of time along time axis equals 300000 *km* along the space axis

2.3. General Theory of Relativity: The Structure of Spacetime

General relativity models gravity into the dynamic structure of spacetime. In general relativity the structure of spacetime is described by a fundamental quantity called the spacetime metric or line element which gives the nature of the geometry of spacetime by finding the distance between two neighbouring points in it. The geometrical structure of spacetime is incarnated in two basic principles [35] 1-principle of general covariance 2-The spacetime contnuum has, at each of its points, a quadratic structure of coordinate differentials $ds^2 = g = g_{\mu\nu} dx^\mu dx^\nu$ known as 'square of the interval' between the two points under consideration. We consider a four dimensional continuum every point of which is distinct from the other with four coordinates-a quadruplet x_1, x_2, x_3, x_4 assigned consecutively to each of them

$$\begin{aligned} x'_1 &= x'_1 (x_1, x_2, x_3, x_4) \\ x'_2 &= x'_2 (x_1, x_2, x_3, x_4) \\ x'_3 &= x'_3 (x_1, x_2, x_3, x_4) \\ x'_4 &= x'_4 (x_1, x_2, x_3, x_4) \end{aligned} \tag{33}$$

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It is denoted by $g_{\mu\nu}$. In matrix form with components it is written as

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & a_{31} & g_{32} & g_{33} \end{pmatrix} \tag{34}$$

The properties of spacetime that intrinsically related to it are completely determined by the spacetime metric.

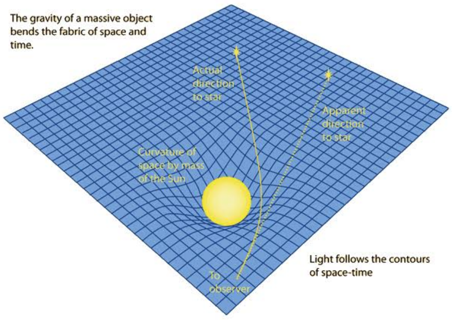

Figure 7. Curved spacetime around the Sun-spacetime in general relativity

2.4. The Basics of General Relativity

It would be convenient to have a retrospective look into the basics of general relativity whose role has been very fundamental to the modern cosmology. We briefly review the structure of the theory specifically in connection with the geometrical structure of spacetime in it. General relativity in its core describes that gravity is the geometry of four dimensional spacetime manifested through its curvature. It is a theory of spacetime and gravitation that are the very basic components of the universe. Einstein's journey towards general relativity in order to introduce gravity in his previous theory sought the fascinating geometry of the structure of spacetime such that gravity as a field force disappeared and was assimilated in the very geometric structure of spacetime. In constructing the framework of new theory, Einstein was influenced and governed by Mach's principle which states that it is a priori existence and distribution of matter which determines the geometry of spacetime and in the absence of it there shall be no geometric structure of a spacetime in the universe. Therefore, there will be no inertial properties in, otherwise, empty universe. In general relativity gravitation and inertia are essentially indistinguishable. The metric tensor $g_{\mu\nu}$ describes the effect of both combinedly and it is arbitrary to ask which one contributes its effect more and which less, therefore to call it with single name is suitable either inertia or gravitation [3]. In general relativity gravitation, Inertia and the geometry of spacetime are coalesced into a single entity represented by a symmetric tensor of second rank $g_{\mu\nu}$ which owes its existence due to presence and distribution of matter which is represented by an other symmetric tensor $T_{\mu\nu}$ known as energy-momentum tensor. The metric tensor $g_{\mu\nu}$ is the fundamental object of study in general relativity and takes into consideration all the causal and geometrical structure of spacetime. General relativity underlies five fundamental principles connoted in it in implicit or explicit manner, which are

1. Mach's principle
2. principle of equivalence
3. principle of covariance
4. principle of minimal gravitational coupling
5. correspondence principle

In the light of the principle of general covariance, the theory requires that the laws of physics might be formulated in a coordinate-independent style. The coordinate-independence requires the

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replacement of partial derivatives by covariant derivatives which introduces connection coefficients $\Gamma^{\lambda}_{\mu\nu}$ as the 2nd kind of Christoffel symbols. All the geometric structure of spacetime is based on the existence of these connection coefficients. The field equations of general relativity read as $G_{\mu\nu} = 8\pi T_{\mu\nu}$, where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor and is expressed in terms of Ricci tensor. metric tensor and Ricci scalar and $T_{\mu\nu}$ is energy momentum tensor. The spacetime contnuum of general relativity is postulated as a 4-dimensional Lorentzian manifold (M, g) where M denotes the Manifold and g is metric defined over it. The geometry of a spacetime is encoded in its metric which has a geodesic structure, though complex and frequently solved numerically for a specific bunch of geodesics. These geodesics specify the physical properties of the geometry of spacetime which are interpreted by drawing graphically in a certain spacelike volume. Gravity is the geometry of spacetime itself which is described through its dynamic structure in the framework of general relativity. Interaction between spacetime and the content it contains which mutually form warp and woof of the universe is the pith and marrow of general relativity. Matter tells spacetime how to curve and spacetime tells the matter how to move. General relativity thus transforms gravitation from being a force to being it a property of spacetime, so that gravity does remain a force but curvature of the geometric structure of spacetime. Einstein worked out a relation between matter-energy content of the universe and its gravitating effects in the form of geometry of spacetime. He employed the language of tensors to describe it. The invariant interval between two events separated infinitesimally with coordinates (t, x, y, z) and $(t + dt, x + dx, y + dy, z + dz)$ has been defined according to special relativity

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu} \tag{35}$$

Which defines a Lorentz invariant Minkowski flat spacetime whose geometry of spacetime is encoded in $\eta_{\mu\nu}$. Under the change of coordinates ds^2 remains invariant and is spacelike for $ds^2 > 0$, timelike for $ds^2 < 0$ and lightlike for $ds^2 = 0$. Photon path is described by $ds = 0$ and baryonic matter follows a path between two events for which

$$\int ds = 0 \tag{36}$$

i.e. it generates stationary values and conforms to the shortest distance between two points to be straight line which means that there are no external forces to set their path deviated. General relativity was based on five principles incorporated in it explicitly or implicitly namely equivalence principle, relativity principle, Mach's principle, Correspondence principle. Tensors are geometric objects defined on a manifold M, which remain invariant under the change of coordinates. It is composed of a set of quantities which are called its components, therefore a it is the generalization of a vector which means that it has more than three components. They represent mathematical entities which conform to certain laws of transformations. The properties of components of a tensor do not depend on a coordinate system which is used to describe the tensors. Transformation laws of a tensor relate its components in two different coordinate systems. The mathematical representation of a tensor is displayed through considering usually a bold face alphabetical letter like A, B, T, P etc. with an index or a set of indices in the form of superscripts or subscripts or both in mixed form. These superscripts and subscripts in case of a tensor are called contravariant and covariant indices. Contravariant indices of a tensor are used to give the meaning of a contravariant components of it like A^{μ}, $A^{\mu\nu}$, $A^{\mu\nu\xi.....}$. Covariant indices of a tensor are used to signify the meaning of a contravariant components of it like A_{μ}, $A_{\mu\nu}$, $A_{\mu\nu\xi.....}$. The indices of both types-contravariant and covariant are used to specify the components of a mixed tensor like A^{ν}_{μ}, $A^{\nu}_{\mu\xi}$, $A^{\nu\sigma}_{\mu}$, $A^{\nu\sigma...}_{\mu\xi\upsilon...}$. A mixed tensor is a tensor which has contravariant as well as covariant components. The number of indices appearing in the symbol representing certain type of a tensor is known as its rank. The appearing indices in the symbol representing a tensor can be contravariant or covariant or both type of indices in it. The order of a tensor is the same thing as rank, only the name differs. The number of components of a tensor is related with its rank or order and the dimensions of the space in which the is being described. In an n-dimensional space, a tensor of rank, say, k will have number of components equal to Number of components of a tensor in n-dimensional space is

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equivalent to $n^k = (number\ of\ dimensions\ of\ space)^{rank}$. However, the spacetime of general relativity is pseudo-Riemannian having four dimensions, three spatial and one temporal. Coordinate patches are necessarily considered to map whole of the spacetime. Each point-event of a coordinate patch in the four dimensional pseudo-Riemannian spacetime is labelled by a general coordinate system , conventionally runs over 0, 1, 2, 3 where 0 stands for time and the rest for space. An inertial or otherwise frame of reference characterized by a coordinate system can be attached to every point event of the spacetime and coordinate transformations between any two coordinate systems and can be written

$$
\begin{aligned}
A'_{\mu} &= \frac{\partial x^{\nu}}{\partial x'^{\mu}} A_{\nu} \\
B'^{\mu} &= \frac{\partial x'^{\mu}}{\partial x^{\nu}} B^{\nu} \\
A'^{\mu}_{\nu} &= \frac{\partial x'^{\mu}}{\partial x^{\zeta}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} A^{\zeta}_{\sigma}
\end{aligned}
\tag{37}
$$

While switching to Riemannian geometry for non-Euclidean spaces ordinary partial differentiation is generalized to covariant differentiation and is defined using a semi-colon ; as

$$
\begin{aligned}
B_{\nu;\mu} &= \partial_{,\mu} B_{\nu} - \Gamma^{\sigma}_{\nu\mu} B_{\sigma} \\
B^{\nu}{}_{;\mu} &= \partial_{,\mu} B^{\nu} + \Gamma^{\nu}_{\mu\sigma} B^{\sigma}
\end{aligned}
\tag{38}
$$

Where comma , denotes an ordinary partial differentiation with respect to the corresponding variable. In covariant differentiation indices can also be raised or lowered with metric tensor, however covariant differentiation of it vanishes. The interval between infinitesimally separated events x^{μ} and $x^{\mu} + dx^{\mu}$ is given by

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{39}$$

The corresponding contravariant tensor of $g_{\mu\nu}$ is given by $g^{\mu\nu}$ and they result in Kronecker delta. Moreover, indices can be lowered or raised using the metric tensor in either form as

$$
\begin{aligned}
g_{\mu\nu} g^{\mu\zeta} &= \delta^{\zeta}_{\nu} \\
g_{\mu\nu} B^{\nu} &= B_{\mu} \\
g^{\mu\nu} B_{\nu} &= B^{\mu}
\end{aligned}
\tag{40}
$$

In general relativity all the geometry of curved spacetime is contained in the second rank symmetric tensor $g_{\mu\nu}$ known as fundamental or metric tensor and is the function of four coordinates $g_{\mu\nu} = g_{\mu\nu}(x_0, x_1, x_2, x_3)$ and $g_{\mu\nu}$ encodes all the information about gravitational field induced by presence of matter. It governs the other matter as a response mimicking the role of gravitational potential similar to that of Newtonian gravity so that the paths remain no more straight and the action in Eq. (36) determines the path of a free particle known as geodesic

$$\frac{d^2 x^{\mu}}{ds^2} + \Gamma^{\mu}_{\nu\zeta} \frac{dx^{\nu}}{ds} \frac{dx^{\zeta}}{ds} = 0 \tag{41}$$

where

$$\Gamma^{\mu}_{\nu\zeta} = g^{\mu\lambda} \Gamma_{\nu\zeta\lambda} = \frac{1}{2} g^{\mu\lambda} \left(\frac{\partial g_{\nu\lambda}}{\partial x^{\zeta}} + \frac{\partial g_{\zeta\lambda}}{\partial x^{\nu}} + \frac{\partial g_{\nu\zeta}}{\partial x^{\lambda}} \right) \tag{42}$$

are the Christoffel symbols which through the geodesic equation specify the worldlines of free particles. The "acceleration due to gravity" in Newtonian gravitation law is described by these symbols in Einstein's picture of gravity as the geometric properties of spacetime encoding the similar information. Locally these symbols vanish in the inertial frame of reference in free fall and under coordinate

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transformation from x^μ and x'^μ do not constitute components of a tensor and therefore do not represent a tensor.

$$\Gamma'^{\sigma}_{\mu\nu} = \frac{\partial x'^{\sigma}}{\partial x^{\lambda}} \frac{\partial x^{\zeta}}{\partial x'^{\mu}} \frac{\partial x^{\rho}}{\partial x'^{\nu}} \Gamma^{\lambda}_{\zeta\rho} + \frac{\partial^2 x^{\zeta}}{\partial x'^{\mu} \partial x'^{\nu}} \frac{\partial x'^{\sigma}}{\partial x^{\zeta}} \tag{43}$$

The Riemann tensor is defined as

$$R^{\sigma}_{\mu\nu\lambda} = \frac{\partial}{\partial x^{\nu}} \Gamma^{\sigma}_{\mu\lambda} - \frac{\partial}{\partial x^{\lambda}} \Gamma^{\sigma}_{\mu\nu} + \Gamma^{n}_{\mu\lambda} \Gamma^{\sigma}_{n\nu} - \Gamma^{n}_{\mu\nu} \Gamma^{\sigma}_{n\lambda} \tag{44}$$

It has symmetry properties and satisfies the following Bianchi identity

$$R^{\sigma}_{\mu\nu\lambda;\zeta} + R^{\sigma}_{\mu\zeta\nu;\lambda} + R^{\sigma}_{\mu\lambda\zeta;\nu} = 0 \tag{45}$$

The Ricci tensor is obtained from Riemann tensor contracting

$$R_{\mu\nu} = g_{\lambda\sigma} R^{\sigma}_{\mu\nu\lambda} = \frac{\partial}{\partial x^{\nu}} \Gamma^{\sigma}_{\mu\lambda} - \frac{\partial}{\partial x^{\lambda}} \Gamma^{\sigma}_{\mu\nu} + \Gamma^{n}_{\mu\lambda} \Gamma^{\sigma}_{n\nu} - \Gamma^{n}_{\mu\nu} \Gamma^{\sigma}_{n\lambda} \tag{46}$$

Another expression of Ricci tensor is written in the form given below when determinant of the metric tensor $g_{\mu\nu}$ is envisaged as a matrix and denoted by g

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \left(\ln \sqrt{-g}\right)_{,\mu\nu} + \left(\ln \sqrt{-g}\right)_{,\lambda} \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\pi\mu} \Gamma^{\pi}_{\lambda\nu} \tag{47}$$

The Ricci scalar or scalar curvature is described as

$$R = g^{\mu\nu} R_{\mu\nu} \tag{48}$$

Contraction of the Bianchi Identity in Eq. (45) gives

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \tag{49}$$

which is Einstein tensor. Now we can write basic equations of general relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \tag{50}$$

or

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{51}$$

$$G_{\mu\nu} \propto T_{\mu\nu} \tag{52}$$

These are written with cosmological constant also. From Eq.(52)

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{53}$$

Energy-momentum tensor $T_{\mu\nu}$ is the source term for the metric tensor $g_{\mu\nu}$ which for a most general matter-energy fluid that is consistent with the assumption of homogeneity and isotropy represents a perfect fluid and has the form

$$T_{\mu\nu} = (\rho + p)\, u_{\mu} u_{\nu} - p g_{\mu\nu} \tag{54}$$

Where $u^{\mu} = (1, 0, 0, 0)$ is the four velocity in a comoving frame of reference and

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{55}$$

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3. Relativistic Cosmology

Relativistic cosmology was founded on three fundamental principles

1. Cosmological princple;
2. Weyl's principle;
3. General relativity.

which are explicated below.

3.1. Cosmological Principle

The principle states that on sufficiently large scale, the universe is homogenous and isotropic at any time. The principle is the generalization of Copernican principle and almost all the standard cosmological models of the spacetime underpin it. It has two forms (1) Cosmological principle with respect to spatial invariance (2) Cosmological principle with respect to temporal invariance In spatial invariance we suppose the invariance of space with respect to translational and rotational properties known as homogeneity and isotropy respectively and the principle may be regarded as cosmological principle. Under both the invariant properties the space remains isomorphic. A perfect cosmological principle incorporates temporal homogeneity and isotropy which was employed by the steady state theory of the eternal universe and was not supported by the observation and was disfavored. For a local observer the principle might not be satisfied as the Earth and the solar system are not homogeneous and isotropic since the matter clumps together to form objects like planets, stars, galaxies with voids of vacuum-like in between them but on the larger scales of about MP>1000 Pc the universe obeys the cosmological principle. The uniformity of CMBR in all directions (homogeneity and isotropy) provides the confirmatory proof of the cosmological principle. It is the generalization of Copernican Principle which incorporates homogeneity and isotropy. Homogeneity means location independence *i.e.* all places in the universe at galactic scales are indistinguishable. Isotropy gives direction independence *i.e.* in whatever direction we look in the universe it appears same. Certainly Isotropy connotes homogeneity but vice versa is not true. To better understand its geometric properties, we begin with 1-dimensional spaces and revise to the four dimensional spaces and then observe how the four dimensional spacetime geometrical properties can be understood in this perspective. It is necessary to understand what we mean by embedding of a geometric object in an n-dimensional space because of the reason FLRW metric incorporates example of embedding three dimensional spaces in four dimensional spacetime.

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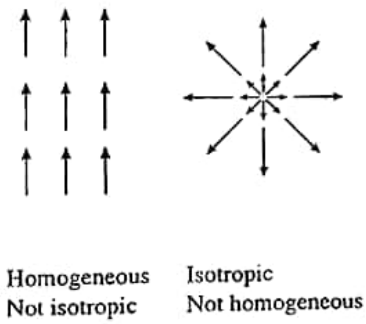

Figure 8. Homogeneity and isotropy of space as implied by cosmological principle

3.2. *Weyl's Principle*

Weyl's principle helps us consider the universal stuff as consisting of fluid, particles of which are constituted by galaxies. In the cosmological spacetime, the world lines of the fundamental observers form a smooth bundle of time-like geodesics which would never meet except in the past singularity from where the universe emerged or at the future singularity if it would happen. The fundamental observers are those who comove with the cosmic fluid. The world lines of galaxies as fluid particles are always and everywhere orthogonal to family of spatial hypersurfaces. The postulate was presented by Hermann Weyl (1885-1955) in 1923 which is essentially about the nature of matter in the universe [36]. He regarded the material content of the universe in the form of fluid whose constituent particles make a substratum.

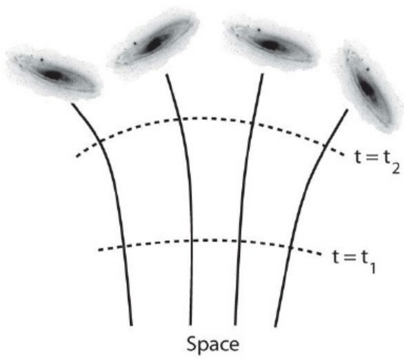

Figure 9. Illustration-of-the-Weyl-postulate

It mans that in the substratum of spacetime it allows us to consider the structure of the universe as fluid. The Weyl principle introduces further symmetry in the structure of spacetime described by the metric tensor by considering the galaxies as test particles and postulates that the geodesics on which these galaxies move do not intersect. It states that the world lines of galaxies considered as 'test particles' form a 3-bundle of nonintersecting geodesics orthogonal to a series of spacelike hypersurfaces.

3.3. *General Relativity*

General relativity provides the best existing theory of gravitation on cosmological scales and models it structured into the geometric structure of spacetime. In section III we have discussed its basic ingredients.

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4. The Standard Model of Cosmology

The standard model in cosmology has been established on the most general homogeneous and isotropic spacetime. The standard model that propounds the hot big bang model of the universe is known as Friedmann-Lemaitre-Robertson-walker (FLRW) line element which reads as in the Cartesian coordinates

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{56}$$

and in the spherical coordinates, we have

$$\begin{aligned} ds^2 = g = g_{\mu\nu} dx^\mu dx^\nu &= -dt^2 \\ &+ a(t)^2 \left(\frac{1}{1-kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \end{aligned} \tag{57}$$

Or equivalently

$$ds^2 = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2(t)r^2 & 0 \\ 0 & 0 & 0 & a^2(t)r^2\sin^2\theta \end{pmatrix} \tag{58}$$

The predictions for the quantitative behavior of the expanding universe is enunciated suitably by the metric tensor and the scale factor as a function of time *i.e.* $a(t)$ describes the scale of coordinate grid interrelating the coordinate distance with physical distance *i.e.* in a smooth and homogeneously expanding universe.

4.1. Geometric Properties of the FLRW Line Element

From the line element in Eq. (57)

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{1}{1-kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right] \tag{59}$$

Since the time flows only in one direction and the space obeys cosmological principe, therefore we are allowed to separate the metric in temporal and spatial parts. To understand the four dimensional spacetime geometry of FLRW universe we begin with the geometry of spatial part of the line element that is

$$a(t)^2 \left(\frac{1}{1-kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \tag{60}$$

This is the spatial part of the metric in Eq. (59) and is characterized by the scale factor $a(t)$, which is the function of time and 2nd curvature of the space k. These are obviously determined by the self-gravitating properties of the matter-energy content in the universe . The spatial part of the metric incorporates cosmological principle implying homogeneity and isotropy which provides the kinematics for the geometry of spacetime while we will observe afterwards that Einstein equations provide the dynamics into it through the scale factor $a(t)$.

4.2. Comoving Coordinates and Peculiar Velocities

The coordinates (r, θ, ϕ) form the cosmological rest frame and are known as comoving coordinates. They can be considered constant because the particles remain at rest in these coordinates. Peculiar velocity is the motion of the particles with respect to comoving coordinates. Peculiar velocities of the galaxies and supernovae are ignored in cosmology in the expanding spacetime. since $p(a) \propto \frac{1}{a(t)}$, therefore momentum in expanding spacetime is redshifted and freely moving particles come to rest

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in comoving coordinates. Physical distance between two points is calculated as thee scale factor $a(t)$ times the coordinate distance. The expression without scale factor inside the bracket is the pure kinematical statement of the geometry of spacetime

$$\frac{1}{1-kr^2}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{61}$$

and represents the line element of the three dimensional space with hidden symmetry of being homogeneous and isotropic. It represents three geometries for three values of k.

4.3. The Geometry of Spherical World

For $k = +1$, the hypersurface is

$$\frac{1}{1-r^2}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{62}$$

and represents a three dimensional sphere embedded in a four dimensional Euclidean space. This space is finite and closed.

4.4. The Geometry of Hyperbolic World

For $k = -1$, the hypersurface is

$$\frac{1}{1+r^2}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{63}$$

and represents a three dimensional hypersphere or hyperbola embedded in a four dimensional pseud-Euclidean space. This space is infinite and open.

4.5. The Geometry of Euclidean World

For $k = 0$, the hypersurface is

$$dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{64}$$

and represents a three dimensional Euclidean flat space. This space is also infinite and open. Now to determine Friedmann equations, we write first the components of the metric tensor, since the metric is diagonal due to homogeneity and isotropy therefore we have these diagonal components

$$\begin{aligned} g_{00} &= g_{tt} = -1 \\ g_{11} &= g_{rr} = \frac{a^2(t)}{1-kr^2} \\ g_{22} &= g_{\theta\theta} = a^2(t) r^2 \\ g_{33} &= g_{\phi\phi} = a^2(t) r^2 \sin^2\theta \end{aligned} \tag{65}$$

Now we turn to solve the FLRW metric and begins with finding Christoffel symbols of 2nd kind or the affine connections which are given by

$$\Gamma^{\sigma}_{\mu\nu} = g^{\sigma\lambda}\Gamma_{\mu\nu\lambda} = \frac{1}{2} g^{\sigma\lambda} \left(\frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} + \frac{\partial g_{\nu\lambda}}{\partial x^{\mu}} + \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} \right) \tag{66}$$

In four dimensions these will have $(4)^3 = 64$ components. The four generalized cases emerge in four dimensions for μ, ν, λ and σ
Case I: $\mu = \nu = \lambda$

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$$\Gamma^{\mu}_{\mu\mu} = \frac{1}{2}\frac{\partial}{\partial x^{\mu}} \log g_{\mu\mu} \tag{67}$$

In four dimensions

$$\begin{matrix} \Gamma^{0}_{00} & \Gamma^{1}_{11} \\ \Gamma^{2}_{22} & \Gamma^{3}_{33} \end{matrix} \tag{68}$$

will emerge.

Case II: $\sigma = \mu$, $\mu \neq \nu$

$$\Gamma^{\mu}_{\mu\lambda} = \frac{1}{2}\frac{\partial}{\partial x^{\lambda}} \log g_{\mu\mu} \tag{69}$$

In four dimensions the following twelve cases

$$\begin{matrix} \Gamma^{0}_{01} & \Gamma^{0}_{02} & \Gamma^{0}_{03} & \Gamma^{1}_{10} \\ \Gamma^{1}_{12} & \Gamma^{1}_{13} & \Gamma^{2}_{20} & \Gamma^{2}_{21} \\ \Gamma^{2}_{23} & \Gamma^{3}_{30} & \Gamma^{3}_{31} & \Gamma^{3}_{32} \end{matrix} \tag{70}$$

will emerge.

Case III: $\sigma = \mu$, $\mu = \lambda$

$$\Gamma^{\mu}_{\lambda\lambda} = -\frac{1}{2g_{\mu\mu}}\frac{\partial g_{\lambda\lambda}}{\partial x^{\mu}} \tag{71}$$

In four dimensions the following twelve cases

$$\begin{matrix} \Gamma^{0}_{11} & \Gamma^{0}_{22} & \Gamma^{0}_{33} & \Gamma^{1}_{00} \\ \Gamma^{1}_{22} & \Gamma^{1}_{33} & \Gamma^{2}_{00} & \Gamma^{2}_{11} \\ \Gamma^{2}_{33} & \Gamma^{3}_{00} & \Gamma^{3}_{11} & \Gamma^{3}_{22} \end{matrix} \tag{72}$$

will emerge.

Case IV: $\sigma \neq \mu \neq \nu$

$$\Gamma^{\sigma}_{\mu\nu} = g^{\sigma\lambda}\Gamma_{\mu\nu\lambda} = \frac{1}{2}g^{\sigma\lambda}\left(\frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} + \frac{\partial g_{\nu\lambda}}{\partial x^{\mu}} + \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}}\right) = 0 \tag{73}$$

In four dimensions the following twenty four cases

$$\begin{matrix} \Gamma^{0}_{12} & \Gamma^{0}_{21} & \Gamma^{0}_{13} & \Gamma^{0}_{31} \\ \Gamma^{0}_{23} & \Gamma^{0}_{32} & \Gamma^{1}_{02} & \Gamma^{1}_{20} \\ \Gamma^{1}_{03} & \Gamma^{1}_{30} & \Gamma^{1}_{23} & \Gamma^{1}_{32} \\ \Gamma^{2}_{01} & \Gamma^{2}_{10} & \Gamma^{2}_{03} & \Gamma^{2}_{30} \\ \Gamma^{2}_{13} & \Gamma^{2}_{31} & \Gamma^{3}_{01} & \Gamma^{3}_{10} \\ \Gamma^{3}_{12} & \Gamma^{3}_{21} & \Gamma^{3}_{20} & \Gamma^{3}_{02} \end{matrix} \tag{74}$$

emerge and vanish.

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4.6. Non-vanishing Christoffel Symbols

$$\begin{aligned}
&\Gamma^1_{11} = \tfrac{kr}{1-kr^2} \\
&\Gamma^1_{10} = \Gamma^1_{01} = \Gamma^2_{20} = \Gamma^2_{02} = \Gamma^3_{30} = \Gamma^3_{03} = \tfrac{\dot{a}(t)}{a(t)} \\
&\Gamma^2_{21} = \Gamma^2_{12} = \Gamma^3_{31} = \Gamma^3_{13} = \tfrac{1}{r} \\
&\Gamma^3_{32} = \Gamma^3_{23} = \tfrac{\sin\theta}{\cos\theta} \\
&\Gamma^0_{11} = \tfrac{a(t)\dot{a}(t)}{1-kr^2} \\
&\Gamma^0_{22} = a(t)\dot{a}(t)r^2 \\
&\Gamma^0_{33} = a(t)\dot{a}(t)r^2\sin^2\theta \\
&\Gamma^1_{22} = -r\left(1-kr^2\right) \\
&\Gamma^1_{33} = -r\sin^2\theta\left(1-kr^2\right) \\
&\Gamma^2_{33} = -\sin\theta\cos\theta
\end{aligned} \tag{75}$$

4.7. Riemann Curvature Tensor

The Riemann curvature tensor $R^{\sigma}_{\mu\nu\lambda}$ has $(4)^4 = 256$ components in four dimensions from which only twenty components can possibly be non-vanishing. The Riemann tensor is given by

$$R^{\sigma}_{\mu\nu\lambda} = \frac{\partial}{\partial x^{\nu}}\Gamma^{\sigma}_{\mu\lambda} - \frac{\partial}{\partial x^{\lambda}}\Gamma^{\sigma}_{\mu\nu} + \Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} - \Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} \tag{76}$$

The possibly non-vanishing twenty components are given by

$$\begin{matrix}
R^0_{110} & R^0_{220} & R^0_{330} & R^0_{221} \\
R^0_{331} & R^1_{001} & R^1_{221} & R^1_{331} \\
R^1_{332} & R^2_{002} & R^2_{112} & R^2_{332} \\
R^2_{021} & R^3_{003} & R^3_{113} & R^3_{223} \\
R^3_{031} & & &
\end{matrix} \tag{77}$$

The non-vanishing components are

$$\begin{aligned}
&R^0_{110} = -\tfrac{a(t)\ddot{a}(t)}{1-kr^2} \\
&R^0_{101} = \tfrac{a(t)\ddot{a}(t)}{1-kr^2} \\
&R^1_{010} = -\tfrac{\ddot{a}(t)}{a(t)} \\
&R^1_{001} = \tfrac{\ddot{a}(t)}{a(t)} \\
&R^0_{220} = -a(t)\ddot{a}(t)r^2 \\
&R^0_{202} = a(t)\ddot{a}(t)r^2 \\
&R^2_{020} = R^3_{030} = -\tfrac{\ddot{a}(t)}{a(t)} \\
&R^2_{002} = R^3_{003} = \tfrac{\ddot{a}(t)}{a(t)} \\
&R^0_{330} = -a(t)\ddot{a}(t)r^2\sin^2\theta \\
&R^0_{303} = a(t)\ddot{a}(t)r^2\sin^2\theta \\
&R^1_{221} = R^3_{223} = -r^2\left(k+\dot{a}^2(t)\right) \\
&R^1_{212} = R^3_{232} = r^2\left(k+\dot{a}^2(t)\right) \\
&R^2_{121} = R^3_{131} = \tfrac{k+\dot{a}^2(t)}{1-kr^2} \\
&R^2_{112} = R^3_{113} = -\tfrac{k+\dot{a}^2(t)}{1-kr^2} \\
&R^1_{331} = R^2_{332} = -r^2\sin^2\theta\left(k+\dot{a}^2(t)\right) \\
&R^1_{313} = R^2_{323} = r^2\sin^2\theta\left(k+\dot{a}^2(t)\right)
\end{aligned} \tag{78}$$

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4.8. Ricci Curvature Tensor and Ricci Scalar

Ricci tensor $(R_{\mu\nu})$ is obtained by contracting Riemann tensor $R^{\sigma}_{\mu\nu\lambda}$. We contract it by placing $\lambda=\sigma$, so that $R^{\sigma}_{\mu\nu\lambda}=R^{\lambda}_{\mu\nu\lambda}=R_{\mu\nu}$ In four dimensions it has $(4)^2=16$ components. These are

$$\begin{array}{ccccc} R_{00} & R_{11} & R_{22} & R_{33} & R_{01} \\ R_{10} & R_{02} & R_{20} & R_{03} & R_{30} \\ R_{12} & R_{21} & R_{31} & R_{13} & R_{23} \\ R_{32} & & & & \end{array} \tag{79}$$

The non-vanishing components are

$$\begin{aligned} R_{00} &= 3\frac{\ddot{a}}{a} \\ R_{11} &= -\frac{a(t)\ddot{a}(t)+2k+2\dot{a}^2}{1-kr^2} \\ R_{22} &= -r^2\left(a(t)\ddot{a}(t)+2k+2\dot{a}^2\right) \\ R_{33} &= -r^2\sin^2\theta\left(a(t)\ddot{a}(t)+2k+2\dot{a}^2\right) \end{aligned} \tag{80}$$

Ricci scalar (R) is obtained by contracting Ricci tensor

$$R = g^{\mu\nu}R_{\mu\nu} \tag{81}$$

Using double sums and simplifying in four dimensions, we have

$$R = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33} \tag{82}$$

$$R = -6\left[\frac{\ddot{a}(t)}{a(t)} + \left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{k}{a^2(t)}\right] \tag{83}$$

4.9. Einstein Tensor $(G_{\mu\nu})$

Einstein tensor is defined in terms of Ricci tensor $R_{\mu\nu}$, Ricci scalar R and the metric tensor $g_{\mu\nu}$. It is expressed as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{84}$$

In four dimensions it has $(4)^2=16$, components. These are

$$\begin{array}{ccccc} G_{00} & G_{11} & G_{22} & G_{33} & G_{01} \\ G_{10} & G_{02} & G_{20} & G_{03} & G_{30} \\ G_{12} & G_{21} & G_{31} & G_{13} & G_{23} \\ G_{32} & & & & \end{array} \tag{85}$$

The non-vanishing components are

$$\begin{aligned} G_{00} &= -3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \\ G_{11} &= g_{11}\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \\ G_{22} &= g_{22}\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \\ G_{33} &= g_{33}\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \end{aligned} \tag{86}$$

In Eq. (86) the spatial components of Einstein tensor can be written in a single equation of tensorial nature.

$$G_{\mu\nu} = g_{\mu\nu}\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \tag{87}$$

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where $\mu = \nu = 1,2,3$, and mixed Einstein tensor can be found by $g^{\zeta\nu}G_{\mu\nu} = G^{\zeta}_{\mu}$

$$\begin{aligned} G^0_0 &= 3\left[\left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] \\ G^1_1 &= g_{11}\left[2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] \\ G^2_2 &= g_{22}\left[2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] \\ G^3_3 &= g_{33}\left[2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] \end{aligned} \tag{88}$$

Now we calculate the energy-momentum tensor of a perfect fluid in mixed form. Cosmological principle and Weyl's postulate imply the material content of the universe to be regarded as perfect fluid [1]

$$g^{\zeta\nu}T_{\mu\nu} = T^{\zeta}_{\mu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \tag{89}$$

Non-vanishing components of energy-momentum tensor are

$$\begin{aligned} T^0_0 &= \rho \qquad & T^1_1 &= -p \\ T^2_2 &= -p \qquad & T^3_3 &= -p \end{aligned} \tag{90}$$

Putting the values of Einstein tensor $G_{\mu\nu}$ and energy-momentum tensor $T_{\mu\nu}$ from Eq. (88) and Eq. (89) respectively in Einstein field equations

$$G^{\zeta}_{\mu} = 8\pi T^{\zeta}_{\mu} \tag{91}$$

$$\begin{aligned} 3\left[\left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] &= 8\pi G\rho \\ g_{11}\left[2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] &= -8\pi Gp \\ g_{22}\left[2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] &= -8\pi Gp \\ g_{33}\left[2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k}{a^2}\right] &= -8\pi Gp \end{aligned} \tag{92}$$

5. Derivation of Friedmann's Equations

Now using the Einstein field equations, we set to derive the Friedmann's Equations that describe the evolution of the universe by relating the large-scale geometrical characteristics of spacetime to the large-scale distribution of matter-energy and momentum. From Eq. (92), we can write

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] = 8\pi G\rho \tag{93}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi Gp \tag{94}$$

For other two components listed in Eq. (92) the 2nd and 3rd components repeat, therefore we will write only one time from the three components. From Eq. (93) and Eq. (94) we derive the Friedmann's Equations and an equation for the conservation of matter. Substituting Eq. (93) in Eq. (94) and performing simplification we get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{95}$$

and from Eq. (93) which is the time-time component of the Einstein Equations.

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{96}$$

for

$$\frac{\dot{a}}{a} = H \tag{97}$$

which is Hubble parameter and gives expansion rate, The above Eq. (96) can be written as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{98}$$

differentiating Eq. (97) with respect to time 't'

$$\partial_t H = \partial_t \left(\frac{\dot{a}}{a}\right) \tag{99}$$

We obtain

$$\dot{H} = \frac{\ddot{a}}{a} - H^2 \tag{100}$$

which gives

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} \tag{101}$$

So that Eq. (95) takes the form in terms of Hubble parameter.

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3p) \tag{102}$$

We can also find

$$\dot{H} = -\frac{4\pi G}{3}(\rho + 3p) - H^2 \tag{103}$$

From Eq. (98) $H^2 = \frac{8\pi G}{3}\rho$ with $k = 0$, for flat universe substituting it in Eq. (103) above

$$\dot{H} = -\frac{4\pi G}{3}(\rho + 3p) - \frac{8\pi G}{3}\rho \tag{104}$$

which results in

$$\partial_t H = -4\pi G(\rho + p) \tag{105}$$

Now differentiating Eq. (93) with respect to time after shifting the factor 3 on the right side, we have

$$\frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\left(\frac{\dot{a}}{a}\right)^2 - 2\frac{k}{a^2}\right] = \frac{8\pi G}{3}\dot{\rho} \tag{106}$$

subtracting now Eq. (93) from Eq. (94), we obtain

$$2\frac{\ddot{a}}{a} - 2\left(\frac{\dot{a}}{a}\right)^2 - 2\frac{k}{a^2} = -8\pi G(\rho + p) \tag{107}$$

substituting Eq. (107) in Eq. (106), after simplification we have

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{108}$$

Cosmological principle compels us to consider a fluid in which inhomogeneities will be considered smoothed out and evolution of the universe shall be considered in the form of perfect fluid characterized by energy density ρ and isotropic pressure p. Further we consider that the pressure of the fluid depends only on the density neglecting its impact on the volume and the temperature *i.e.* $p = p(\rho)$ which defines a barotropic fluid. In addition, pressure and density bear a linear relationship

$$p \propto \rho \Rightarrow p = w\rho \tag{109}$$

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where $w = \frac{p}{\rho}$ is a dimensionless constant known as equation of state parameter. Substituting Eq. (109) in Eq. (108), we have another form of energy conservation for the equation of state parameter w,

$$\frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1+w) = 0 \tag{110}$$

Now, Eq. (95), Eq. (96) and Eq. (108) represent two Friedmann's Equations namely acceleration and evolution equations and equation of conservation respectively. According to this equation the evolution of all kinds of matter is determined by the conservation of energy and momentum.

5.1. Friedmann Equations with Cosmological Constant Λ

We have to incorporate dark matter and dark energy in the matter-energy content due to the significance of their role in current accelerated expansion and the present Minkowskian flat geometry of the universe. Therefore, their role is however unavoidable in the evolution of the universe. The solution of FLRW line element gives the Friedman's equations using Einstein field equations with cosmological constant Λ written usually in the form

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{111}$$

and Friedmann's equations with cosmological constant Λ can be worked out

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{112}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{113}$$

The equation of energy conservation can also be calculated from these Friedman equations in the presence of cosmological constant Λ. Multiplying Eq. (112) with $3a^2$, differentiating it with respect to time and then dividing by $\dot{a}$, we have

$$6\ddot{a} = 8\pi G a\left(2\rho + \frac{a}{\dot{a}}\dot{\rho}\right) + 2\Lambda a \tag{114}$$

dividing Eq. (114) by a.

$$6\frac{\ddot{a}}{a} = 8\pi G\left(2\rho + \frac{a}{\dot{a}}\dot{\rho}\right) + 2\Lambda \tag{115}$$

Substituting now the 2nd Friedman Equation from Eq. (113) in it, we have

$$6\left(-\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}\right) = 8\pi G\left(2\rho + \frac{a}{\dot{a}}\dot{\rho}\right) + 2\Lambda \tag{116}$$

after simplification, we obtain

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{117}$$

Where ρ and p are contributed by all whatever exists and constitutes the universe.

6. A Geometric Object Embedded in an n-Dimensional Space

An object cannot be placed in a space whose dimensions are equal or less than the object to be placed, rather the space must have larger number of dimensions in order to let the object allow rest in it. The presence of an object in a space having larger dimensions than the object is called embedding of it in that space.

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6.1. Intrinsic Geometry

The properties of the geometry that we have access to, based on visualization of the two dimensional beings are called intrinsic because two dimensional beings cannot observe how surfaces are shaped in three or higher dimensional spaces.

6.2. Extrinsic Geometry

The properties of the geometry that we have access to, based on visualization of higher dimensional creature are called extrinsic because higher dimensional creature can observe how surfaces are shaped in three or higher dimensional spaces. The geometrical properties related to an object describing how it has been embedded in some higher dimensional space. Extrinsic geometrical properties depend on how the bodies are placed in the space and how they affect it. The geometry which comes into existence due to interaction between space and the body placed in it describes the extrinsic properties. General relativity considers the geometry of spacetime as the extrinsic property of an object and owes its existence due to the body being present in it.

6.3. The Geometry of 2-Sphere Embedded in Three Dimensional Space

We consider a three dimensional Euclidean space where three dimensions namely length, width and height are represented by three coordinate axes respectively, as we know this space consists of points separate from time, and therefore we do not call its points as events. We assign the triplet of three Cartesian coordinates (x, y, z) to each point of it, where x, y and z are measured along the three axes of it.

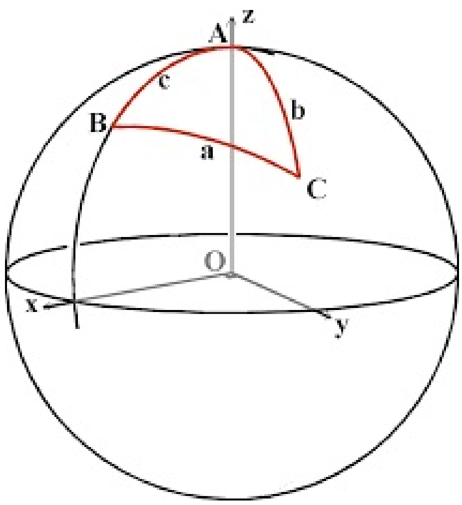

Figure 10. The geometry of 2-sphere embedded in three dimensional Euclidean space

The line element in this space is given by

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{118}$$

Considering now a sphere with its center at the origin of this coordinate system and envisaging its radius to be a, the surface in Cartesian coordinates (x, y, z) where x, y and z are along the three axes of three dimensional Euclidean space. The equation of sphere of this sphere is

$$x^2 + y^2 + z^2 = a^2 \tag{119}$$

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Differentiating Eq. (119) with respect to time

$$2x\frac{dx}{dt}+2x\frac{dx}{dt}+2x\frac{dx}{dt}=0 \tag{120}$$

And in differential form

$$2xdx+2ydy+2zdz=0 \tag{121}$$

Solving Eq. (121) for dz, we have

$$dz=-\frac{xdx+ydy}{z} \tag{122}$$

Finding the value of z from Eq. (119)

$$z=\sqrt{a^2-(x^2+y^2)} \tag{123}$$

Substituting in Eq. (122)

$$dz=-\frac{xdx+ydy}{\sqrt{a^2-(x^2+y^2)}} \tag{124}$$

The value of $dz=-\frac{xdx+ydy}{[a^2-(x^2+y^2)]^{\frac{1}{2}}}$ comes up with a sort of constraint on dz which despite of being displaced by infinitesimally small amounts dx and dy from an arbitrary point on the surface of the sphere holds us on the surface of the sphere. Squaring dz in Eq. (124)

$$dz^2=\frac{(xdx+ydy)^2}{a^2-(x^2+y^2)} \tag{125}$$

Putting in Eq. (118), the line element takes the form by substituting for dz^2

$$ds^2=dx^2+dy^2+\frac{(xdx+ydy)^2}{a^2-(x^2+y^2)} \tag{126}$$

The value of the line element in Eq. (126) represents the line element for a sphere in terms of Cartesian coordinates (x,y,z). We further observe that the line element in Eq. (126) has a coordinate singularity at $a^2=x^2+y^2$ in correspondence with the equator of the sphere and in relation to the point A, otherwise at the equator in the intrinsic geometry of 2-sphere there exists no such physical situation. The embedding scenario manifests how the coordinates (x,y) cover the whole surface of the sphere uniquely up to this point A. The geometry of 2-sphere in these coordinates becomes geometrically meaningful in three dimensional Euclidean space. We can transform the line element in Eq. (126) above into spherical polar coordinates by taking

$$\begin{aligned} x&=r\sin\theta\cos\phi \\ y&=r\sin\theta\sin\phi \end{aligned} \tag{127}$$

where we differentiate each of x and y with respect to θ and ϕ alternately to find

$$\begin{aligned} dx&=\sin\theta\cos\phi dr+r\cos\phi\cos\theta d\theta-r\sin\theta\sin\phi d\phi \\ dy&=\sin\theta\sin\phi dr+r\sin\phi\cos\theta d\theta+r\sin\theta\cos\phi d\phi \end{aligned} \tag{128}$$

Adding the values of x, y given in Eq. (127) after taking square of both equations in it, we get

$$x^2+y^2=r^2\sin^2\theta \tag{129}$$

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Adding dx and dy in Eq. (128) after taking square of both equations in it, we possess

$$dx^2 + dy^2 = (\sin\theta dr + r\cos\theta d\theta)^2 + r^2\sin^2\theta d\phi^2 \tag{130}$$

we find the expression

$$\begin{aligned} &xdx + ydy = (\sin\theta dr + r\cos\theta d\theta)\,(x\cos\phi + y\sin\phi) \\ &-r\sin\theta d\phi\,(x\sin\phi - y\cos\phi) \end{aligned} \tag{131}$$

Squaring Eq. (131), we have

$$(xdx + ydy)^2 = \begin{pmatrix} (\sin\theta dr + r\cos\theta d\theta)\,(x\cos\phi + y\sin\phi) \\ -r\sin\theta d\phi\,(x\sin\phi - y\cos\phi) \end{pmatrix}^2 \tag{132}$$

Now substituting Eq. (129), Eq. (130) and Eq. (132) in Eq. (126) and simplifying to have the following form

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{133}$$

The value of the line element in Eq. (133) gives the line element for a sphere in terms of Spherical polar coordinates (r, θ, ϕ). The line element in Eq. (126) results in an alternative form for

$$\begin{aligned} x &= \xi\cos\phi \\ y &= \xi\sin\phi \end{aligned} \tag{134}$$

$$ds^2 = \frac{a^2}{a^2 - \xi^2)} d\xi^2 + \xi^2 d\phi \tag{135}$$

The line element in Eq. (135) above gives us, in addition, freedom to choose an arbitrary point on the surface of the sphere by $\xi = 0$ as the origin of the coordinate system. This freedom connotes in it as a hidden symmetry. We can develop ξ and ϕ coordinate curves on the surface of the sphere by generating a standard coordinate system (ξ, ϕ) on the tangent plane at the point A that projects vertically downward onto the surface of the sphere. We further observe that the line element in Eq. (135) has a coordinate singularity at $a = \xi$ in correspondence with the equator of the sphere in relation to the point A, otherwise at the equator in the intrinsic geometry of 2-sphere there exists no shade of occurence of such situation. The embedding picture manifests how the coordinates (ξ, ϕ) cover the whole surface of the sphere uniquely up to this point A. The geometry of 2-sphere in these coordinates becomes geometrically meaningful in three dimensional Euclidean space.

6.4. The Geometry of 3-Sphere Embedded in Four Dimensional Euclidean Space

Spaces with dimensions higher than three are now significant in mathematical sciences to have proper description of the physical universe. We consider a four dimensional Euclidean space which can be considered mathematical extension of three dimensional Euclidean space. Minkowski used a four dimensional spacetime to explain the phenomena of the physical world as required by special relaticvity. The structure of Euclidean four dimensional space is simple as compared to the Minkowskian structure of spacetime. Minkowskian four dimensional spacetime is pseudo-Euclidean space. In four dimensional Euclidean we assign the quadruplet of four Cartesian coordinates (x, y, z, w) to each point of it, where x, y, z and w are along the four axes of it. The line element in this space is given by

$$ds^2 = dx^2 + dy^2 + dz^2 + dw^2 \tag{136}$$

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Considering now a sphere with its center at the origin of this coordinate system with radius a, the surface in Cartesian coordinates (x, y, z, w) where x, y, z and w are along the four axes of four dimensional Euclidean space. The equation of the sphere reads as

$$x^2 + y^2 + z^2 + w^2 = a^2 \tag{137}$$

Differentiating Eq. (137) with respect to time,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} + 2w\frac{dw}{dt} = 0 \tag{138}$$

And in differential form

$$2xdx + 2ydy + 2zdz + 2wdw = 0 \tag{139}$$

Finding out the value of dw from Eq. (138), we get

$$dw = -\frac{xdx + ydy + zdz}{w} \tag{140}$$

Now finding the value of w from Eq. (137)

$$w = \sqrt{a^2 - (x^2 + y^2 + z^2)} \tag{141}$$

Substituting in Eq. (140), we obtain

$$dw = -\frac{xdx + ydy + zdz}{\sqrt{a^2 - (x^2 + y^2 + z^2)}} \tag{142}$$

The value of $dw = -\frac{xdx+ydy+zdz}{[a^2-(x^2+y^2+z^2)]^{\frac{1}{2}}}$ provides a sort of constraint on dw which, though displaced by infinitesimally small amounts dx, dy, dz from an arbitrary point on the surface of the sphere holds us stuck on the surface of the sphere. Squaring Eq. (142)

$$dw^2 = -\frac{(xdx + ydy + zdz)^2}{a^2 - (x^2 + y^2 + z^2)} \tag{143}$$

Substituting now in Eq. (136), the line element takes the form for the value of dw^2

$$ds^2 = dx^2 + dy^2 + dz^2 + \frac{(xdx + ydy + zdz)^2}{a^2 - (x^2 + y^2 + z^2)} \tag{144}$$

The value of the line element in Eq. (144) gives the line element for a sphere in terms of Cartesian coordinates (x, y, z, w), We further observe that the line elements in Eq. (144) has a coordinate singularity at $a^2 = x^2 + y^2 + z^2$ in correspondence with the equator of the sphere relative to the point A, otherwise at the equator in the intrinsic geometry of the 3-sphere there does not exist any situation like this. The embedding picture manifests how the coordinates (x, y, z) cover the whole surface of the sphere uniquely up to this point A. The geometry of 3-sphere in these coordinates becomes geometrically meaningful in four dimensional Euclidean space. We transform the line element in Eq. (144) into spherical polar coordinates which are given below

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned} \tag{145}$$

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Where we differentiate x, and y with respect to θ and with respect to ϕ each and differentiate z with respect to θ only to find

$$\begin{aligned} dx &= \sin\theta\cos\phi dr + r\cos\phi\cos\theta d\theta - r\sin\theta\sin\phi d\phi \\ dy &= \sin\theta\sin\phi dr + r\sin\phi\cos\theta d\theta + r\sin\theta\cos\phi d\phi \\ dz &= r\cos\theta \end{aligned} \tag{146}$$

Adding x, y and z in Eq. (145) after taking square of all three equations in it to obtain

$$x^2 + y^2 + z^2 = r^2 \tag{147}$$

Adding now dx, dy and dz in Eq. (146) after taking square of all three equations in it to have

$$dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{148}$$

and the expression we calculate

$$\begin{aligned} xdx + ydy + zdz &= (\sin\theta dr + r\cos\theta d\theta)(x\cos\phi + y\sin\phi) \\ &-r\sin\theta d\phi(x\sin\phi - y\cos\phi) + r\cos\theta(\cos\theta dr - r\sin\theta d\theta) \end{aligned} \tag{149}$$

Squaring Eq. (149), we have

$$(xdx + ydy + zdz)^2 = \begin{pmatrix} (\sin\theta dr + r\cos\theta d\theta)(x\cos\phi + y\sin\phi) \\ -r\sin\theta d\phi(x\sin\phi - y\cos\phi) \\ +r\cos\theta(\cos\theta dr - r\sin\theta d\theta) \end{pmatrix}^2 \tag{150}$$

Now substituting Eq. (147), Eq. (148) and Eq. (150) in Eq. (144), fter simplification we get

$$ds^2 = \frac{a^2}{a^2 - r^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{151}$$

It can further be expressed in the form

$$ds^2 = \frac{1}{\left(1 - \frac{r^2}{a^2}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{152}$$

It is important to note here that for $a \to \infty$ in the Eq. (152) above, It reduces to the metric of ordinary three dimensional Euclidean space

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{153}$$

Which we calculated in Eq. (133). The metric in Eq. (151) has a singularity at $r = a$, which is just a coordinate singularity and has nothing to do with physical reality of the sphere as we can observe. The line element in Eq. (152) results in an alternative form for

$$\begin{aligned} x &= \xi\cos\phi \\ y &= \xi\sin\phi \end{aligned} \tag{154}$$

$$ds^2 = \frac{a^2}{a^2 - \xi^2)} d\xi^2 + \xi^2 d\phi \tag{155}$$

The line element in Eq. (155) gives us, in addition, freedom to choose an arbitrary point on the surface of the sphere by $\xi = 0$ as the origin of the coordinate system. This freedom is implied by it as a hidden symmetry in it. We can develop ξ and ϕ coordinate curves on the surface of the sphere by generating a standard coordinate system (ξ, ϕ) on the tangent plane at the point A that projects vertically downward

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onto the surface of the sphere. We further observe that the line element in Eq. (155) has a coordinate singularity at $a = \xi$ with respect to the equator of the sphere in relation with the point A, otherwise at the equator in the intrinsic geometry of 2-sphere there exists no such situation. The embedding picture manifests how the coordinates (ξ, ϕ) cover the whole surface of the sphere uniquely up to the point A. The geometry of the 2-sphere in these coordinates becomes geometrically meaningful in three dimensional Euclidean space.

7. Einstein's Static Universe

It was Einstein who applied general relativity himself to the large scale of spacetime [2] and presented the very first relativistic model of the universe laying the foundational stone of the modern theoretical cosmology. The model was later on called as Einstein world or universe. For this purpose Einstein modified his field equations by proposing an inbuilt energy density known as cosmological constant *Lambda* in the geometrical structure of spacetime itself that provides repulsive gravity to keep the universe from expanding

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{156}$$

Eq. (156) when solved for the most homogeneous and isotropic geometry of FLRW spacetime produces Friedmann equations as we derived earlier

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{157}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{158}$$

Since for a static universe $H = 0$, which implies that $\frac{\ddot{a}}{a} = 0$. Now a static universe possesses cold matter which means it does not has pressure *i.e.* $p = 0$, so Eq. (157) and Eq. (158) reduce to the form respectively

$$\frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2} = 0 \tag{159}$$

and

$$-\frac{4\pi G}{3}\rho + \frac{\Lambda}{3} = 0 \tag{160}$$

From above Eq. (160), we have

$$\Lambda = 4\pi G\rho \tag{161}$$

Substituting this value of Λ in Eq. (159), and having the equation simplified we again get the value of Λ in terms of the curvature term k and the scalae factor $a(t)$, that is

$$\Lambda = \frac{k}{a^2} \tag{162}$$

The line element for the static Einstein universe can be written now using FLRW metric. From above Eq. (162) for $k = +1$, we have $a^2(t) = \Lambda^{-1}$, substituting in Eq. (59), the static solution for closed universe becomes

$$ds^2 = -dt^2 + \Lambda^{-1}\left(\frac{1}{1-kr^2}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

Using the Schwarzschild coordinates with the rescale of radial coordinate and by defining $R = ra$, we have

$$ds^2 = -dt^2 + \frac{1}{1-\Lambda R^2}dR^2 + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

Case-I (empty universe)

substituting $\Lambda = 0$ in Eq. (161) gives $4\pi G\rho = 0 \Rightarrow \rho = 0$ which implies that $k = 0$ a Euclidean flat universe. It does not belong to Einstein static universe because it is empty.

Case II (non-empty universe)

Einstein universe belongs to $\Lambda \neq 0$ and $\rho \neq 0$ implying that $k > 0$ which represents a universe with hypersurface of Riemannian geometry. In Einstein's universe $\rho > 0$, therefore a positive cosmological constant $\Lambda > 0$ would be allowed which also implies $k > 0$.

7.1. Instability of Einstein's Universe

Equation of energy conservation can be had from Eq. (157) and Eq. (158) by multiplying Eq. (158) with $3a^2$, differentiating it with respect to time and then dividing by $\dot{a}$, we have

$$6\ddot{a} = 8\pi Ga\left(2\rho + \frac{a}{\dot{a}}\dot{\rho}\right) + 2\Lambda a \tag{163}$$

Dividing Eq. (163) by a and substituting the 2nd Friedman Equation from Eq. (159) in it, we have

$$6\frac{\ddot{a}}{a} = 8\pi G\left(2\rho + \frac{a}{\dot{a}}\dot{\rho}\right) + 2\Lambda \tag{164}$$

$$6\left(-\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}\right) = 8\pi G\left(2\rho + \frac{a}{\dot{a}}\dot{\rho}\right) + 2\Lambda \tag{165}$$

after simplification, we get

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{166}$$

For the cold matter universe $p = 0$, with this the resulting equation is a separable universe

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho = 0 \tag{167}$$

$$\int \frac{\dot{\rho}}{\rho}dt = -3\int \frac{\dot{a}}{a}dt \tag{168}$$

$$\ln\rho = -3\ln a + \ln Z \tag{169}$$

$$\ln\rho = \ln a^{-3} + \ln Z \tag{170}$$

$$\ln\rho - \ln a^{-3} = \ln Z \tag{171}$$

$$\ln\frac{\rho}{a^{-3}} = \ln Z \tag{172}$$

which gives

$$\rho a^3 = Z \tag{173}$$

where Z is some positive constant of integration $Z > 0$

$$\rho = \frac{Z}{a^3} \tag{174}$$

Further, since the universe does not expand so that $a(t) = a(t_0) = a_0$, therefore replacing $a(t)$ with a_0 in Eq. (174)

$$\rho = \frac{Z}{a_0^3} \tag{175}$$

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Substituting the value of ρ from Eq. (176) in Eq. (158) *i.e.* 2nd Friedmann Equation with $p = 0$, we obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi GZ}{3{a_0}^3} + \frac{\Lambda}{3} \tag{176}$$

and substituting in Eq. (161) gives

$$\Lambda = 4\pi G\rho = 4\pi G\frac{Z}{a_0^3} \tag{177}$$

where $4\pi G\frac{Z}{a_0^3} > 0$ since $Z > 0$, Now we perturb the solution slightly with the following perturbation

$$\varepsilon << 1 \tag{178}$$

$$a(t) = a(t_0) + \varepsilon(t)a(t_0) = a_0(1+\varepsilon(t)) \tag{179}$$

substituting this in Eq. (175), we have

$$\frac{\frac{d^2}{dt^2}\left(a_0(1+\varepsilon(t))\right)}{a_0(1+\varepsilon(t))} = -\frac{4\pi GZ}{3\left(a_0(1+\varepsilon(t))\right)^3} + \frac{\Lambda}{3} \tag{180}$$

Or

$$\frac{d^2}{dt^2}\left(a_0(1+\varepsilon(t))\right) = -\frac{4\pi GZ}{3a_0^2}(1+\varepsilon(t))^{-2} + \frac{\Lambda}{3}a_0(1+\varepsilon(t)) \tag{181}$$

Using the Maclaurin series expansion as $\varepsilon << 1$, and $(1+\varepsilon(t))^{-2} = 1 - 2\varepsilon + O(\varepsilon^2)$, Now Eq. (181) becomes by neglecting $O(\varepsilon^2)$ since $\varepsilon << 1$, so that

$$a_0\frac{d^2\varepsilon}{dt^2} = -\frac{4\pi GZ}{3a_0^2}(1-2\varepsilon(t)) + \frac{\Lambda}{3}a_0(1+\varepsilon(t)) + O(\varepsilon^2) \tag{182}$$

$$\ddot{\varepsilon} = -\frac{4\pi GZ}{3a_0^3} + \frac{8\pi GZ}{3a_0^3}\varepsilon + \frac{\Lambda}{3}\varepsilon + \frac{\Lambda}{3} = \left(\frac{8\pi GZ}{3a_0^3} + \frac{\Lambda}{3}\right)\varepsilon + \frac{\Lambda}{3} - \frac{4\pi GZ}{3a_0^3} \tag{183}$$

Using the value of $\Lambda = \frac{4\pi GZ}{a_0^3}$ from Eq. (177) in Eq. (183), it can be expressed in the form

$$\frac{d^2\varepsilon}{dt^2} - \Lambda\varepsilon = 0 \tag{184}$$

As the cosmological constant is $\Lambda > 0$, the solution of above equation will read as

$$\varepsilon = P\exp(\sqrt{\Lambda}t) + Q\exp(-\sqrt{\Lambda}t) \tag{185}$$

Due to existence of the 1st term in the above solution as positive and in the case of an arbitrary perturbation considered initially, both of the constants $P \neq 0$, $Q \neq 0$ will help the perturbation grow and it will not remain small which will imply that the static solution is unstable, although $P = 0$ can be possible only for specialized initial conditions such as singular one.

7.2. de Sitter Universe

In Einstein's static model with positive cosmological constant when energy density of the matter is removed de Sitter model results. The de Sitter model of the universe presented in 1917 was proposed just after Einstein presented his static closed model of the universe. Einstein resorting to the Mach's principle was of the view that it is merely matter density in universe that is the cause of inertia and gravitation. For checking the status of this Einstein's belief de Sitter posed the 2nd model of the universe devoid of matter density $T_{\mu\nu} = 0$, however retaining the cosmological constant that is $G_{\mu\nu} = g_{\mu\nu}\Lambda$. de Sitter model is the maximally symmetric solution of the Einstein's field equations

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with vanishinf matter density. The geometric theoratic structure of sacetime of de Sitter model is comparatively more complicated than that of Einstein model of the universe. The characteristic of the de Sitter model is that it predicts redshift despite it contains neither matter density nor radiation. we reviewe de Sitter model using Fiedmann's equations, however these equations were worked out after the development of de Sitter model. We derived Friedmann equations above in the presence of cosmological constant term Λ which are

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{186}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{187}$$

de Sitter universe corresponds to $\rho = 0$, so that $k(\rho) = 0$, Eq. (186) takes the form

$$\frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} \tag{188}$$

Integrating with respect to time

$$\int \frac{\dot{a}(t)}{a(t)}dt = \sqrt{\frac{\Lambda}{3}} \int dt \tag{189}$$

$$a(t) = e^{\sqrt{\frac{\Lambda}{3}}t} \tag{190}$$

From Eq. (188), $H = \frac{\dot{a}}{a}$, so the Eq. (190) can be expressed as

$$a(t) = e^{Ht} \tag{191}$$

Which corresponds to the modified Einstein field equations

$$G_{\mu\nu} = -g_{\mu\nu}\Lambda \tag{192}$$

8. The Conformal FLRW Line Element

The metric in Eq. (57) can be conformaly recast by defining conformal time as

$$d\tau = \frac{dt}{a(t)} \tag{193}$$

so that

$$dt = a(t)d\tau \tag{194}$$

After substituting Eq. (194) in Eq. (57) and simplifying, we get the line element in the form

$$ds^2 = -a(\tau)^2 \left[-d\tau^2 - \left(1 - kr^2\right)^{-1} dr^2 - r^2\Omega^2\right] \tag{195}$$

Due to conformal time the scale factor $a(\tau)$ becomes a factor of spatial as well as temporal components in the metric. Now a function $f(t)$ which depends upon time can be differentiated as

$$\begin{aligned} \dot{f}(t) &= \frac{f'(\tau)}{a(\tau)} \\ \ddot{f}(t) &= \frac{f''(\tau)}{a^2(\tau)} - \frac{f'(\tau)}{a^2(\tau)}\mathcal{H} \end{aligned} \tag{196}$$

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Where dot "." and "," represent derivatives with respect to cosmic and conformal times respectively and $\mathcal{H} = \frac{a'(\tau)}{a(\tau)}$. Now replacing $f(t)$ and its derivatives with $a(t)$ both in correspondence with cosmic time $'t'$ and conformal time $'\tau'$

$$\dot{a}(t) = \frac{a'(\tau)}{a(\tau)} \tag{197}$$

$$\ddot{a}(t) = \frac{a''(\tau)}{a^2(\tau)} - \frac{\mathcal{H}^2}{a(\tau)} \tag{198}$$

and

$$H = \frac{\dot{a}}{a} = \frac{\mathcal{H}}{a(\tau)} \tag{199}$$

$$\dot{H} = \frac{\mathcal{H}'}{a^2(\tau)} - \frac{\mathcal{H}^2}{a^2(\tau)} \tag{200}$$

Similarly

$$\mathcal{H}^2 = \frac{8\pi G a^2}{3}\rho - k \tag{201}$$

and

$$\rho' + 3\mathcal{H}(\rho + p) = 0 \tag{202}$$

Now we solve the energy conservation equation From Eq. (108)

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{203}$$

in order to get the relation between energy density ρ, scale factor a and equation of state parameter $w = \frac{p}{\rho}$ we solve

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) = -3\frac{\dot{a}}{a}\rho\left(1 + \frac{p}{\rho}\right) \tag{204}$$

$$\Rightarrow \frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a}(1 + w) \tag{205}$$

where $\frac{p}{\rho} = w$. Integrating Eq. (205)

$$\int \frac{1}{\rho} d\rho = -3(1 + w)\int \frac{1}{a} da \tag{206}$$

which gives

$$\rho = a^{-3(1+w)} \tag{207}$$

Now from 1st Friedmann equation, after simplification and doing integration, we find

$$a = t^{\frac{2}{3(1+w)}} \tag{208}$$

For $w = -1, 0, \frac{1}{3}$, we find pressure, energy density and scale factor characterizing the expansion of the universe which depicts three phases of the universe namely vacuum dominated, radiation dominated and matter dominated respectively.

8.1. Vacuum Domination (Λ-dominated era

) For $w = -1$

$$\rho = a^{-3(1+w)} = a^0 \tag{209}$$

and

$$a = t^{\frac{2}{3(1-1)}} = t^{\infty} \tag{210}$$

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8.2. Radiation Domination

For $w = \frac{1}{3}$

$$\rho = a^{-3\left(1+\frac{1}{3}\right)} = a^{-4} \tag{211}$$

and

$$a = t^{\frac{2}{3\left(1+\frac{1}{3}\right)}} = t^{\frac{1}{2}} \tag{212}$$

8.3. Matter Domination

For $w = 0$

$$\rho = a^{-3(1+0)} = a^{-3} \tag{213}$$

and

$$a = t^{\frac{2}{3(1+0)}} = t^{\frac{2}{3}} \tag{214}$$

8.4. Critical Density (ρ_c) and Density Parameter (Ω)

Now from 1st Friedman Eq. (112) with $\Lambda = 0$ and $H = \partial_t \ln a$, we relate the curvature of spacetime k and the expansion characterized by the scale factor $a\,(t)$ to the energy density $\rho\,(t)$ of the universe and find the expression for the critical density required to keep the current rate of the expansion.

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{215}$$

For critical density ρ_c the curvature of spacetime geometry k must vanish, So that Eq. (215) reduces to

$$H^2 = \frac{8\pi G}{3}\rho \tag{216}$$

where we obtain the expression for critical density

$$\rho = \rho_c = \frac{3H^2}{8\pi G} \tag{217}$$

From Eq. (215) dividing both sides by H^2 and rearranging

$$1 = \frac{8\pi G}{3H^2}\rho - \frac{k}{a^2H^2} = \frac{\rho}{\left(\frac{3H^2}{8\pi G}\right)} - \frac{k}{a^2H^2} \tag{218}$$

Where $\frac{3H^2}{8\pi G} = \rho_c$, therefore Eq. (218) becomes

$$1 = \frac{\rho}{\rho_c} - \frac{k}{a^2H^2} = \Omega - \frac{k}{a^2H^2} \tag{219}$$

$$\Rightarrow \Omega - 1 = \frac{k}{a^2H^2} \tag{220}$$

where $\Omega = \frac{\rho}{\rho_c}$ is the density parameter and we can predict in terms of it about the geometry of universe. The local geometry of the universe is investigated by this parameter by observing whether the relative density is smaller than unity, greater than or equal to it.

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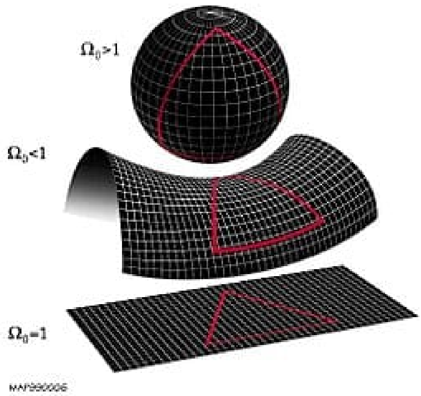

Figure 11. The spherical geometry $\Omega_0 > 1$ and for hyperbolic geometry $\Omega_0 < 1$ and $\Omega_0 = 1$ represents flat geometry

Eq. (220) can also be derived from Eq. (215) in an alternative style. Writing Eq. (215) by multiplying and dividing the 1st term on the right side with ρ_c

$$H^2 = \frac{8\pi G\rho}{3}\frac{\rho_c}{\rho_c} - \frac{k}{a^2} \tag{221}$$

Using the density parameter $\Omega = \frac{\rho}{\rho_c}$, in Eq. (221) we can write

$$H^2 = \frac{8\pi G}{3}\rho_c\Omega - \frac{k}{a^2} \tag{222}$$

Now from from the critical density expression in Eq. (217)

$$\begin{aligned} &\Rightarrow \frac{3}{8\pi G} = \frac{\rho_c}{H^2} \\ &\Rightarrow \frac{8\pi G}{3} = \frac{H^2}{\rho_c} \end{aligned} \tag{223}$$

Substituting the 2nd part in Eq. (223 in Eq. (222) and using the density parameter we get

$$H^2 = H^2\Omega - \frac{k}{a^2} \tag{224}$$

which gives the following form similar to Eq. (220)

$$\Omega - 1 = \frac{k}{a^2H^2} \tag{225}$$

Now

$$\Omega = \frac{\rho}{\rho_c} \tag{226}$$

is considered decisive in describing the evolution of the universe. The present value of it is denoted by Ω_0 and it gives following three geometries of the universe

$$\Omega_0 \succ 1 \tag{227}$$

a closed universe implying the universe with spherical geometry

$$\Omega_0 \prec 1 \tag{228}$$

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an open universe implying the universe with hyperbolic geometry and

$$\Omega_0 = 1 \tag{229}$$

a flat universe implying the universe with Euclidean or Minkowskan geometry. The present value of critical density can be calculated with present value of Hubble constant H_0, gravitational constant G and π.

$$\begin{aligned} \rho_{c,0} &= \tfrac{3H_0^2}{8\pi G} \\ &= \tfrac{3()^2}{8()()} \\ &= 1.1 \times 10^{-5} h^2 \end{aligned} \tag{230}$$

where the scaled Hubble parameter h is defined by $H = 100hkm\ s^{-1}\ Mpc^{-1}$ and $H^{-1} = 9.778h^{-1}\ Gyr$ $H^{-1} = 2998h^{-1}\ Mpc$.

8.5. Particle Horizon

When the scale factor $a(t)$ is multiplied with the co-moving coordinates we get the proper distance. In cosmology causality is one directional since we just receive photons from the outer world that serves to be self-sufficient approach. The horizon or horizon distance of the universe is defined as the maximum distance that light could have travelled to our reference Earth since the time after the beginning of the universe when for the first time it became exposed to electromagnetic radiation [36], thus horizon represents the causal distance in the universe.

$$d_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} \tag{231}$$

Such that $d_H(t) \sim H^{-1}(t)$ Particle horizon is defined to be the distance travelled by a photon from the time of big bang upto a certain later time t. Particle horizon puts limits on communication from the deep inward past.

8.6. Event Horizon

An event horizon defines such a set of points from which light signals sent at some given time will never be received by an observer in the future. It sets limits on the horizon distance and on communication to the future so that as long as it exists, the size of the causal patch of the universe will be finite.

8.7. Deceleration Parameter (q_0)

A Taylor series is a series expansion of a function about a given point. We require here a one dimensional Taylor series which is the expansion of a real function $f(x)$ about a point $x = a$ and is given by

$$\begin{aligned} &f(t) = f(x)|_{x=a} = f(a) + f'(a)(x-a) + \tfrac{f''(a)}{2!}(x-a)^2 \\ &+ \tfrac{f'''(a)}{3!}(x-a)^3 + \cdots + \tfrac{f^n(a)}{n!}(x-a)^n + \cdots \end{aligned} \tag{232}$$

We take the function $f(x) = a(t)$ which is scale factor and find its Taylor series expansion about the present time $t = t_0$

$$\begin{aligned} &a(t) = a(t)|_{t=t_0} = a(t_0) + \dot{a}(t_0)(t-t_0) + \tfrac{\ddot{a}(t_0)}{2!}(t-t_0)^2 \\ &+ \tfrac{(t_0)}{3!}(t-t_0)^3 + \cdots + \tfrac{a^n(t_0)}{n!}(t-t_0)^n + \cdots \end{aligned} \tag{233}$$

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dividing Eq. (233) by $a\,(t_0)$ throughout, we have

$$\begin{aligned}\frac{a(t)}{a(t_0)} &= \frac{a(t_0)}{a(t_0)} + \frac{\dot{a}(t_0)}{a(t_0)}\,(t-t_0) + \frac{1}{2}\frac{\ddot{a}(t_0)}{a(t_0)}(t-t_0)^2 \\ &+ \frac{1}{6}\frac{(t_0)}{a(t_0)}(t-t_0)^3 + \cdots + \frac{1}{n!}\frac{a^n(t_0)}{a(t_0)}(t-t_0)^n + \cdots\end{aligned} \tag{234}$$

Ignoring the higher terms we have the following remaining expression

$$\frac{a\,(t)}{a\,(t_0)} = 1 + \frac{\dot{a}\,(t_0)}{a\,(t_0)}\,(t-t_0) + \frac{1}{2}\frac{\ddot{a}\,(t_0)}{a\,(t_0)}(t-t_0)^2 \tag{235}$$

Multiplying and dividing now by $\dot{a}\,(t)$ with 3rd term of Eq. (235) on the right hand side

$$\frac{a\,(t)}{a\,(t_0)} = 1 + \frac{\dot{a}\,(t_0)}{a\,(t_0)}\,(t-t_0) + \frac{1}{2}\frac{\dot{a}\,(t_0)}{a\,(t)}\frac{\ddot{a}\,(t_0)}{\dot{a}\,(t_0)}(t-t_0)^2 \tag{236}$$

Multiplying again the 3rd term on the right hand side of Eq. (236) with $\frac{\dot{a}(t_0)}{a(t_0)}$ and its reciprocal $\frac{a(t_0)}{\dot{a}(t_0)}$ we have

$$\frac{a\,(t)}{a\,(t_0)} = 1 + \frac{\dot{a}\,(t_0)}{a\,(t_0)}\,(t-t_0) + \frac{1}{2}\left(\frac{\dot{a}\,(t_0)}{a\,(t_0)} \times \frac{a\,(t_0)}{\dot{a}\,(t_0)}\right)\frac{\dot{a}\,(t_0)}{a\,(t)}\frac{\ddot{a}\,(t_0)}{\dot{a}\,(t_0)}(t-t_0)^2 \tag{237}$$

Putting for $\frac{\dot{a}(t_0)}{a(t_0)} = H_0$, the present value of Hubble parameter and $\frac{a(t_0)\ddot{a}(t_0)}{[\dot{a}(t_0)]^2} = -q_0$, Eq. (237) reduces to the following

$$\frac{a\,(t)}{a\,(t_0)} = 1 + H_0\,(t-t_0) + \frac{1}{2}H_0^2\,(-q_0)\,(t-t_0)^2 \tag{238}$$

where

$$q_0 = -\frac{a\,(t_0)\,\ddot{a}\,(t_0)}{[\dot{a}\,(t_0)]^2} = -\frac{\ddot{a}\,(t_0)}{\dot{a}\,(t_0)}H_0^{-1} = -\frac{\ddot{a}\,(t_0)}{a\,(t_0)}H_0^{-2} \tag{239}$$

is called the deceleration parameter. It tells us that greater the value of q_0, the faster will be speed of deceleration. It can be further related with the acceleration equation

$$\frac{\ddot{a}\,(t)}{a\,(t)} = -\frac{4\pi G}{3}\,(\rho + 3p) \tag{240}$$

Putting Eq. (240) in Eq. (239)

$$q_0 = -\left(-\frac{4\pi G}{3}\,(\rho + 3p)\right)H_0^{-2} \tag{241}$$

With $p = 0$ for a universe having matter-domination and present energy density $\rho = \rho_0$ with dividing and multiplying by 2, we possess

$$q_0 = \frac{1}{2}\frac{8\pi G}{3H_0^2}\rho_0 \tag{242}$$

Now the critical density is given by $\rho_c = \frac{3H_0^2}{8\pi G}$ from the 1st Friedmann equation. Therefore Eq. (242) takes the form

$$q_0 = \frac{1}{2}\left(\frac{1}{\rho_c}\right)\rho_0 = \frac{1}{2}\frac{\rho_0}{\rho_c} = \frac{1}{2}\Omega_0 \tag{243}$$

The measurement of deceleration parameter q_0 determines how bigger was the universe in earlier times. The explorations of redshift measures of supernovae of Type Ia to measure the value of q_0 has shown astoundingly that $q_0 < 0$ at the present which means that the expansion of the universe is accelerating rather than to be decelerating which affirms that the concept of dark energy must be acknowledged. Accelerated expansion of the universe corresponds to $q_0 < 0$ whereas $q_0 > 0$

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corresponds decelerated expansion. It is interesting to notice that for all of these components we have $H > 0$ *i.e.* an increasing scale factor which gives the expansion rate of the universe. Moreover, to get a better understanding of the properties of each species, it is useful to introduce the deceleration parameter q_0 as:

$$\begin{aligned} q_0 &= -\frac{\ddot{a}\dot{a}}{\dot{a}^2} \\ &= -\frac{\ddot{a}}{\dot{a}}\frac{a}{\dot{a}} \\ &= -\frac{\ddot{a}}{\dot{a}}H^{-1} \end{aligned} \tag{244}$$

such that for both matter-dominated or radiation-dominated universe the expansion is decelerating. It is also interesting to notice that components with $w < -\frac{1}{3}$ give an accelerated expansion.

8.8. Friedmann Equations in terms of Density Parameter

We found earlier Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{245}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) \tag{246}$$

In Eq. (245) in order to incorporate vacuum energy we can write energy density as the sum of all energy components $\rho = \rho_m + \rho_r + \rho_\Lambda$ such that the equation can be written as

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r + \rho_\Lambda\right) - \frac{k}{a^2} \tag{247}$$

where $\frac{\dot{a}}{a} = H$ is the Hubble parameter, writing ρ_Λ as $\rho_\Lambda = \Lambda = \frac{\Lambda}{8\pi G}$ and $\rho = \rho_m + \rho_r$ which further can be written as the contributing ingredients $\rho_m = \rho_b + \rho_{CDM}$ and $\rho_r = \rho_\gamma + \rho_\nu$, also we found earlier the critical density to be $\frac{3H^2}{8\pi G} = \rho_{cd}$, which for the present value can be expressed as $\rho_{c,0} = \frac{3H_0^2}{8\pi G}$ we find from it the value of $8\pi G = \frac{3H_0^2}{\rho_{cd,0}}$ and substitute in Eq. (247) which comes to be

$$H^2 = H_0^2\left(\frac{\rho_m}{\rho_{c,0}} + \frac{\rho_r}{\rho_{c,0}} + \frac{\rho_\Lambda}{\rho_{c,0}}\right) - \frac{k}{a^2} \tag{248}$$

or

$$H^2 = H_0^2\left(\Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0}\right) - \frac{k}{a^2} \tag{249}$$

where

$$\Omega_{m,0} = \frac{\rho_m}{\rho_{c,0}}, \Omega_{r,0} = \frac{\rho_r}{\rho_{c,0}}, \Omega_{\Lambda,0} = \frac{\rho_\Lambda}{\rho_{c,0}} \tag{250}$$

It might be suitable to write the curvature term k in terms of density parameter $k = \Omega_{k,0} = \frac{\rho_k}{\rho_{c,0}}$, further the present value of the scale factor $a\left(t\right) = 1$ so that Eq. (249) takes the form

$$H^2 = H_0^2\left(\Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0}\right) - \Omega_{k,0} \tag{251}$$

Now for the present value of Hubble parameter *i.e.* $H = H_0$, Eq. (251) can be written for the curvature density parameter

$$\Omega_{k,0} = H_0^2\left(\Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} - 1\right) \tag{252}$$

Eq. (249) can be written in general form *i.e.* $H \neq H_0$ and $a \neq a_0 = 1$

$$H^2 = H_0^2\left(\Omega_m + \Omega_r + \Omega_\Lambda\right) - \frac{\Omega_k}{a^2} \tag{253}$$

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Eq. (253) can also be written for the present values of all the energy density parameters

$$H^2 = H_0^2 \left(\Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0}\right) - \frac{\Omega_{k,0}}{a^2} \tag{254}$$

We know that energy density ρ for matter, radiation and vacuum domination eras changes with the scale factor that characterizes the expansion of the universe according to

$$\begin{aligned} \rho &\propto a^{-3} \\ \rho &\propto a^{-4} \\ \rho &\propto a^{0} \end{aligned} \tag{255}$$

respectively. So that Eq. (254) takes the following form using Eq. (255)

$$H^2 = H_0^2 \left(\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{\Lambda,0} a^{0}\right) - \Omega_{k,0} a^{-2} \tag{256}$$

The Eq. (256) represents Friedmann equation in terms of density parameters.

For $a = \frac{a(t)}{a(t_0)} = \frac{1}{1+z}$, Eq. (256) can be expressed in terms of redshift as follows

$$\begin{aligned} H^2 = H_0^2 &\left(\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0}(1+z)^0\right) \\ &-\Omega_{k,0}(1+z)^2 \end{aligned} \tag{257}$$

We can discuss various models for the universe using Eq. (256) for matter, radiation, Λ and curvature dominated eras.

For matter domination Eq. (256) with $\Omega_{m,0} = 1$ and with the rest vanishing gives

$$\begin{aligned} a &= \left(\tfrac{3}{2} H_0 t\right)^{\frac{2}{3}} \\ t &= \frac{2a^{\frac{3}{2}}}{3H_0} \end{aligned} \tag{258}$$

Which gives an expanding universe with expansion rate inversely proportional to time *i.e.*.....

For radiation domination Eq. (256) with $\Omega_{r,0} = 1$ and with the rest vanishing, gives

$$\begin{aligned} a &= \sqrt{2H_0 t} \\ t &= \frac{a^2}{2H_0} \end{aligned} \tag{259}$$

Which gives an expanding universe with expansion rate inversely proportional to time *i.e.*....

For Λ domination Eq. (256) with $\Omega_{\Lambda,0} = 1$ and the rest vanishing gives

$$\begin{aligned} a &= e^{H_0 t} \\ t &= \infty \end{aligned} \tag{260}$$

Which gives an expanding universe with expansion rate inversely proportional to time *i.e.*.....

For k domination or otherwise empty universe Eq. (256) with $\Omega_{k,0} = 1$ and with the rest vanishing gives

$$\begin{aligned} a &= H_0 t \\ t &= \infty \end{aligned} \tag{261}$$

Which gives an expanding universe with expansion rate inversely proportional to the time *i.e.* "t"

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8.9. Cosmological Redshift

we considering the FLRW geometry

$$ds^2 = -dt^2 + a(t)^2 \left[\left(1 - kr^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right] \tag{262}$$

It is to be remembered here that the coordinates (r, θ, ϕ) in the metric Eq. (262) are comoving spatial coordinates, therefore galaxies which are considered as point particles constituting the particles of cosmological fluid in cosmology remain at fixed coordinates and it is the geometry of the spacetime that expands itself and is characterized by the scale factor $a\,(t)$ completely. Three intervals namely *i.e* spacelike, timelike and lightlike or null expressed in the form $ds^2 > 0$, $ds^2 < 0$, $ds^2 = 0$ respectively. In the spacetime geometry light propagates following the interval $ds^2 = 0$ or $ds = 0$ which means that it does not travel at all any distance through the spacetime. We consider a ray of light propagating along the radius since all the points in space are equivalent at a given time from some zero value radius to some certain value of it in later times. Since the light ray travels radially therefore only one spatial dimension is retained and vanishing of time dimension follows from $ds = 0$ and other two spatial dimensions vanish due to radial propagation of light therefore $dt = d\theta = d\phi = 0$, then Eq. (262) gives

$$0 = -dt^2 + a(t)^2 \left(1 - kr^2\right)^{-1} dr^2 \tag{263}$$

or

$$\frac{dt}{a(t)} = \frac{1}{\sqrt{(1 - kr^2)}} dr \tag{264}$$

In order to calculate the total time elapsed from $r = 0$ to some certain later time value $r = r_0$, we shall integrate Eq. (264) between emission and reception times t_e and t_r respectively.

$$\int_{t=t_{emi}}^{t=t_{rec}} \left(\frac{1}{a(t)}\right) dt = \int_{r=0}^{r=r_0} \left(\frac{1}{\sqrt{1 - kr^2}}\right) dr \tag{265}$$

A ray of light, now, given off after a short interval of time dt_{emi} so that time of emission of light ray becomes $t_{emi} + dt_{emi}$ and accordingly we can have the time of reception to be $t_{rec} + dt_{rec}$ from an integral of the same nature given in Eq. (265) because of comoving coordinates the galaxies remain at the same coordinates, so that

$$\int_{t=t_{emi}}^{t=t_{rec}} \left(\frac{1}{a(t)}\right) dt = \int_{t=t_{emi}+dt_{emi}}^{t=t_{rec}+dt_{rec}} \left(\frac{1}{a(t)}\right) dt \tag{266}$$

now

$$\int_{t=t_{emi}}^{t=t_{emi}+dt_{emi}} \left(\frac{1}{a(t)}\right) dt = \int_{t=t_{rec}}^{t=t_{rec}+dt_{rec}} \left(\frac{1}{a(t)}\right) dt \tag{267}$$

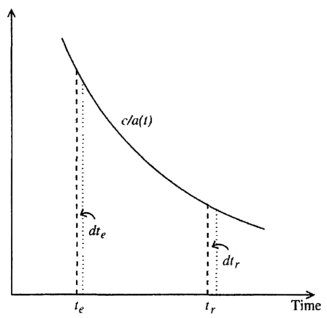

Figure 12. diagrammatic scheme for deriving redshift

The slices are very narrow, so the area is just the area of a rectangle *i.e.* width times height *i.e.*

$$\frac{dt_{rec}}{a\left(t_{rec}\right)} = \frac{dt_{emi}}{a\left(t_{emi}\right)} \tag{268}$$

For an expanding universe

$$a\left(t_{rec}\right) > a\left(t_{emi}\right) \tag{269}$$

, it implies from Eq. (268) $dt_{rec} > dt_{emi}$ that as the universe expands the time interval between two rays increases. We consider now successive crests or troughs of a single ray instead of two rays as we did earlier so that wave length λ is directly proportional to the time interval between two successive crests or troughs $\lambda \propto dt$ and $dt \propto a(t)$ and we have

$$\frac{\lambda_{rec}}{\lambda_{emi}} = \frac{a\left(t_{rec}\right)}{a\left(t_{emi}\right)} \tag{270}$$

We define now the redshift

$$1 + z \equiv \frac{a\left(t_{rec}\right)}{a\left(t_{emi}\right)} \tag{271}$$

8.10. Luminosity ((L)), Brightness, Luminosity Distance ((d$_L$*)) and Angular Diameter distance (d*$_A$*)*

We can deduce relations from the properties of electromagnetic radiation and the quantities contained in FLRW line element. The velocity of electromagnetic waves is constant and finite. Light-an electromagnetic radiation acts as cosmological messenger and all the distances measured cosmologically are extracted from the properties of it. The velocity of light being finite has to take time to reach us and universe might have expanded significantly during this time.

8.11. Luminosity L

Luminosity is defined as the absolute measure of the electromagnetic power or energy radiated per unit time by an astronomical object like star, galaxy or cluster of galaxies. It is denoted by L and is measure in Loule per second $\left(Js^{-1}\right)$ which is also known as watts. Usually luminosity is measured in terms of the luminosity of the sun denoted by $L_{\odot}$

8.12. Brightness

It refers to how bright an object appears to an observer and depends upon luminosity, distance between the observer and the object and absorption of light along the path between observer and the object.

8.13. Luminosity Distance ((d$_L$*))*

We consider a point source S radiating electromagnetic light equally in all directions spherically, the amount of light passing through elements of surface areas varies with the distance of it from the light source.

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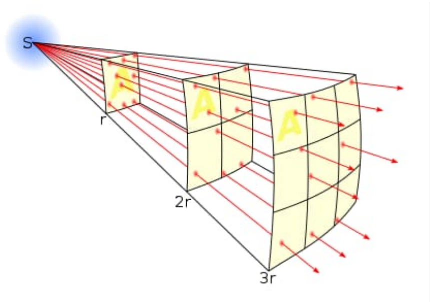

Figure 13. A source S radiating electromagnetic enrgy

In above Fig.13, the light of luminosity L is being radiated. We consider a spherical hollow centered on the point source S as shown in the Fig. above. The interior of hollow sphere gets illuminated thoroughly. As radius of the sphere increases, the surface area of the imagined hollow sphere also increases such that a constant or absolute measure of luminosity has to spread in expanding sphere illuminating it i.e. as the radius increases the constant luminosity has more and more surface area to illuminate which leads to decrease in observed brightness. If an observer at a distance equivalent to the radius of sphere receives the electromagnetic radiation L per unit time and F be the energy flux per unit time per unit area from the source the point source say O, so that in Euclidean geometry we would have

$$F = \frac{L}{A} = \frac{L}{4\pi r^2} \tag{272}$$

where, $F =$ Flux density of the illuminated sphere, $L =$ luminosity and $A =$ area of the illuminated sphere From Eq. (272) for $r = d_L$

$$F = \frac{L}{4\pi {d_L}^2} \tag{273}$$

which gives

$$d_L = \sqrt{\frac{L}{4\pi F}} \tag{274}$$

We next look how the luminosity distance is related with expansion of the universe. In expanding sphere we might have its radius as the product of scale factor and the radius *i.e.* $a(t)r = a(t)d_L$, so that the energy emitted gets diluted

$$4\pi r^2 \rightarrow 4\pi (a\,(t)\,r)^2 \tag{275}$$

and a photon loses energy as $F \propto \frac{a(t_e)}{a(t_0)}$ and redshift relation we have $1 + z = \frac{\lambda(t_0)}{\lambda(t_e)} = \frac{a(t_0)}{a(t_e)}$ which implies $F \propto \frac{a(t_e)}{a(t_0)} \propto \frac{1}{1+z}$ Eq. (273) becomes

$$F = \frac{L}{4\pi (a(t)r)^2} \tag{276}$$

further

$$\frac{a(t_e)}{a(t_0)} = \frac{L}{4\pi (a(t)d_L)^2} \tag{277}$$

If L is known for a source, it is known as standard candle. Supernovae type Ia were used as standard candles for lager cosmic redshifts which led to accelerated expansion.

8.14. Angular Diameter Distance (d_A)

It is the ratio of the proper distance measured when the light left the surface of an object to the later measured distance by redshifting of light in some later time. Certainly the redshift of light

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measured would be smaller measured at the time when the light left the surface of the object to be measured in later times.

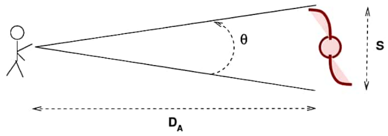

Figure 14. Angular diameter distance

It is defined in terms of objects physical distance known as proper distance and the angular size of the object seen from the surface of earth. If size of the source be S and angular size θ, then

$$\theta = \frac{S}{D_A} \tag{278}$$

Where D_A is the angular diameter distance of the source. From FLRW line element for photons $dr^2 \approx d\phi^2 \approx 0$, we have

$$ds^2 = a^2(t)\left(r^2 d\theta^2\right) \tag{279}$$

$$ds = D = a(t) r d\theta \tag{280}$$

$$d\theta = \frac{D}{ra(t)} = \frac{d_A}{ra(t)} \tag{281}$$

9. Problems Faced by the Standard Model of Cosmology

From 1st Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{282}$$

we see that curvature k is negligible depending on observation and $\Omega \simeq 1$ which means it would have been created tuned finely in the very early universe. From 2nd Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{283}$$

we see that if $(\rho + 3p)$ remains positive, the acceleration is negative which means that the expansion of the universe will go on slowing down. Further far flung parts of the universe display same properties as observation evidence despite they have not been in causal contact with each other.

9.1. Monopole Problem

The problem is about the question of why do we not observe magnetic monopoles in the universe today. It results from combining the big bang model with GUT in particle physics, thus it is related with particle cosmology where during symmetry breaking phase transitions are considered. In the very early universe when the phase transitions are considered to occur, it is expected that these phase transitions will create magnetic monopoles with enormous energy density which might dominate the total energy density of the universe. During symmetry breaking when phase transitions take place, these give rise to flaws known to be as topological defects. GUT predict that during GUT phase transitions these point-like topological defects are created which act as magnetic monopoles. It is considered that the radiation and matter dominated eras could not take place since these monopoles do not get diluted as they are supposed to be non-relativistic and their energy density would decay like

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a^{-3} [38] but as we observe the universe evolved to the later eras so question arises how this occurred which is at the heart of this problem.

9.2. Horizon Problem

On the basis of the standard big bang model it is difficult to understand the uniform distribution of the temperature of CMB to 1 part in 10^5. The horizon problem is related with the issue of the causal contact as it has been revealed by the uniform distribution of temperature of the cosmic background radiation (CMB) across all parts of the universe. In order to understand the problem we have to understand the horizon size and causal contact or communication. At any instant of time horizon size is defined as the largest distance *i.e* maximall distance over which two events could be in causal with each other. Therefore it is the maximum distance a photon could have travelled since the birth of the universe or since the time when the universe became transparent. It can be found from the FLRW metric to be $ds^2 = R_H = c\int_0^t \frac{dt}{a(t)}$ which reveals the fact that size of the horizon depends upon the history of the universe as it evolves through time. It is also called comoving horizon as causal contact develops between two events and the universe is expanding so that they are getting separated apart mutually. In the standard big bang theory the universe was matter dominated at the time of last scattering (t_{ls}) so that the horizon distance at that time can be approximated by the value $d_H(t_{ls}) = 2cH^{-1}(t_{ls})$. Now the Hubble distance at the time of last scattering was $cH^{-1}(t_{ls}) \approx 0.2Mpc$ and the horizon distance at last scattering was $d_H(t_{ls}) \approx 0.4Mpc$. Therefore the points which were separated more than $0.4Mpc$ distance apart at the time of last scattering (t_{ls}) were not connected causally in the big bang scenario. Further angular diameter distance (d_A) to the last scattering surface is $13Mpc$, therefore points on the last scattering surface that were separated by a horizon distance shall have angular separation $\theta_H = \frac{d_H(t_{ls})}{d_A} \approx \frac{0.4Mpc}{13Mpc} \approx 0.03rad \approx 2^0$ as viewed today from the Earth. It means that the points separated by an angle as small as $\sim 2^0$ on the last scattering surface were not in causal contact with each other when CMB emitted with temperature fluctuations. However, we come to know that $\frac{\delta T}{T}$ is as small as 10^{-5} on the scales with angular separation $\theta_H > 2^0$. So here we state the problem that the regions which were not connected through causal contact with each other at the time of last scattering have similar properties homogeneously.

9.3. Flatness Problem

When we consider Friedmann's equations evolve in a universe where only radiation and baryonic matter exist without vacuum energy density present there, then flatness problem arises in such a universe [39]. From 1st Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{284}$$

$$1 = \frac{8\pi G}{3H^2}\rho - \frac{k}{a^2H^2} = \frac{\rho}{\left(\frac{3H^2}{8\pi G}\right)} - \frac{k}{a^2H^2} \tag{285}$$

Where $\frac{3H^2}{8\pi G} = \rho_c$, therefore Eq. (285) becomes

$$1 = \frac{\rho}{\rho_c} - \frac{k}{a^2H^2} = \Omega - \frac{k}{a^2H^2} \tag{286}$$

$$\Rightarrow \Omega - 1 = \frac{k}{a^2H^2} \tag{287}$$

so that the spatial curvature of the universe is related to the density parameter Ω through Friedmann's equation. Observational evidence shows that the universe is nearly flat today *i.e.* $\rho = \rho_c, \Rightarrow \Omega = \frac{\rho}{\rho_c} \approx 1$. This means that the value of Ω would have to be very close to 1 at Planck era t_{pl}. This means that the initial conditions of the universe were tuned finely. Due to this reason flatness problem is also

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known as fine tuning problem and the flatness problem arises because in a comoving volume the entropy remains conserved. Further From Eq. (284) above, the energy density of the universe without considerations of vacuum energy as is the case of big bang model is $\rho = \rho_R + \rho_M$ and we can also write

$$H^2 = \frac{8\pi G}{3}(\rho_R + \rho_M) - \frac{k}{a^2} \tag{288}$$

The term $-\frac{k}{a^2}$ is clearly proportional to a^{-2}, while the energy density terms ρ_R and ρ_M fall off faster than scale factor $a(t)$ *i.e.* $\rho_R \propto \frac{1}{a^3(t)}$ and $\rho_M \propto \frac{1}{a^4(t)}$. This ratio $\frac{\left(\frac{k}{a^2(t)}\right)}{\left(\frac{8\pi G}{3}(\rho_R+\rho_M)\right)} = \frac{\left(\frac{k}{a^2(t)}\right)}{\left(\frac{\rho}{3M_{pl}^2}\right)}$ then is much smaller than unity when the scale factor $a(t)$ has increased by a factor of 10^{30} since the Planck era.

9.4. Entropy Problem

The adiabatic expansion of the universe following the first law of thermodynamics is related to the flatness problem [40] discussed above. Temperature plays a significant role in the early universe because at early epoches the age and expansion rate $H = \partial_t \ln a$ are described in terms of it with the number of relativistic degrees of freedom. From 1st Friedmann's equation we have the expression for density parameter $\Omega - 1 = \frac{k}{a^2(t)H^2}$ and expansion rate in radiation-dominated era in terms of temperature is $H_{\rho R}^2 \approx 8\pi G T^4 = \frac{T^4}{M_{pl}^2}$, so that the density parameter expression becomes $\Omega - 1 = \frac{kM_{pl}^2}{a^2(t)T^4}$. Now the entropy density is $s \sim T^3$ and the entropy per commoving volume $S \propto a^3(t)s \propto a^3(t)T^3$ and we have $\Omega - 1 = \frac{kM_{pl}^2}{S^{\frac{2}{3}}T^2}$. The entropy per commoving volume Sremains constant throughout the evolution of the universe as the hypothesis of adiabaticity requires so that we $|\Omega - 1|_{t=t_{pl}} = \frac{(1)M_{pl}^2}{S_U^{\frac{2}{3}}T_{pl}^2} \approx 10^{-60}$. It comes clear that at early epochs the value of $\Omega - 1$ is very close to zero as the total entropy of the universe is very large.

10. Introduction to Inflation

Inflation is the period of superluminally accelerated expansion of the universe taking place sometime in the very early history of the universe. It is now a widely accepted paradigm which is described as the monumental outgrowth gushing out during the tiniest fraction of the first second between $\left(10^{-36} - 10^{-32}\right)$ seconds. Inflation maintains that just after the occurrence of the big bang, exponential stretching of spacetime geometry took place *i.e.* becoming twice in size again and again at least about $(60 - 70)$ times over before slowing down. Alexei Strobinsky approached the exponentially expanding phase in the early universe by modifying Einstein Field Equations whereas Alan Guth approached the scenario in the realm of particle physics proposing a new picture of the time elapsed in the very small fraction of the first second in the 1980s. He suggested that the universe spent its earliest moments growing exponentially faster than it does today. There is a large number of inflation models in hand today but every model has its own limitations to draw the true picture of what happened actually in the early universe.

As the theory of inflation is recognized today, it has myriad models describing inflationary phase in the early universe. From amongst the heap of these competing models which differ slightly from one to the other, no model can claim a complete and all embracing prospectus of what happened actually in the universe so that the fast expansion of or in spacetime takes place. All the energy density that can be adhered to the early exponentially expanding phase of the universe was in the very fabric of spacetime itself despite to be in the form of radiation or particles. The early accelerating phase can be now best described with de Sitter model with slightly time symmetry broken. With the creation of spacetime that purports to be the earliest patch of the universe that comes to being would be stretched apart in an incredibly small time span of the order of a tiniest fraction of first second to such a colossaly larger size that its geometry and topology would be hardly indiscernible from Euclidean geometry. It will logically ensue similar initial conditions for the energy density to be dispersed at every point in the fabric of spacetime and the same will be the condition of temperature in this early phase. That's why the quantum fluctuations which seed in later times the structure formation in the universe impart the uniform temperature to all parts of the universe thereby resolving the homogeneity problem of the universe. This is because all the quantum fluctuations which cause the observable universe were once causally connected in the deep past of the universe. It might have attained a highest temperature which was within or lesser than the limits of Planck scale (that 10^{19} GeV. The energy scale mentioned earlier when the inflation comes to an end and transforms into the uniform, very hot, largely dense that is a cooling and expanding state we ascribe to the hot big bang. This will take place for a universe inflating from a lower entropy state to an entropy state at higher level in the panorama of the hot big bang, where the entropy would carry on to get larger as it happens in our observed universe. The point of time in the earliest where the universe can be viewed approximately and hardly as classical is known as the Planck Era. It is thought that prior to this era the universe might be described as the hitherto unsuspected theory of certain quantum nature like quantum gravity etc. This era corresponds to $E_P \sim 10^{19} GeV > E > E_{GUT} \sim 10^{15} GeV$ and the energies, temperature and times of particles are $E_P \sim 10^{19} GeV$, $T_P \sim 10^{32} K$, $t_P \sim 10^{-43} s$ respectively. Grand unified theories describe that at high energies as described above the electroweak and strong force are unified into a single force and that these interactions bring the particles present into thermal equilibrium Electroweak Era corresponds to phase transitions that occur through spontaneous symmetry breaking(SSB) which can be characterized by the acquisition of certain non-zero values by scalar parameters known as Higgs fields. Until the Higgs field has zero values symmetry remains observable and spontaneously breaks the moment at least one of the Higgs field becomes non-zero. The idea of phase transitions in the very early universe suggests the existence of the scalar fields and provides the motivation for considering their effect on the expansion of the universe.

The power spectrum of CMBR is calculated by measuring the magnitude of temperature variations versus the angular size of hot and cold spots. During these measurements, a series of peaks with different strengths and frequencies are determined which conforms to the predictions of inflation theory which confirms that all sound waves were indeed produced at the same moment by inflation. It is believed that inflation might have given rise to sound waves-waves traveling in the primordial vacuum-like medium with different frequencies after the big bang at $10^{-35} s$ starting in phase and would have been oscillating in radiation era for 380000 years. Now in the acoustic oscillations of the early universe, these must be measurable as power spectrum similar to that of measuring the sound spectrum of a musical instrument

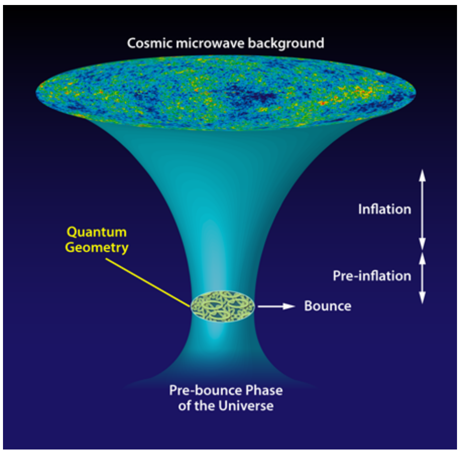

Figure 15. Inflationary universe

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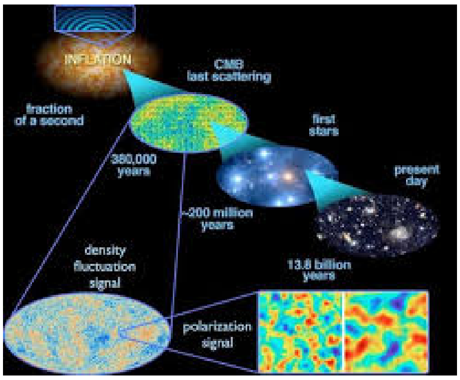

Figure 16. History of the universe beginning with big bang and expanding with inflation

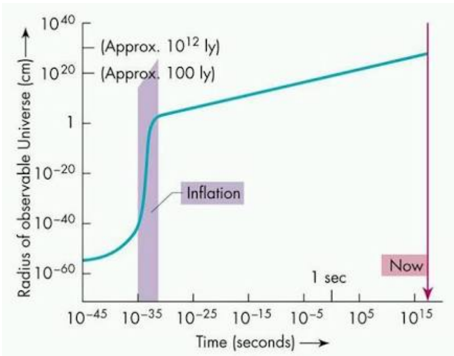

Figure 17. How scalar field drive inflationary era

10.1. Starobinsky R^2-Inflation

Alexei Starobinsky suggested a cosmological inflationary phase of the universe shortly before Alan Guth in 1980 working in the framework of general relativity. The model has been founded on the semiclassical Einstein field equations which provide a self-consistent solution for an exponentially accelerating era [41]. Starobinsky modified the general relativity to describe the behavior of very early universe undergoing an exponential period by suggesting quantum corrections to the energy momentum tensor. Quantum corrections are calculated by taking the expectation value of the energy momentum tensor. We begin with Einstein equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{289}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi < T_{\mu\nu} > \tag{290}$$

Where $< T_{\mu\nu} >$ represents the expectation value of the energy momentum tensor. The expectation value of energy momentum tensor is the probabilistic value of a result or measurement which is fundamentally rooted in all quantum mechanical systems. Intuitively it is the arithmetic mean of a large number of independent values of that variable. The energy momentum tensor $T_{\mu\nu}$ usually takes care of classical stuff of the universe in the form of matter and radiation and there is concerned with flat spacetime as the observations evidenced in recent data. In the case of curved spacetimes , nonetheless $T_{\mu\nu}$ might be vanishing and $< T_{\mu\nu} >$ must be imparted contributions from quantum regime non-trivially. In absence of classical stuff of universe in the form of matter and radiation, the curvature of spacetime from quantum fluctuations of matter fields contribute to $< T_{\mu\nu} >$ non-trivially which starobinsky utilized and are known as quantum corrections to $T_{\mu\nu}$. Quantum fluctuations of matter fields give non-trivial contributions to the expectation value of the energy momentum tensor

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$< T_{\mu\nu} >$ in the presence of cosmologically curved spacetime despite matter and radian do not exist in classical style. In the background we consider FLRW spacetime

$$ds^2 = -dt^2 + a(t)^2 \left(\left(1 - kr^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \tag{291}$$

The spatial part $\left(\frac{1}{1-kr^2}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$ of the metric represents the three geometries depending on the values of k. For $k = +1$, it represents a spherical geometry of 3-sphere which is finite, closed and without boundary. For $k = 0$, it represents a flat Euclidean geometry of 3-planes which is, in principle, infinite in extent, open and without boundary. $k = -1$, it represents a hyperbolic geometry of 3-hyperboloids which is infinite, open and without boundary. In the presence of conformally-invariant, free and massless fields, the quantum corrections adapt a simple form such that we can express the expectation value of energy momentum tensor as

$$< T_{\mu\nu} >= k_1 H_{\mu\nu} + k_2 H_{\mu\nu} \tag{292}$$

where k_1 and k_2 are numerical coefficients in standard notation. In order to find $< T_{\mu\nu} >$ we have to compute constants these constants k_1 and k_2 and $H_{\mu\nu}$ and $H_{\mu\nu}$. The coefficient k_1 is determined experimentally and can assume any value $H_{\mu\nu}$ is a tensor and is conserved identically when expressed as the action given below and varied with respect to metric tensor $\sqrt{-g}$, *i.e.*

$$H_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int \left(d^4x \sqrt{-g}\right) R^2 = 0 \tag{293}$$

$$H_{\mu\nu} = 2R_{,\mu,\nu} - 2g_{\mu\nu} R^{,\lambda}_{,\lambda} + 2RR_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^2 \tag{294}$$

The coefficient k_2 of $H_{\mu\nu}$ is defined uniquely in following form

$$k_2 = \left(N_0 + \frac{11}{2} N_{1/2} + 31 N_1 \right) \frac{1}{1440\pi^2} \tag{295}$$

Where N_0, $N_{1/2}$ and N_1 denote the number of quantum fields with the subscripts of all three $N's$ 0, $\frac{1}{2}$ and 1 representing spins of zero, half and one respectively. In certain GUT theories due to larger multiplier factor of N_1, the value of k_2 is majorly contributed by vector fields. Now $H_{\mu\nu}$ is also a tensor and it does not conserve generally but conserves only in those spacetimes which are conformally flat like FLRW spacetimes in particular and cannot be obtained by varying a local action as in the case of $H_{\mu\nu}$. The Eq. (292) multiplying with $8\pi G$ to both sides can be written as

$$8\pi G < T_{\mu\nu} >= 8\pi G k_1 H_{\mu\nu} + 8\pi G k_2 H_{\mu\nu} \tag{296}$$

or

$$8\pi G < T_{\mu\nu} >= \frac{48\pi G}{6} k_1 H_{\mu\nu} + 8\pi G k_2 H_{\mu\nu} \tag{297}$$

Now we introduce the following parameters for convenience

$$M = \sqrt{\frac{1}{48\pi G k_1}} \qquad H_0 = \sqrt{\frac{1}{8\pi G k_2}} \tag{298}$$

Where both the parameters are positive *i.e.* $H_0 > 0$ and $M > 0$. Now Eq. (297) takes the form

$$8\pi G < T_{\mu\nu} >= \frac{1}{6} M^{-2} H_{\mu\nu} + H_0^{-2} H_{\mu\nu} \tag{299}$$

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Eq. (299) can serve as reasonable approximation in case of certain GUT models for the limit $R > \mu^2$, where μ represents the unified energy scale. Conformally invariant field equations usually describe the spinor and massless vector fields and contribute to $< T_{\mu\nu} >$ in the form of Eq. (299). Further, if the number of matter fields is sufficiently bigger then the corrections to Einstein's field equations due to gravitons can also be ignored.

10.2. Trace-anomaly

The trace of expectation value of energy-momentum tensor $< T_{\mu\nu} >$ does not vanish instead has a non-zero anomalous trace and this is what we call as trace-anomaly. It is although interesting here to note that the trace of energy-momentum tensor without expectation value *i.e* $T_{\mu\nu}$, vanishes for those classical fields which are conformally invariant. Thus the trace of $< T_{\mu\nu} >$ is given by

$$< T_{\nu}^{\nu} >= M_{pl}^{-2}\left[H_0^{-2}\left(\frac{1}{3}R^2 - R_{\nu\sigma}R^{\nu\sigma}\right) - M^{-2}R_{;\nu}^{;\nu}\right] \tag{300}$$

The masses of the fields can be looked over in the limit of higher curvature *i.e.* when $R \gg m^2$ and in the same limit it remains true for the case of asymptotically free gauge theories where interactions between the fields become negligible. In de sitter space we can have

$$R_{\mu\nu} = \frac{1}{4}Rg_{\mu\nu} \tag{301}$$

where R is constant. Substituting now Eq. (300) and Eq. (301) in Eq. (290), we have $R = 12H_0^2$ for non-trivial solution and the corresponding de Sitter solutions come about for $k = 0, +1, -1$ respectively

$$a(t) = a_0 e^{tH_0} \tag{302}$$

$$a(t) = \frac{1}{H_0}\cosh(tH_0) \tag{303}$$

$$a(t) = \frac{1}{H_0}\sinh(tH_0) \tag{304}$$

Eq. (302) corresponds to $k = 0$ and gives a flat universe, Eq. (303) gives a closed solution for $k = +1$ and the 3rd Eq. (304) for $k = -1$ propounds the open de Sitter model of the universe. These solutions are impelled completely by the quantum corrections rendered to classical EFE and serve the purpose of inflationary epoch in the very early universe. Starobinsky inflation corresponds to a potential parameterized in terms of scalar field ϕ is $V(\phi) = \frac{3}{4}\left(1 - e^{-\sqrt{\frac{2}{3}\phi}}\right)^2$.

10.3. Inflation and de Sitter Universe

In a very shorter period of time about 10^{-35} after the springing out of the spacetime into being, the inflationary era of accelerating superluminal expansion known to be de Sitter phase took place. de Sitter phase removed all the wrinkles of curvature and warpage of spacetime so that the universe is to be observed flat. It further smoothed out all energy density stuff for the distribution of radiation and matter. One significant remnant as the traces of this fast expansion remains there known later on to be cosmic background radiation. In de Sitter universe there exists no ordinary matter, however, de Sitter retained cosmological constant which represents vacuum energy smeared out into the structure of spacetime. We can define the energy density of this non-relativistic matter

$$\rho_{\Lambda} = \frac{\Lambda}{8\pi G} \tag{305}$$

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Since $p_\Lambda = -\rho_\Lambda$ which gives an exotic form of matter with negative pressure, that is where the scale factor $a(t)$ goes on increasing but $\dot{a}(t)$ is decreasing. We write

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_\Lambda - \frac{k}{a^2} \tag{306}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_\Lambda + 3p_\Lambda\right) \tag{307}$$

And

$$\dot{\rho}_\Lambda + 3H\left(\rho_\Lambda + p_\Lambda\right) = 0 \tag{308}$$

From Eq. (308) with $\dot{\rho}_\Lambda = \frac{d\rho_\Lambda}{dt} = 0$ and $\frac{p_\Lambda}{\rho_\Lambda} = w$

$$3H\left(\rho_\Lambda + p_\Lambda\right) = 0 \tag{309}$$

or

$$3H\rho_\Lambda\left(1 + w\right) = 0 \tag{310}$$

Now from Eq. (307) for $\frac{p_\Lambda}{\rho_\Lambda} = w$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_\Lambda\left(1 + 3w\right) \tag{311}$$

For $w = -1$, Eq. (311) becomes

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_\Lambda\left(1 - 3\right) = \frac{8\pi G}{3}\rho_\Lambda \tag{312}$$

or

$$\frac{d^2a}{dt^2} - \frac{8\pi G}{3}\rho_\Lambda a = 0 \tag{313}$$

Eq. (313) is the equation of a harmonic oscillator. From Eq. (306) for vanishing curvature *i.e.* $k = 0$ where Λ dominates and $\frac{\dot{a}}{a} = H$

$$H_\Lambda^2 = \frac{8\pi G}{3}\rho_\Lambda \tag{314}$$

Finding the value of ρ_Λ

$$\rho_\Lambda = \frac{3H_\Lambda^2}{8\pi G} \tag{315}$$

Substituting Eq. (315) in Eq. (313), and simplifying we have

$$\frac{d^2a}{dt^2} - \frac{8\pi G}{3}\left(\frac{3H_\Lambda^2}{8\pi G}\right)a = 0 \tag{316}$$

or

$$\frac{d^2a}{dt^2} - 3H_\Lambda^2 a = 0 \tag{317}$$

We can write the solution of above Eq. (317) as

$$a(t) = C_1 \exp\left(H_\Lambda t\right) + C_2 \exp\left(-H_\Lambda t\right) \tag{318}$$

Differentiating Eq. (318) twice with respect to time $'t'$

$$\dot{a}(t) = C_1 H_\Lambda \exp\left(H_\Lambda t\right) - C_2 H_\Lambda \exp\left(-H_\Lambda t\right) \tag{319}$$

again

$$\ddot{a}(t) = C_1 H_\Lambda^2 \exp\left(H_\Lambda t\right) + C_2 H_\Lambda^2 \exp\left(-H_\Lambda t\right) \tag{320}$$

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Using Eq. (318) in Eq. (320), we can write

$$\ddot{a}(t) = H_\Lambda^2 \left(C_1 \exp\left(H_\Lambda t \right) + C_2 \exp\left(-H_\Lambda t \right) \right) = H_\Lambda^2 a(t) \tag{321}$$

Substituting the value of ρ_Λ from Eq. (315) in Eq. (306), we have

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left(\frac{3H_\Lambda^2}{8\pi G} \right) - \frac{k}{a^2} = H_\Lambda^2 - \frac{k}{a^2} \tag{322}$$

simplifying Eq. (322) gives

$$k = H_\Lambda^2 a^2 - \dot{a}^2 \tag{323}$$

Substituting the values of $a(t)$ and $\dot{a}(t)$ from Eq. (318) and Eq. (319) in above Eq. (323)

$$\begin{aligned} k = {} & H_\Lambda^2 \left(C_1 \exp\left(H_\Lambda t \right) + C_2 \exp\left(-H_\Lambda t \right) \right)^2 \\ & - \left(C_1 H_\Lambda \exp\left(H_\Lambda t \right) - C_2 H_\Lambda \exp\left(-H_\Lambda t \right) \right)^2 \end{aligned} \tag{324}$$

Simplification gives

$$k = 4H_\Lambda^2 C_1 C_2 \tag{325}$$

Eq. (325) means that the curvature term k depends upon the constants of integration C_1 and C_2. For flat universe either $C_1 = 0$ or $C_2 = 0$. The solution in Eq. (318) becomes accordingly

$$a(t) = C_2 \mathrm{e}^{-H_\Lambda t} \tag{326}$$

and

$$a(t) = C_1 \mathrm{e}^{H_\Lambda t} \tag{327}$$

Further Einstein equations are given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{328}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \tag{329}$$

and the form of solution of these equations upon which big bang standard cosmology is based, as worked out by Alexander Friedman (1922), George Lemaitre (1927) and afterwards by Robertson and Walker (1935) independently on the base of cosmological principle which put to use the homogeneity and isotropy, is

$$ds^2 = -dt^2 + a(t)^2 \left(\left(1 - kr^2 \right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \tag{330}$$

Where $\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The metric in Eq. (330) is characterized by scale factor $a(t)$ and the curvature of spacetime k which are obviously determined by the self-gravitation of all the matter-energy content in the universe. We have incorporated dark matter and dark energy in the matter-energy content because their role is not avoidable at all in accelerated expansion and the present Minkowskian flat geometry of the universe. The solution of this line element gives Friedman equations using Einstein field equations that govern the time evolution of the universe and are given as

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} \tag{331}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3p \right) + \frac{\Lambda}{3} \tag{332}$$

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The presence of cosmological term Λ in above equations would be equivalent to that of a fluid having an equation of state $p = -\rho$ which is satisfied by

$$\rho + 3p > 0 \tag{333}$$

Looking at the things classically we may approach the classical period of exponential expansion by using the first Friedmann equation by vanishing density ρ of radiation and baryons and the entailing curvature k in Λ-dominated Era which corresponds to equivalently having a fluid with $p = -\rho$, thus Eq. (331) becomes

$$\begin{aligned}
&\tfrac{\dot{a}^2}{a^2} + \tfrac{(0)}{a^2} = \tfrac{8\pi G}{3}(0) + \tfrac{\Lambda}{3} \\
&\tfrac{\dot{a}^2}{a^2} = \tfrac{\Lambda}{3} \\
&\dot{a} = \tfrac{da}{dt} = \sqrt{\tfrac{\Lambda}{3}}a \\
&\tfrac{da}{a} = \sqrt{\tfrac{\Lambda}{3}}dt
\end{aligned} \tag{334}$$

After integrating and simplifying, we get

$$a = e^{\sqrt{\frac{\Lambda}{3}}t} \tag{335}$$

Eq. (335) gives the exponential expansion of the scale factor. It describes the fact that when the universe was dominated by cosmological constant Λ, the rate expansion was much faster than the present day scenario. From Eq. (332)

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)\, a + \frac{\Lambda}{3}a \tag{336}$$

Considering a closed volume with energy $U = \rho V = \rho\frac{4\pi}{3}a^3$ and now we see how inflationary period is obtained in the perspective of particle physics where a negative pressure is achieved for it to take place. Friedmann solved EFE with $\Lambda = 0$, so

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{337}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{338}$$

Eq. (338) is known as acceleration equation. The inflationary period, as its definition implies, is the acceleratingly expanding phase of the universe in a very small fraction of first second, since the expansion is characterized by the scale factor a, therefore, we have such an era as

$$\ddot{a} > 0 \tag{339}$$

thus Inflationary era

$$\Leftrightarrow \ddot{a} > 0 \tag{340}$$

Dividing both sides of Eq. (317) by scale factor a

$$\frac{\ddot{a}}{a} > 0 \tag{341}$$

Which is LHS of Eq. (338) Eq. (341) imposes the condition on RHS of Eq. (338)

$$\begin{aligned}
&-\tfrac{4\pi G}{3}(\rho + 3p) > 0 \\
&\Rightarrow \rho + 3p < 0 \\
&\Rightarrow p < -\tfrac{1}{3}\rho \\
&\Rightarrow \rho > -3p
\end{aligned} \tag{342}$$

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For the inflation to occur and set the universe in an accelerating phase we require the matter to possess an equation of state with negative pressure. The possibility of this negative pressure p which is less than negative of one-third of density is in perspective of symmetry breaking in modern models of particle physics. From

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{343}$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k \tag{344}$$

For $\ddot{a} > o$, the scale factor shall increase faster than $a\,(t) \propto t$ and the term $\frac{8\pi G}{3}\rho a^2$ shall increase during this accelerated era such that the curvature term k will become negligibly small and shall vanish. Inflationary era is also defined by considering the shrinking of Hubble Sphere [43] due to its direct linkage to the horizon problem and due to providing a fundamental role in producing of quantum fluctuations. Shrinking Hubble Sphere is defined as

$$\frac{d\left[(aH)^{-1}\right]}{dt} < 0 \tag{345}$$

$$\frac{d\left[(aH)^{-1}\right]}{dt} = \frac{d\left[\left(a\frac{\dot{a}}{a}\right)^{-1}\right]}{dt} = \frac{d\left[(\dot{a})^{-1}\right]}{dt} = -\frac{\ddot{a}}{a^2} \tag{346}$$

$$-\frac{\ddot{a}}{a^2} < 0 \tag{347}$$

which will imply accelerated expansion

$$\ddot{a} > 0 \tag{348}$$

At $t = 0$, the scale factor a characterizing expansion of the universe comes out to be of a specific value. In Eq. (337), when $\rho = \rho_\phi$ is of very larger value and the scale factor a dominates over the curvature term k, then we have

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_\phi \tag{349}$$

$$a = a_0 e^{Ht} \tag{350}$$

de Sitter line element is given by

$$ds^2 = -dt^2 + e^{2Ht}\left(dx^2 + dy^2 + dz^2\right) \tag{351}$$

inflation has to terminate and H is constant which means that de Sitter phase cannot give perfect inflationary era, however for $\frac{\dot{H}}{H^2}$ it would compensate. It would be interesting here to note that Z. G. Lie and Y.S. Piao have shown that the universe we observe today may have emerged from a de Sitter background without having the requirement of a large tunneling in potential and with low energy scale. [42]

10.4. The Conditions Under Which the Inflation Occurs

Shrinking Hubble sphere has been considered as basic definition of inflationary era due to its direct connection to the horizon problem and with mechanism of quantum fluctuation generations [43]. differentiating the comoving Hubble radius $(aH)^{-1}$ with respect to time we find the acceleratedly expanding Hubble sphere

$$\partial_t (aH)^{-1} = -\frac{\ddot{a}}{\dot{a}^2} \tag{352}$$

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We see that $-\frac{\ddot{a}}{\dot{a}^2} < 0$, multiplying the inequality by -1 and simplifying, we have

$$\ddot{a} > 0 \tag{353}$$

which means that shrinking comoving Hubble sphere $(aH)^{-1}$ points toward accelerated expansion $\ddot{a} > 0$. Since Hubble sphere H remains nearly constant, in order to understand the meaning of nearly constant we see how its slow roll variation takes place, so taking H as variable

$$\partial_t \left(\frac{1}{aH}\right) = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}\left(1 + \frac{\dot{H}}{H^2}\right) \tag{354}$$

where $\frac{\dot{H}}{H^2} = -\varepsilon$ known as slow roll parameter. It can be inferred that $\frac{\dot{H}}{H^2} < 0$ implies shrinking Hubble sphere.

10.5. Slow Roll Inflation-The Dynamics of Scalar Field

Elementary particles in modern physics are represented by quantum fields and oscillations of these fields are translated as particles. Scalar fields represent spin zero particles in field theories and look like vacuum states because they have same quantum numbers as vacuum. The matter with negative pressure $\rho = -p$ represents physical vauum-like state where quantum fluctuations of all types of physical fields exist. These fluctuations can be considered as waves of all possible wavelengths related with physical fields *i.e.* wavy physical fields moving freely in all directions. The negative pressure violates the strong energy condition which is necessary for the inflation to occur. To keep things simpler a single scalar field namely inflaton $\phi = \phi(x, t)$ is considered present in the very early universe. Since the value of the scalar field depends upon position x in space which assigns potential energy to each field value. It is also dynamical due to being function of time t and has kinetic energy as well *i.e* energy density $\rho(\phi)$ associated with the inflaton ϕ is $\rho(\phi) = \rho_p + \rho_k$. The ratio of the potential and kinetic energy terms of $\phi = \phi(x, t)$, decides the evolution of the universe. The Langrangian of the scalar inflaton field ϕ is expressed as the energy difference between its kinetic and potential terms.

$$L = \frac{1}{2}\left(g_{\mu\nu}\partial^2\phi - V(\phi)\right) \tag{355}$$

It is assumed that the background of FLRW universe has been sourced by energy-momentum associated with the inflaton which dominates the universe in the beginning. We shall observe under what conditions this causes accelerated expansion of the FLRW universe.

$$S = \int d^4x\sqrt{-g}L = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(g_{\mu\nu}\partial^2\phi - V(\phi)\right)\right] \tag{356}$$

The energy-momentum tensor of the inflaton field is given as

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(L) \tag{357}$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(L) \tag{358}$$

Which for $\mu = 0, \nu = 0$ results as

$$T_{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}\nabla^2\phi + V(\phi) \tag{359}$$

and for $\mu = \nu = j$

$$T_{jj} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}\nabla^2\phi - V(\phi) \tag{360}$$

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The gradient term vanishes, in otherwise condition, the pressure gained is much less than the required value to impart impetus for inflation to take place, therefore we obtain the following values for energy density and pressure

$$\rho_\phi = T_{00} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{361}$$

and

$$p_\phi = T_{jj} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{362}$$

The condition $V(\phi) >> \dot{\phi}^2$ corresponds to the negative pressure condition $\rho_\phi = -p_\phi$ which means that the potential (vacuum) energy of the inflaton derives inflation. Now using Euler-Langrange equations

$$\partial^\mu \frac{\delta\left(\sqrt{-g}L\right)}{\delta \partial^\mu \phi} - \frac{\delta\left(\sqrt{-g}L\right)}{\delta \phi} = 0 \tag{363}$$

we can find equation for inflaton field that comes to be

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2(t)}\nabla^2\phi + V_{,\phi}(\phi) = 0 \tag{364}$$

It can also be computed from the energy density and the pressure terms given in Eq. (361) and Eq. (362) respectively by substituting in equation of energy conservation

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0 \tag{365}$$

Eq. 365 in terms of inflation field ϕ

$$\frac{d\rho_\phi}{dt} + 3H\left(\rho_\phi + p_\phi\right) = 0 \tag{366}$$

By substituting Eq. (361) and eq. (362) in Eq. (366), we have

$$\frac{d\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)}{dt} + 3H\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\dot{\phi}^2 - V(\phi)\right) = 0 \tag{367}$$

$$\left(\ddot{\phi} + V'(\phi) + 3H\dot{\phi}\right)\dot{\phi} = 0 \tag{368}$$

$$\ddot{\phi} + V'(\phi) + 3H\dot{\phi} = 0 \tag{369}$$

Where $V'(\phi) = \frac{dV(\phi)}{d\phi}$ and $3H\dot{\phi}$ is known as friction term which offers friction to the inflaton field when it rolls down ($\dot{\phi}$) its potential during expansion of the universe $H = \frac{\dot{a}}{a}$.

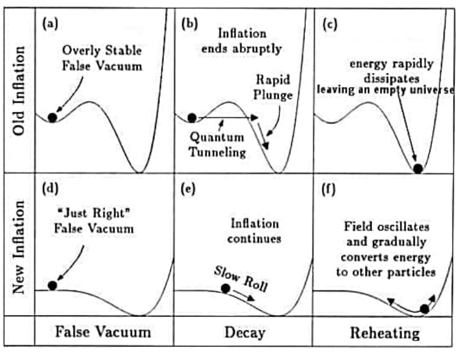

Figure 18. how the universe springs into being through sclar field

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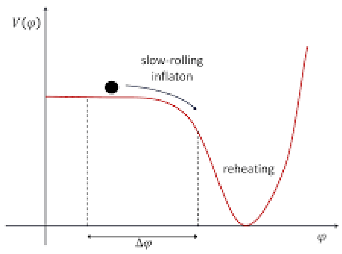

Figure 19. how inflation ends-slow roll inflation

10.6. Conditions of the Slow Roll Inflation

According to the big bang model, that is the currently accepted model, the universe is about 14 billion years old. At the point of creation the curvature of spacetime was very large or equivalently can described in other words that space was largely warped and curved where only quantum effects can prevail and the question of time to exist is likely to become absurd. From this state how the very brief era of exponential expansion can be had is fulfilled by assumption of scalar field which take the responsibility of such state mentioned. we know from the 2nd Friedmann's equation which is acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_\phi + 3p_\phi\right) \tag{370}$$

For $\ddot{a} > 0$

$$\rho_\phi + 3p_\phi < 0 \Rightarrow p_\phi < \frac{1}{3}\rho_\phi \tag{371}$$

From Eq. (361) and Eq. (362), substituting for p_ϕ and ρ_ϕ in Eq. (371)

$$\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right) < -\frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \tag{372}$$

Solving the inequality and keeping in mind that $\dot{\phi}$ is a squared term, we have

$$\dot{\phi}^2 << V(\phi) \tag{373}$$

which means that the inflaton field is slowly rolling down its potential. Differentiating Eq. (373) with respect to time, we have

$$\ddot{\phi} < \frac{1}{2}V'(\phi) \tag{374}$$

Now from Eq. (369), we obtain

$$\ddot{\phi} + V'(\phi) = -3H\dot{\phi} \tag{375}$$

We neglect the acceleration providing term $\ddot{\phi} = \frac{d^2\phi}{dt^2}$ as the inflaton field has to roll now slowly to escape from graceful exit problem in inflation i.e. deceleratingly, so we write

$$V'(\phi) = -3H\dot{\phi} \tag{376}$$

plugging Eq. (376) in Eq. (374)

$$\ddot{\phi} < \frac{1}{2}(-3H\dot{\phi}) \tag{377}$$

On neglecting the constant factor, it gives

$$\ddot{\phi} << 3H\dot{\phi} \tag{378}$$

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differentiating now Eq. (376) with respect to time,

$$3\left(\dot{H}\dot{\phi}+H\ddot{\phi}\right)=-V''(\phi)\dot{\phi} \tag{379}$$

Since H remains constant during inflation, therefore $\dot{H}$ vanishes and we have

$$\ddot{\phi}=-\frac{V''(\phi)\dot{\phi}}{3H} \tag{380}$$

Putting Eq. (380) in Eq. (378), we have

$$-\frac{V''(\phi)\dot{\phi}}{3H}<<3H\dot{\phi} \tag{381}$$

It gives

$$V''(\phi)<<H^2 \tag{382}$$

10.7. Parameters for the Slow Roll Inflation

Two slow roll parameters ε and η are defined in terms of Hubble parameter H as well as potential V which quantify slow roll inflation.

$$\varepsilon_H=-\frac{\dot{H}}{H^2} \tag{383}$$

Using the relation $a(t)\propto e^{-N}\Rightarrow N=\ln a$, it can also be expressed in the form

$$\varepsilon_H=-\frac{d\left(\ln H\right)}{dN} \tag{384}$$

where N is the number of e-folds And 2nd is defined as

$$\eta_H=-\frac{1}{2}\frac{\ddot{H}}{\dot{H}H} \tag{385}$$

From 1st Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2-\frac{k}{a^2}=\frac{8\pi G}{3}\rho \tag{386}$$

For $\rho=\rho_\phi$ and from Eq. (361) $\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi)$, Since during inflation $V\left(\phi\right)>>\dot{\phi}^2$, so that $\rho_\phi=V(\phi)$ also curvature term k is negligibly small, so that Eq. (386) becomes

$$H^2=\frac{8\pi G}{3}V\left(\phi\right) \tag{387}$$

differentiating Eq. (387) with respect to time and simplifying

$$\dot{H}=\frac{4\pi G}{3H}V'\left(\phi\right)\left(\dot{\phi}\right) \tag{388}$$

And from Eq. (376) substituting in Eq. (388), we have

$$\dot{H}=-4\pi G\left(\dot{\phi}^2\right) \tag{389}$$

Substituting above in Eq. (383), we have

$$\varepsilon_H=-\frac{\dot{H}}{H^2}=-\frac{-4\pi G\left(\dot{\phi}^2\right)}{H^2}=\frac{4\pi G}{H^2}\dot{\phi}^2 \tag{390}$$

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Again from Eq. (376) we have

$$\dot{\phi} = -\frac{V'(\phi)}{3H} \tag{391}$$

squaring

$$\dot{\phi}^2 = -\frac{V'^2(\phi)}{9H^2} \tag{392}$$

substituting in Eq. (390)

$$\varepsilon_H = \frac{4\pi G}{H^2}\left(-\frac{V'^2(\phi)}{9H^2}\right) = \frac{4\pi G V'^2(\phi)}{9(H^2)^2} \tag{393}$$

From Eq. (387) putting for H^2

$$\varepsilon_V = \frac{4\pi G V'^2(\phi)}{9\left(\frac{8\pi G}{3}V(\phi)\right)^2} = \frac{1}{16\pi G}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 = \frac{M_{pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \tag{394}$$

η_H can also be expressed as

$$\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} \tag{395}$$

$$\eta_V = \frac{1}{8\pi G}\left(\frac{V''(\phi)}{V(\phi)}\right) = M_{pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right) \tag{396}$$

From Eq. (387) $H^2 = \frac{8\pi G}{3}V(\phi)$, which gives $8\pi G V(\phi) = 3H^2$ substituting above in Eq. (396), we have

$$\eta_V = \frac{V''(\phi)}{3H^2} \tag{397}$$

10.8. Number of e-folds

It is usual practice to have the inflation quantified and the quantity which does this is called number of e-fold denoted by N before the inflation ends. As the time goes by N goes on decreasing and becomes zero when inflation ends. It is counted or measured backwards in time from the end of inflation which means that $N = 0$ at the end of inflation grows to maximal value towards the beginning of inflation. It measures the number of times the space grows during inflationary period. The amount of e-folds necessarily required to resolve the big bang problems of Horizon, Flatness, Monopole and Entropy etc is $N \sim 60 - 75$ depending upon the different models and on the reasonable estimation of the observational parameters. To find the number of e-folds between beginning and end of inflation we know that during inflation the scale factor evolves as

$$a(t) = a(t_0)\, e^{Ht} \tag{398}$$

or

$$a(t) = a(t_0)\, e^{H(t-t_i)} \tag{399}$$

The factor Ht constitute the number of e-folds denoted by N *i.e.*

$$N = Ht \tag{400}$$

Differentiating Eq. (400) with respect to time

$$\frac{dN}{dt} = H = \partial_t \ln a \tag{401}$$

$$N = \int_{t_i}^{t_f} H dt = \int_{t_i}^{t_f} \frac{\dot{a}}{a} dt = \ln\left(\frac{a_{t_f}}{a_{t_i}}\right) \tag{402}$$

Further, the relation between Hubble parameter H and the number of e-folds N can be written We have derived earlier the evolution equation for inflaton field to be

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \tag{403}$$

During slow roll inflation $\ddot{\phi} = 0$, so that Eq. (403) becomes

$$3H\dot{\phi} + V_{,\phi} = 0 \tag{404}$$

$$3H\dot{\phi} = -V_{,\phi} \tag{405}$$

And during slow roll the Friedmann's 1st equation evolves as with $k = 0$ and $\rho = V(\phi) + \frac{1}{2}\dot{\phi}^2$

$$H^2 = \frac{8\pi G}{3}\left(V(\phi) + \frac{1}{2}\dot{\phi}^2\right) \tag{406}$$

During slow roll $(\dot{\phi})^2 << V(\phi)$and only $\dot{\phi}$ works, thus Eq. (406) becomes

$$H^2 = \frac{8\pi G}{3} V(\phi) \tag{407}$$

Dividing Eq. (405) by Eq. (407)

$$\frac{\dot{\phi}}{H} = -\frac{V_{,\phi}}{8\pi G V(\phi)} \tag{408}$$

Now from Eq. (400), we can write because $t = t_f - t_i$, so $t = \int_{t_i}^{t_f} dt$ and with dividing and multiplying by $d\phi$

$$N = Ht = \int_{t_i}^{t_f} H dt = \int_{t_i}^{t_f} H \frac{dt}{d\phi} d\phi \tag{409}$$

Where $\dot{\phi} = \frac{d\phi}{dt}$, Eq. (409) takes the form

$$N = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi \tag{410}$$

Substituting from Eq. (408) after inverting

$$N = \int_{\phi_i}^{\phi_f} \left(-\frac{8\pi G V(\phi)}{V_{,\phi}}\right) d\phi = -8\pi G \int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V_{,\phi}} d\phi \tag{411}$$

or

$$N = 8\pi G \int_{\phi_f}^{\phi_i} \frac{V(\phi)}{V_{,\phi}} d\phi \tag{412}$$

Thus number of e-folds can be found in terms of potential of the inflaton field. Further slow roll parameter ε_H can be described in terms of number of e-fold N, we know

$$\varepsilon_H = -\frac{\dot{H}}{H^2} = -\frac{1}{H^2}\frac{dH}{dt} = -\frac{1}{H^2}\frac{dH}{dN}\frac{dN}{dt} \tag{413}$$

$$\varepsilon_H = -\frac{1}{H^2}\frac{d\ln N}{dt} \tag{414}$$

11. Inflationary Solutions to the Big Bang Problems

Horizon, flatness, entropy and monopole problems are initial value problems which inflation solves in one go. Inflation explains why the observable universe is spatially flat, isotropically homogeneous and so large in size.

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11.1. Inflation and Horizon Problem

We consider that the inflation begins at a time (t_i) and comes to an end at some time $\left(t_f\right)$ and the expansion rate $H = \partial_t \ln a$, curvature term k and energy density of matter and radiation $\rho = \rho_M + \rho_R$ during inflation vanishes, we know that

$$a(t) = a(t)e^{Ht} = a(t)e^{H(t-t_i)} \tag{415}$$

and

$$a(t) = a(t)e^{Ht} = a(t)e^{H(t_f-t)} \tag{416}$$

We will find how long the inflation must sustain to resolve the horizon problem. We can find the corresponding e-folding number N that is

$$N = Ht = \int_{t_i}^{t_f} Hdt \tag{417}$$

Since $H = \frac{\dot{a}}{a}$,

$$N = \int_{t_i}^{t_f} \frac{\dot{a}}{a}dt = \int_{t_i}^{t_f} \frac{da}{a} \tag{418}$$

$$N = \ln a|_{t_i}^{t_f} = \ln\left(a_f - a_i\right) \tag{419}$$

or

$$\frac{a_f}{a_i} = e^N \tag{420}$$

or

$$\frac{a_i}{a_f} = e^{-N} \tag{421}$$

Now the horizon scale observed today H_0^{-1} was reduced during inflation to a value of $\lambda_{H_0}(t_i)$ which is smaller than the horizon length during inflation.

$$\lambda_{H_0}(t_i) = R_{H_0}\left(\frac{a_{t_i}}{a_{t_0}}\right) \tag{422}$$

Dividing and multiplying Eq. (422) by a_{t_f}

$$\lambda_{H_0}(t_i) = R_{H_0}\left(\frac{a_{t_i}}{a_{t_0}} \times \frac{a_{t_f}}{a_{t_f}}\right) \tag{423}$$

Now from Eq. (421) $\frac{a_i}{a_f} = e^{-N}$ and using the relation between scale factor and temperature during this phase $a \sim \frac{1}{T} \Rightarrow a_i \sim \frac{1}{T_i}$ and $\Rightarrow a_f \sim \frac{1}{T_f}$ so that we have

$$\lambda_{H_0}(t_i) = H_0^{-1}\frac{T_0}{T_f}e^{-N} \tag{424}$$

Where $R_{H_0} = H_0^{-1}$. Now $\lambda_{H_0}(t_i) < H_I^{-1}$ where H_I^{-1} is the horizon length during inflation. So Eq. (424) can be expressed as

$$H_0^{-1}\frac{T_0}{T_f}e^{-N} \leq H_I^{-1} \tag{425}$$

$$\frac{H_0^{-1}}{H_I^{-1}}\frac{T_0}{T_f} \leq e^N \tag{426}$$

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or

$$e^N \geq \frac{\left(\frac{T_0}{H_0}\right)}{\left(\frac{T_f}{H_I}\right)} \Rightarrow N \geq \ln \frac{\left(\frac{T_0}{H_0}\right)}{\left(\frac{T_f}{H_I}\right)} \tag{427}$$

$$N \geq \ln\left(\frac{T_0}{H_0}\right) - \ln\left(\frac{T_f}{H_I}\right) \tag{428}$$

or

$$N \approx 67 + \ln\left(\frac{H_I}{T_f}\right) \tag{429}$$

$$N \geq 70 \tag{430}$$

11.2. Inflation and Flatness Problem

From 1st Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{431}$$

We found the density parameter expression

$$\Omega - 1 = \frac{k}{H^2 a^2(t)} \tag{432}$$

During the inflationary period Hubble parameter giving expansion rate remains almost constant so that Eq. (432) is

$$\Omega - 1 = \frac{k}{H^2 a^2(t)} \propto \frac{1}{a^2(t)} \tag{433}$$

We observed earlier that

$$|\Omega - 1|_{t=t_{pl}} \approx 10^{-60} \tag{434}$$

Which means that to have the value of the density parameter as observed today *i.e.* Ω_0 to be of the order of unity, the initial value of Ω at the beginning of the radiation-dominated era must be same as given in Eq. (434) above. and from Eq. (432) we can write for the time at the beginning of inflationary era

$$|\Omega - 1|_{t=t_i} = \frac{k}{H_I^2 a_i^2(t)} \tag{435}$$

and for the time when inflationary period comes to end

$$|\Omega - 1|_{t=t_f} = \frac{k}{H_I^2 a_f^2(t)} \tag{436}$$

Further the beginning of the radiation-dominated era can be recognized with the beginning of inflationary phase such that it is required

$$|\Omega - 1|_{t=t_f} = 10^{-60} \tag{437}$$

Dividing now Eq. (436) by Eq. (435)

$$\frac{|\Omega - 1|_{t=t_f}}{|\Omega - 1|_{t=t_i}} = \frac{\frac{k}{H_I^2 a_f^2(t)}}{\frac{k}{H_I^2 a_i^2(t)}} = \left(\frac{a_i^2(t)}{a_f^2(t)}\right) \tag{438}$$

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We calculated $\frac{a_i}{a_f} = e^{-N}$, the above Eq. (438) takes the form

$$\frac{|\Omega - 1|_{t=t_f}}{|\Omega - 1|_{t=t_i}} = \left(e^{-N}\right)^2 \tag{439}$$

$$e^{-2N} = \frac{|\Omega - 1|_{t=t_f}}{|\Omega - 1|_{t=t_i}} \tag{440}$$

With taking $|\Omega - 1|_{t=t_i} \approx 1$,

$$N \approx -\frac{1}{2} \ln |\Omega - 1|_{t=t_f} \tag{441}$$

$$N \simeq 70 \tag{442}$$

11.3. Inflation and Entropy Problem

Entropy problem can be resolved if a large amount of entropy is created in the very early universe non-adiabatically [39] which is accomplished by inflationary era in a finite time in the early history of the universe. Let the entropy at the end of inflation is S_f and in the beginning it was S_i such that $S_f \propto S_i$, then

$$S_f = M^3 S_i \tag{443}$$

Where M is the numerical factor with value $M^3 = 10^{10} \Rightarrow M = 10^{30}$. Now $S_f = S_U$. We know that $S \sim (aT)^3$, so that we can write for

$$S_i \sim (a_i T_i)^3 \tag{444}$$

and for

$$S_f \sim \left(a_f T_f\right)^3 \tag{445}$$

where T_i and T_f are the measures of temperature at the beginning and end of the inflationary period. Dividing Eq. (445) by Eq. (444) we have

$$\frac{S_f}{S_i} \approx \left(\frac{a_f}{a_i}\right)^3 \left(\frac{T_f}{T_i}\right)^3 \tag{446}$$

or

$$\left(\frac{a_f}{a_i}\right)^3 \approx \frac{S_f}{S_i} \left(\frac{T_i}{T_f}\right)^3 \tag{447}$$

$$\frac{a_f}{a_i} \approx \left(\frac{S_f}{S_i}\right)^3 \frac{T_i}{T_f} \tag{448}$$

Now $\frac{a_f}{a_i} = e^N$, and considering that at the beginning of inflationary phase the total entropy of the universe was of the order 1 *i.e.* $S_i \sim 1$, and $S_f = S_U$ thus Eq. (448) takes the form

$$e^N \approx (S_U)^3 \frac{T_i}{T_f} \tag{449}$$

$$N \approx \ln (S_U)^3 \frac{T_i}{T_f} \tag{450}$$

$$N \sim 70 \tag{451}$$

therefore, entropy problem is resolved by inflationary period.

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11.4. Inflation and Monopole Problem

In grand unified theories(GUT), the standard model $SU(3) \times SU(2) \times U(1)$ in particle physics emerges out of a simple symmetry group breaking. In these theories some very density particles known as magnetic monopoles are predicted to be created. Cosmological monopoles prior the period of inflation takes place are allowed supposedlyly to exist. Monopoles are supposed to form in symmetry breaking during phase transitions and where the inflationary era is supposed to take place just after it. Inflation dilutes the density of these magnetic monopoles $n_{mp} \propto \frac{N_{mp}}{a^3} \rightarrow 0$ to the negligibly small size such that these become so small to be detected today [44]. During inflation monopoles collapse in an exponential way and their abundant presence falls to the level of being hardly detectable.

12. The Line Element of the Perturbed Universe

When the perturbations of inflaton field ϕ are considered the energy momentum tensor $T_{\mu\nu}$ is also perturbed. The perturbations of inflaton field thus are consequently reflected through the metric tensor $g_{\mu\nu}$ such that $\delta\phi \Leftrightarrow \delta g_{\mu\nu}$. We discuss here only the scalar perturbations for which the metric takes the form as

$$ds^2 = a^2(t)\begin{bmatrix} (-1-2A)\,d\tau^2 + 2\partial_i B d\tau dx^i + (1-2\psi)\,\delta_{ij} \\ +D_{ij}E)dx^i dx^j \end{bmatrix} \tag{452}$$

$$\delta g_{\mu\nu} = a^2(t)\begin{pmatrix} \delta g_{00} & \delta g_{0i} \\ \delta g_{i0} & \delta g_{ij} \end{pmatrix} = a^2(t)\begin{pmatrix} 1-2A & \partial_i B \\ \partial_i B & ((1-2\psi)\,\delta_{ij} + D_{ij}E) \end{pmatrix} \tag{453}$$

12.1. Inverse of $\delta g_{\mu\nu}$

Let the inverse of $\delta g_{\mu\nu}$ be $\delta g^{\mu\nu}$ and we write

$$\begin{aligned} \delta g^{\mu\nu} &= \tfrac{1}{a^2(t)}\begin{pmatrix} \delta g^{00} & \delta g^{0i} \\ \delta g^{i0} & \delta g^{ij} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{a^2(t)}(-1+X) & \frac{1}{a^2(t)}\partial^i Y \\ \frac{1}{a^2(t)}\partial^i Y & \frac{1}{a^2(t)}\left((1+2Z)\,\delta^{ij} + D^{ij}K\right) \end{pmatrix} \end{aligned} \tag{454}$$

We find the inverse of Eq. (454) that is $g^{\mu\nu}$, so that we have

$$\begin{aligned} & g^{\mu\zeta} g_{\zeta\nu} = g_0^{\mu\zeta} g_{\zeta\nu}^0 = \delta_\nu^\mu \\ & \left(g_0^{\mu\zeta} + \delta g^{\mu\zeta}\right)\left(g_{\zeta\nu}^0 + \delta g_{\zeta\nu}\right) = \delta_\nu^\mu \\ & g_0^{\mu\zeta} g_{\zeta\nu}^0 + g_0^{\mu\zeta}\delta g_{\zeta\nu} + \delta g^{\mu\zeta} g_{\zeta\nu}^0 + \delta g^{\mu\zeta}\delta g_{\zeta\nu} = \delta_\nu^\mu \end{aligned} \tag{455}$$

where δ_ν^μ is the Kronecker delta function and is defined as

$$\delta_\nu^\mu = \begin{cases} 1, & \text{if } \mu = \nu \\ 0, & \text{if } \mu \neq \nu \end{cases} \tag{456}$$

and $g_{\zeta\nu}$ is the simply unperturbed FLRW line element described as

$$ds^2 = a^2(t)\left[-dt^2 + \delta_{ij} dx^i dx^j\right] \tag{457}$$

$$g_{\mu\nu}^0 = g_{\mu\nu} = a^2(t)\begin{pmatrix} g_{00} & g_{0i} \\ g_{i0} & g_{ij} \end{pmatrix} = a^2(t)\begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} \end{pmatrix} \tag{458}$$

The inverse of $g^0_{\mu\nu} = g_{\mu\nu}$ is simply $g_0^{\mu\nu} = g^{\mu\nu}$ since for diagonal unperturbed metric $g_{\mu\nu} = \frac{1}{g_{\mu\nu}}$, so that we can write

$$g_0^{\mu\nu} = g^{\mu\nu} = \frac{1}{a^2(t)}\begin{pmatrix} g^{00} & g^{0i} \\ g^{i0} & g^{ij} \end{pmatrix} = \begin{pmatrix} \frac{1}{a^2(t)} & 0 \\ 0 & \frac{1}{a^2(t)}\delta_{ij} \end{pmatrix} \tag{459}$$

For $\mu = 0, \nu = 0$ in Eq. (455), we have

$$\begin{aligned} &\left(g_0^{00} g_{00}^{(0)} + g_0^{0i} g_{i0}^{(0)}\right) + \left(g_0^{00}\delta g_{00} + g_0^{0i}\delta g_{i0}\right) + \left(\delta g^{00} g_{00}^{(0)} + \delta g^{0i} g_{i0}^{(0)}\right) \\ &+ \left(\delta g^{00}\delta g_{00} + \delta g^{0i}\delta g_{0i}\right) = \delta_0^0 \end{aligned} \tag{460}$$

which give

$$\delta g^{00}\delta g_{00} + \delta g^{0i}\delta g_{i0} = 1 \tag{461}$$

substituting the values, we have

$$-\frac{1}{a^2(t)}\left(-1 + X\right) a^2(t)\left(-1 - 2A\right) + \frac{1}{a^2(t)}\partial^i Y\left(a^2(t)\partial_i B\right) = 1 \tag{462}$$

Simplifying and neglecting second order product terms $-2AX$ and $\partial^i Y \cdot \partial_i B$, we get

$$X = 2A \tag{463}$$

Again from Eq. (455) for $\mu = 0, \nu = i$, we have after simplification

$$\begin{aligned} &\left(g_0^{00} g_{0i}^{(0)} + g_0^{0i} g_{ji}^{(0)}\right) + \left(g_0^{00}\delta g_{0i} + g_0^{0i}\delta g_{ji}\right) + \left(\delta g^{00} g_{0i}^{(0)} + \delta g^{0i} g_{ji}^{(0)}\right) \\ &+ \left(\delta g^{00}\delta g_{0i} + \delta g^{0i}\delta g_{ji}\right) = \delta_i^0 \end{aligned} \tag{464}$$

$$\delta g^{00}\delta g_{0i} + \delta g^{0j}\delta g_{ji} = 0 \tag{465}$$

Substituting the values

$$\frac{1}{a^2(t)}\left(-1 + X\right) a^2(t)\partial_i B + \frac{1}{a^2(t)}\partial^j Y\left(a^2(t)\left(1 - 2\psi\right)\delta_{ji} + D_{ji}E\right) = 0 \tag{466}$$

neglecting the higher product terms $2A \cdot \partial_i B$, $\partial_i Y \cdot 2\psi$ and $\partial^j D_{ji} Y \cdot E$, we have

$$-\partial_i B + \partial_i Y = 0 \tag{467}$$

integrating we get

$$Y = B \tag{468}$$

Now from Eq. (455) for $\mu = i, \nu = j$, we have

$$\begin{aligned} &\left(g_0^{i0} g_{0j}^{(0)} + g_0^{ij} g_{ij}^{(0)}\right) + \left(g_0^{i0}\delta g_{0j}^{(0)} + g_0^{ij}\delta g_{ij}^{(0)}\right) + \left(\delta g^{i0} g_{0j}^{(0)} + \delta g^{ij} g_{ij}^{(0)}\right) \\ &+ \left(\delta g^{i0}\delta g_{0j} + \delta g^{ij}\delta g_{ij}\right) = \delta_j^i \end{aligned} \tag{469}$$

The non-vanishing terms are

$$\delta g^{i0}\delta g_{0j} + \delta g^{ik}\delta g_{kj} = \delta_j^i \tag{470}$$

Substituting values suitable change of indices

$$\begin{aligned} &\tfrac{1}{a^2(t)}\left(\partial^i Y\right) a^2(t)\partial_i B + \tfrac{1}{a^2(t)}\left(\left(1 + 2z\right)\delta^{ik} + D^{ik}E\right) \\ &\cdot a^2(t)\left(\left(1 - 2\psi\right)\delta_{kj} + D_{kj}E\right) = \delta_j^i \end{aligned} \tag{471}$$

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using properties $\delta^{ik}\delta_{kj} = \delta^i_j$, $\delta^{ik}D_{kj} = D^i_j$, $\delta_{kj}D^{ik} = D^i_k$ and neglecting the higher order product terms $\partial^i B \cdot \partial_j B$, $-4Z\psi$, $2ZD^i_jE$, $-2\psi \cdot D^i_jK$ and D^i_jKE, we have

$$(1 - 2\psi + 2Z)\,\delta^i_j + (E + K)\,D^i_j = \delta^i_j + 0D^i_j \tag{472}$$

Comparing the coefficients of δ^i_j and D^i_j, we get $Z = \psi$ and $K = -E$, so that inverse metric of the perturbed line element becomes

$$\begin{aligned}\delta g^{\mu\nu} &= \frac{1}{a^2(t)}\begin{pmatrix}\delta g^{00} & \delta g^{0i}\\ \delta g^{i0} & \delta g^{ij}\end{pmatrix}\\ &= \begin{pmatrix}\frac{1}{a^2(t)}(-1+2A) & \frac{1}{a^2(t)}\partial^i B\\ \frac{1}{a^2(t)}\partial^i B & \frac{1}{a^2(t)}\left((1+2\psi)\,\delta^{ij} - D^{ij}E\right)\end{pmatrix}\end{aligned} \tag{473}$$

12.2. The Unperturbed Line Element

The unperturbed line element is given by

$$ds^2 = a^2(t)\left[-dt^2 + \delta_{ij}dx^i dx^j\right] \tag{474}$$

or

$$g^0_{\mu\nu} = g_{\mu\nu} = a^2(t)\begin{pmatrix}g_{00} & g_{0i}\\ g_{i0} & g_{ij}\end{pmatrix} = a^2(t)\begin{pmatrix}-1 & 0\\ 0 & \delta_{ij}\end{pmatrix} \tag{475}$$

The inverse of $g^0_{\mu\nu} = g_{\mu\nu}$ is simply $g^{\mu\nu}_0 = g^{\mu\nu}$ since for diagonal unperturbed metric $g_{\mu\nu} = \frac{1}{g_{\mu\nu}}$, so that we can write

$$g^{\mu\nu}_0 = g^{\mu\nu} = \frac{1}{a^2(t)}\begin{pmatrix}g^{00} & g^{0i}\\ g^{i0} & g^{ij}\end{pmatrix} = \begin{pmatrix}\frac{1}{a^2(t)} & 0\\ 0 & \frac{1}{a^2(t)}\delta_{ij}\end{pmatrix} \tag{476}$$

We calculate now affine connections-the 2nd kind of Christoffel symbols

$$\Gamma^\sigma_{\mu\nu} = g^{\sigma\lambda}\Gamma_{\mu\nu\lambda} = \frac{1}{2}g^{\sigma\lambda}\left(g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu} + g_{\mu\nu,\lambda}\right) \tag{477}$$

We can compute the following possible components

$$\begin{matrix}\Gamma^0_{00} & \Gamma^i_{ii} & \Gamma^j_{jj} & \Gamma^i_{00}\\ \Gamma^j_{00} & \Gamma^i_{0j} & \Gamma^j_{i0} & \Gamma^0_{ij}\\ \Gamma^0_{0i} & \Gamma^0_{0j} & \Gamma^i_{jk} & \Gamma^j_{ik}\\ \Gamma^k_{ij} & & & \end{matrix} \tag{478}$$

For $\sigma = \mu = \nu = 0$, we have

$$\Gamma^0_{00} = -\frac{1}{2a^2(t)}\partial_{,0}g_{00} = \frac{\dot{a}}{a} \tag{479}$$

For $\sigma = i$, $\mu = 0$, $\nu = j$, we have

$$\Gamma^i_{0j} = \frac{1}{2a^2(t)}\delta^{ik}\partial_{,0}\left(a^2(t)\delta_{jk}\right) = \frac{\dot{a}}{a}\delta^i_j \tag{480}$$

For $\sigma = 0$, $\mu = i$, $\nu = j$, we have

$$\Gamma^0_{ij} = \frac{1}{2a^2(t)}\partial_{,0}\left(a^2(t)\delta_{ij}\right) = \frac{\dot{a}}{a}\delta_{ij} \tag{481}$$

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For $\sigma = i$, $\mu = 0$, $\nu = 0$, we have

$$\Gamma^i_{00} = 0 \tag{482}$$

and

For $\sigma = 0$, $\mu = 0$, $\nu = i$, we have

$$\Gamma^0_{0i} = 0 \tag{483}$$

and for $\sigma = i$, $\mu = j$, $\nu = k$, we have

$$\Gamma^i_{jk} = 0 \tag{484}$$

Now we can calculate perturbed element when we have necessary components of perturbed and unperturbed metric and its inverse metric tensor components,

$$\begin{aligned}&\delta\Gamma^\sigma_{\mu\nu} = \delta\left(g^{\sigma\lambda}\Gamma_{\mu\nu\lambda}\right) = \tfrac{1}{2}\delta\left(g^{\sigma\lambda}\left(g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu} + g_{\mu\nu,\lambda}\right)\right)\\ &= \tfrac{1}{2}\delta g^{\sigma\lambda}\left(g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu} - g_{\mu\nu,\lambda}\right)\\ &+\tfrac{1}{2}g^{\sigma\lambda}\left(\left(\delta g_{\mu\lambda}\right)_{,\nu} + \left(\delta g_{\nu\lambda}\right)_{,\mu} + \left(\delta g_{\mu\nu}\right)_{,\lambda}\right)\end{aligned} \tag{485}$$

We are on our stake now to calculate the following components

$$\begin{array}{ccc} \delta\Gamma^0_{00} & \delta\Gamma^i_{ii} & \delta\Gamma^j_{jj} \\ \delta\Gamma^i_{00} & \delta\Gamma^j_{00} & \delta\Gamma^i_{0j} \\ \delta\Gamma^j_{i0} & \delta\Gamma^0_{ij} & \delta\Gamma^0_{0i} \\ \delta\Gamma^0_{0j} & \delta\Gamma^i_{jk} & \delta\Gamma^j_{ik} \\ \delta\Gamma^k_{ij} & & \end{array} \tag{486}$$

The non-vanishing components are For $\sigma = \mu = \nu = 0$, we have

$$\delta\Gamma^0_{00} = \dot{A} \tag{487}$$

For $\sigma = i$, $\mu = 0$, $\nu = j$, we have

$$\delta\Gamma^i_{0j} = -\dot{\psi}\delta^i_j + \frac{1}{2}D_{ij}\dot{E} \tag{488}$$

For $\sigma = 0$, $\mu = i$, $\nu = j$, we have

$$\delta\Gamma^0_{ij} = -2\frac{\dot{a}}{a}A\delta_{ij} - \partial_i\partial_j B - 2\frac{\dot{a}}{a}\psi\delta_{ij} - \psi'\delta_{ij} - \frac{\dot{a}}{a}D_{ij}E + \frac{1}{2}D_{ij}\dot{E} \tag{489}$$

For $\sigma = i$, $\mu = 0$, $\nu = 0$, we have

$$\delta\Gamma^i_{00} = \frac{\dot{a}}{a}\partial^i B + \partial^i\dot{B} + \partial^i A \tag{490}$$

and For $\sigma = 0$, $\mu = 0$, $\nu = i$, we have

$$\delta\Gamma^0_{0i} = \partial_i A + \frac{\dot{a}}{a}\partial_i B \tag{491}$$

and for $\sigma = i$, $\mu = j$, $\nu = k$, we have

$$\begin{aligned}&\delta\Gamma^i_{jk} = -\partial_j\psi\delta^i_k - \partial_k\psi\delta^i_j + \partial^i\psi\delta_{jk} - \tfrac{\dot{a}}{a}\partial^i B\delta_{jk} + \tfrac{1}{2}\partial_j D^i_k E\\ &+\tfrac{1}{2}\partial_k D^i_j E - +\tfrac{1}{2}\partial^i D_{jk}E\end{aligned} \tag{492}$$

Now unperturbed Ricci tensor is given

$$R_{\mu\nu} = g_{\lambda\sigma}R^\sigma_{\mu\nu\lambda} = \partial_{,\nu}\Gamma^\sigma_{\mu\lambda} - \partial_{,\lambda}\Gamma^\sigma_{\mu\nu} + \Gamma^n_{\mu\lambda}\Gamma^\sigma_{n\nu} - \Gamma^n_{\mu\nu}\Gamma^\sigma_{n\lambda} \tag{493}$$

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For $\mu = 0, \nu = 0$, we have

$$R_{00} = \partial_{,\sigma}\Gamma^{\sigma}_{00} - \partial_{,0}\Gamma^{\sigma}_{0\sigma} + \Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} - \Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} = 0 \tag{494}$$

For $\mu = 0, \nu = i$, we have

$$R_{0i} = \partial_{,\sigma}\Gamma^{\sigma}_{00} - \partial_{,0}\Gamma^{\sigma}_{0\sigma} + \Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} - \Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} = 0 \tag{495}$$

For $\mu = i, \nu = j$, we have

$$R_{ij} = \partial_{,\sigma}\Gamma^{\sigma}_{00} - \partial_{,0}\Gamma^{\sigma}_{0\sigma} + \Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} - \Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} = 0 \tag{496}$$

Now we calculate the perturbed Ricci tensor components

$$\begin{aligned}\delta R_{\mu\nu} &= \partial_{,\nu}\delta\Gamma^{\sigma}_{\mu\lambda} - \partial_{,\lambda}\delta\Gamma^{\sigma}_{\mu\nu} + \delta\Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} + \Gamma^{n}_{\mu\lambda}\delta\Gamma^{\sigma}_{n\nu} - \delta\Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} \\ &-\Gamma^{n}_{\mu\nu}\delta\Gamma^{\sigma}_{n\lambda}\end{aligned} \tag{497}$$

For $\mu = 0, \nu = 0$, we have

$$\begin{aligned}\delta R_{00} &= \partial_{,\nu}\delta\Gamma^{\sigma}_{\mu\lambda} - \partial_{,\lambda}\delta\Gamma^{\sigma}_{\mu\nu} + \delta\Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} + \Gamma^{n}_{\mu\lambda}\delta\Gamma^{\sigma}_{n\nu} - \delta\Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} \\ &-\Gamma^{n}_{\mu\nu}\delta\Gamma^{\sigma}_{n\lambda} = \tfrac{\dot{a}}{a}\partial_i\partial^i B + \partial_i\partial^i B' + \partial_i\partial^i A + 3\psi'' + 3\tfrac{\dot{a}}{a}\psi' + 3\tfrac{\dot{a}}{a}A'\end{aligned} \tag{498}$$

For $\mu = 0, \nu = i$, we have

$$\begin{aligned}\delta R_{0i} &= \partial_{,\nu}\delta\Gamma^{\sigma}_{\mu\lambda} - \partial_{,\lambda}\delta\Gamma^{\sigma}_{\mu\nu} + \delta\Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} + \Gamma^{n}_{\mu\lambda}\delta\Gamma^{\sigma}_{n\nu} - \delta\Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} \\ &-\Gamma^{n}_{\mu\nu}\delta\Gamma^{\sigma}_{n\lambda} = \tfrac{\ddot{a}}{a}\partial_i B + \left(\tfrac{\dot{a}}{a}\right)^2\partial_i B + 2\partial_i\psi' + 2\tfrac{\dot{a}}{a}\partial_i A + \tfrac{1}{2}\partial_k D^k_i E'\end{aligned} \tag{499}$$

For $\mu = i, \nu = j$, we have

$$\begin{aligned}&\delta R_{ij} = \partial_{,\nu}\delta\Gamma^{\sigma}_{\mu\lambda} - \partial_{,\lambda}\delta\Gamma^{\sigma}_{\mu\nu} + \delta\Gamma^{n}_{\mu\lambda}\Gamma^{\sigma}_{n\nu} + \Gamma^{n}_{\mu\lambda}\delta\Gamma^{\sigma}_{n\nu} - \delta\Gamma^{n}_{\mu\nu}\Gamma^{\sigma}_{n\lambda} - \Gamma^{n}_{\mu\nu}\delta\Gamma^{\sigma}_{n\lambda} \\ &= \begin{pmatrix} -\tfrac{\dot{a}}{a}\dot{A} - 5\tfrac{\dot{a}}{a}\dot{\psi} - 2\tfrac{\ddot{a}}{a}A - 2\left(\tfrac{\dot{a}}{a}\right)^2 A - 2\tfrac{\ddot{a}}{a}\psi - 2\left(\tfrac{\dot{a}}{a}\right)^2\psi - \ddot{\psi} \\ +\partial_k\partial^k\psi - \tfrac{\dot{a}}{a}\partial_k\partial^k B \end{pmatrix}\delta_{ij} \\ &-\partial_i\partial_j\dot{B} + \tfrac{\dot{a}}{a}D_{ij}\dot{E} + \tfrac{\ddot{a}}{a}D_{ij}E + \left(\tfrac{\dot{a}}{a}\right)^2 D_{ij}E + \tfrac{1}{2}D_{ij}\ddot{E} + \partial_i\partial_j\psi - \partial_i\partial_j A \\ &-2\tfrac{\dot{a}}{a}\partial_i\partial_j B + \tfrac{1}{2}\partial_k\partial_i D^k_j E + \tfrac{1}{2}\partial_k\partial_j D^k_i E - \tfrac{1}{2}\partial_k\partial^k D_{ij}E\end{aligned} \tag{500}$$

Unperturbed Ricci scalar is obtained by contracting unperturbed Ricci tensor

$$R = g^{\mu\nu}R_{\mu\nu} \tag{501}$$

Using double sum and simplifying, we have

$$R = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33} = 6\frac{\ddot{a}}{a^3} \tag{502}$$

and perturbed Ricci scalar, using double sum and simplifying, we have

$$\begin{aligned}&\delta R = \delta g^{\mu\nu}R_{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu} = \delta g^{00}R_{00} + \delta g^{11}R_{11} + \delta g^{22}R_{22} \\ &+\delta g^{33}R_{33} + g^{00}\delta R_{00} + g^{11}\delta R_{11} + g^{22}\delta R_{22} + g^{33}\delta R_{33} \\ &= -6\tfrac{\dot{a}}{a^3}\partial_i\partial^i B - \tfrac{2}{a^2}\partial_i\partial^i B - \tfrac{2}{a^2}\partial_i\partial^i A - \tfrac{6}{a^2}\ddot{\psi} - 6\tfrac{\dot{a}}{a^3}\dot{A} - 18\tfrac{\dot{a}}{a^3}\dot{\psi} \\ &-12\tfrac{\ddot{a}}{a^3}A + \tfrac{4}{a^2}\partial_i\partial^i\psi + \tfrac{1}{a^2}\partial_k\partial^i D^k_i E\end{aligned} \tag{503}$$

now unperturbed Einstein tensor is

$$G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R \tag{504}$$

For $\mu = 0, \nu = 0$, we have

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$$G_{00} = R_{00} + \frac{1}{2} g_{00} R = 3\left(\frac{\dot{a}}{a}\right)^2 \tag{505}$$

For $\mu = 0, \nu = i$, we have

$$G_{0i} = R_{0i} + \frac{1}{2} g_{0i} R = 0 \tag{506}$$

For $\mu = i, \nu = j$, we have

$$G_{ij} = R_{ij} + \frac{1}{2} g_{ij} R = \left(-2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) \delta_{ij} \tag{507}$$

Now perturbed Einstein tensor at first order of perturbation is

$$\delta G_{\mu\nu} = \delta R_{\mu\nu} - \frac{1}{2}\left(\delta g_{\mu\nu} R + g_{\mu\nu} \delta R\right) \tag{508}$$

For $\mu = 0, \nu = 0$, we have

$$\begin{aligned} \delta G_{00} = \delta R_{00} - \tfrac{1}{2}\left(\delta g_{00} R + g_{00}\delta R\right) = -2\tfrac{\dot{a}}{a}\partial_i\partial^i B - 6\tfrac{\dot{a}}{a}\dot{\psi} \\ +2\partial_i\partial^i\psi + \tfrac{1}{2}\partial_k\partial^i D_i^k E \end{aligned} \tag{509}$$

For $\mu = 0, \nu = i$, we have

$$\begin{aligned} \delta G_{0i} = \delta R_{0i} - \tfrac{1}{2}\left(\delta g_{0i} R + g_{0i}\delta R\right) = -2\tfrac{\ddot{a}}{a}\partial_i B + \left(\tfrac{\dot{a}}{a}\right)^2 \partial_i B \\ +2\partial_i\dot{\psi} + \tfrac{1}{2}\partial_k D_i^k \dot{E} \\ +2\tfrac{\dot{a}}{a}\partial_i A \end{aligned} \tag{510}$$

For $\mu = i, \nu = j$, we have

$$\begin{aligned} &\delta G_{ij} = \delta R_{ij} - \tfrac{1}{2}\left(\delta g_{ij} R + g_{ij}\delta R\right) \\ &= \begin{pmatrix} 2\tfrac{\dot{a}}{a}\dot{A} + 4\tfrac{\dot{a}}{a}\dot{\psi} + 4\tfrac{\ddot{a}}{a}A - 2\left(\tfrac{\dot{a}}{a}\right)^2 A + 4\tfrac{\ddot{a}}{a}\psi - 2\left(\tfrac{\dot{a}}{a}\right)^2\psi + 2\ddot{\psi} \\ -\partial_k\partial^k\psi + 2\tfrac{\dot{a}}{a}\partial_k\partial^k B + \partial_k\partial^k\dot{B} + \partial_k\partial^k A + \tfrac{1}{2}\partial_k\partial^\rho D_\rho^k E \end{pmatrix} \delta_{ij} \\ &-\partial_i\partial_j\dot{B} + \tfrac{\dot{a}}{a}D_{ij}\dot{E} - 2\tfrac{\ddot{a}}{a}D_{ij}E - \partial_i\partial_j A + \partial_i\partial_j\psi + \left(\tfrac{\dot{a}}{a}\right)^2 D_{ij}E \\ &+\tfrac{1}{2}D_{ij}\ddot{E} - 2\tfrac{\dot{a}}{a}\partial_i\partial_j B + \tfrac{1}{2}\partial_k\partial_i D_j^k E + \tfrac{1}{2}\partial^k\partial_j D_{ik}E - \tfrac{1}{2}\partial_k\partial^k D_{ij}E \end{aligned} \tag{511}$$

Now unperturbed stress energy tensor is

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi - V(\phi)\right) \tag{512}$$

For $\mu = 0, \nu = 0$, we have

$$T_{00} = \partial_0\phi\partial_0\phi - g_{00}\left(\frac{1}{2}g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi - V(\phi)\right) = \frac{1}{2}\dot{\phi}^2 + a^2(t)V(\phi) \tag{513}$$

For $\mu = 0, \nu = i$, we have

$$T_{0i} = \partial_0\phi\partial_i\phi - g_{0i}\left(\frac{1}{2}g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi - V(\phi)\right) = 0 \tag{514}$$

For $\mu = i, \nu = j$, we have

$$T_{ij} = \partial_i\phi\partial_j\phi - g_{ij}\left(\frac{1}{2}g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi - V(\phi)\right) = \left(\frac{1}{2}\dot{\phi}^2 - a^2(t)V(\phi)\right)\delta_{ij} \tag{515}$$

Now perturbed stress energy tensor at first order of perturbation is

$$\delta T_{\mu\nu} = \begin{array}{l} \partial_\mu (\delta\phi)\,\partial_\nu\phi + \partial_\mu\phi\partial_\nu (\delta\phi) - \delta g_{\mu\nu}\left(\frac{1}{2}g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi + V(\phi)\right) \\ -g_{\mu\nu}\left(\begin{array}{l} \frac{1}{2}\delta g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi + g^{\sigma\rho}\partial_\sigma(\delta\phi)\,\partial_\rho\phi + g^{\sigma\rho}\partial_\sigma\phi\partial_\rho(\delta\phi) \\ +\partial_\phi\delta V(\phi) + \partial_\phi V(\phi)\,\delta\phi \end{array}\right) \end{array} \tag{516}$$

For $\mu = 0, \nu = 0$, we have

$$\delta T_{00} = \dot{\phi}\delta\dot{\phi} + 2a^2(t)AV(\phi) + \delta\phi a^2(t)V_\phi(\phi) \tag{517}$$

For $\mu = 0, \nu = i$, we have

$$\delta T_{0i} = \dot{\phi}\partial_i(\delta\phi) + \frac{\dot{\phi}^2}{2}\partial_i B - a^2(t)V(\phi)\,\partial_i B \tag{518}$$

For $\mu = i, \nu = j$, we have

$$\begin{array}{l} \delta T_{ij} = \left(\dot{\phi}\delta\dot{\phi} - \dot{\phi}^2 A - a^2(t)V_\phi(\phi)\delta\phi - \dot{\phi}^2\psi + 2a^2(t)V(\phi)\psi\right)\delta_{ij} \\ -a^2(t)V(\phi)D_{ij}E + \frac{1}{2}\dot{\phi}^2 D_{ij}E \end{array} \tag{519}$$

Therefore, The perturbed Einstein field equations are

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu} \tag{520}$$

Comparing the corresponding components, we have

$$\begin{array}{l} -2\frac{\dot{a}}{a}\partial_i\partial^i B - 6\frac{\dot{a}}{a}\dot{\psi} + 2\partial_i\partial^i\psi + \frac{1}{2}\partial_k\partial^i D_i^k E \\ = 8\pi\dot{\phi}\delta\dot{\phi} + 2a^2(t)AV(\phi) + \delta\phi a^2(t)V_\phi(\phi) \end{array} \tag{521}$$

$$\begin{array}{l} -2\frac{\ddot{a}}{a}\partial_i B + \left(\frac{\dot{a}}{a}\right)^2\partial_i B + 2\partial_i\dot{\psi} + \frac{1}{2}\partial_k D_i^k\dot{E} + 2\frac{\dot{a}}{a}\partial_i A \\ = 8\pi\left(\dot{\phi}\partial_i(\delta\phi) + \frac{\dot{\phi}^2}{2}\partial_i B - a^2(t)V(\phi)\,\partial_i B\right) \end{array} \tag{522}$$

$$\begin{array}{l} \left(\begin{array}{l} 2\frac{\dot{a}}{a}\dot{A} + 4\frac{\dot{a}}{a}\dot{\psi} + 4\frac{\ddot{a}}{a}A - 2\left(\frac{\dot{a}}{a}\right)^2 A + 4\frac{\ddot{a}}{a}\psi - 2\left(\frac{\dot{a}}{a}\right)^2\psi + 2\ddot{\psi} \\ -\partial_k\partial^k\psi + 2\frac{\dot{a}}{a}\partial_k\partial^k B + \partial_k\partial^k\dot{B} + \partial_k\partial^k A + \frac{1}{2}\partial_k\partial^\rho D_\rho^k E \end{array}\right)\delta_{ij} \\ -\partial_i\partial_j\dot{B} + \frac{\dot{a}}{a}D_{ij}\dot{E} - 2\frac{\ddot{a}}{a}D_{ij}E - \partial_i\partial_j A + \partial_i\partial_j\psi + \left(\frac{\dot{a}}{a}\right)^2 D_{ij}E \\ +\frac{1}{2}D_{ij}\ddot{E} - 2\frac{\dot{a}}{a}\partial_i\partial_j B + \frac{1}{2}\partial_k\partial_i D_j^k E + \frac{1}{2}\partial^k\partial_j D_{ik}E - \frac{1}{2}\partial_k\partial^k D_{ij}E \\ = 8\pi\left(\begin{array}{l} \left(\dot{\phi}\delta\dot{\phi} - \dot{\phi}^2 A - a^2(t)V_\phi(\phi)\delta\phi - \dot{\phi}^2\psi + 2a^2(t)V(\phi)\psi\right)\delta_{ij} \\ -a^2(t)V(\phi)D_{ij}E \end{array}\right) \\ +\frac{1}{2}\dot{\phi}^2 D_{ij}E \end{array} \tag{523}$$

perturbed equations can also be determined in mixed tensor form. Expressing Eq.(520) in mixed form and working out on the same lines as we did earlier

$$\delta G_\mu^\nu = 8\pi\delta T_\mu^\nu \tag{524}$$

13. Summary

Relativistic cosmology was founded on the general theory of relativity with introducing cosmological principle and Weyl's principle implicitly implied. In the beginning, Einstein and de Sitter cosmological models were presented, though now of historical interest, yet they both are very significant as the first initiates the modern cosmology relativistically and scientifically and the latter, later on, was used to provide the initial conditions of the big bang model with a slight change. The first theoretical models for the possibility of dynamic universe evolved beginning with Friedmann, Lemaitre and were observationally determined by E. Hubble. In 1929, E. Hubble found exactly the same expanding universe that Friedmann did theoretically in 1922. Therefore it was Friedmann who championed the cause of dynamical universes, however his work was recognized later when he was no more in the world. The theory of big bang based on the standard cosmological model faces Horizon, Flatness, and Entropy problems etc. To resolve these problems a phase of exponentially expanding universe was introduced in its very early history which occurred in a very small fraction of time (about $\frac{1}{10^{43}}$s of the very 1st second after time creation) known as inflation. The inflation is identified as the initial conditions under which the big bang might have taken place. The introduction of inflation caused the name inflationary cosmology and it is about fourty years since its birth to date. The inflationary paradigm stands now on firm observational footing and is accepted irrevocably in cosmology as the viable description for the early universe. Starobinsky, Guth, and Linde are credited with setting the foundations of inflationary cosmology. The inflationary cosmology is being hailed as successful in explaining the origin of structure formation through cosmological quantum fluctuations as relicts of cosmic inflation. The observations conducted on microwave background radian and the recent discoveries of gravitational waves and black holes lend the confirmatory support to the underlying principles of the inflationary cosmology .

14. Conflicts of interest/Competing interests

The authors declare that they have no conflicts of interest/Competing interests concerning with this manuscript.

15. Author Contributions

All of the authors shared their contribution to the design and structure of the study concept of the manuscript. The 2nd author presented the idea of the article and first author did search for the literature, analysed the data, drafted and revised the data judgmentally and third author contributed in conclusion section with financial support.

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Appendix A Space, Time and Spacetime

A background arena of space and time is necessarily required for all the physical phenomena to play over it and the compatibility of the known physical laws is made with structure of space and time. Space, time and motion are concomitant ingredients cohered to matter and can never be disengaged from each other. universe exists in space and evolves in time so that universe, space and time are in separable from each other and are coherently related to each other. Space is understood as possessing three dimensions and time is speculated to have only one dimension. Therefore Newtonian Mechanics has been formulated in such a way to consider the spatial dimensions existing independently from the only one dimension of time. The Euclidean geometry provides necessary mechanism in dealing with such notions of space and time. In this regard Euclidean space becomes important which proposes three independent perpendicular dimensions of space and the dimension of time does not get affected by it. space and time are envisaged as independent absolute entities which are not affected by each other. The Euclidean structure of space is flat and distances are measured by using the standard Pythagoras theorem for three dimensions as

$$ds^2 = x^2 + y^2 + z^2, \tag{A1}$$

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or in differential of the distances

$$ds^2 = dx^2 + dy^2 + dz^2, \tag{A2}$$

where $ds = (x, y, z)$ or $ds = (dx, dy, dz)$ respectively. The time coordinate does appear anywhere in this distance-measuring formula which means in the geometry of space, the dimension of time will be dealt separately. Newton's notions of space and time as described in Principia Mathematica are given as "Absolute space, in its own nature, without regard to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces which our senses determine by its position to bodies: and which is vulgarly taken for immovable space. Absolute motion is the translation of a body from one absolute place into another: and relative motion, the translation from one relative place into another" and absolute time is defined in these words "Absolute, true and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration. Relative, apparent and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time"

In 1905, Einstein's paper entitled "On the electrodynamics of moving bodies" put forth on the base of two postulates that time might be dealt on equal footing with space as one of the dimensions of space. Minkowski (1864-1909), translated the mixing of space and time coordinates as requiring a four dimensional scenario where physical phenomena take place and the geometry of such four dimensional spacetime, where time is one dimension, is described by spacetime interval which is the generalized form of Pythagoras theorem

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{A3}$$

or

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{A4}$$

where

$$\eta_{\mu\nu} = \begin{pmatrix} \eta_{00} & \eta_{01} & \eta_{02} & \eta_{03} \\ \eta_{10} & \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{20} & \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{30} & \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{A5}$$

Minkowski first understood that the spacetime interval given in Eq. (A3) remains invariant for all the observers and carries the similar meaning for all the observers in uniform relative motion, however, Einstein considered either with respect time or space the interval does not remain identical for all relative observers in uniform motion. Minkowski avowedly said in a conference addressing to the German scientists that "Ladies and gentlemen! the views of space and time which i wish to lay before you have sprung from the soil of experimental physics, therein lies their strength, they are radical. Henceforth space by itself and time by itself are doomed to fade away into mere shadows and only a union of the two will preserve an independent reality" [34]. general relativity was formulated on the base of four dimensional spacetime as Minkowski has laid it but in order to incorporate the gravity into it Einstein utilized the power of tensors and modeled the curved geometry of spacetime describing its curvature as gravity. The geometry of curved spacetime is encoded into a two rank symetric tensor known as fundamental tensor and given as the spacetime metric or line element as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{A6}$$

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Where $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & 0 \\ 0 & 0 & 0 & g_{33} \end{pmatrix} \tag{A7}$$

In the absence of matter, the curvature of spacetime vanishes and the geometry of spacetime becomes flat *i.e.* $g_{\mu\nu} = \mu_{\mu\nu}$, yet non-Euclidean that is required by the special relativity.

Appendix B Maximally Symmetric 3-Space (Spherically Symmetric Space)

In order to have a space more symmetrical we require comparatively lesser number of functions as much as possible to determine its properties. It is the curvature of a space and its nature that determines how much the space is symmetric maximally. If the curvature K of a space under consideration does not depend upon the coordinates of the points constituting it and has a constant value, then the space shall be maximally symmetric and the spaces possessing the curvature of this kind logically entail cosmological principle *i.e.* homogeneity and isotropy. Spacelike coordinates $\left(x^1, x^2, x^3\right)$ obviously span 3-space which we require to be maximally symmetric. The Riemann curvature tensor $R^{\sigma}_{\mu\nu\rho}$ in three dimensional space has $3^4 = 81$ components which depend on the coordinates. From these only six components are independent and are the functions of coordinates and require six functions to be described in order to specify intrinsically the geometric properties of the three dimensional space. The Riemann curvature tensor $R^{\sigma}_{\mu\nu\rho}$ depends on curvature K and the metric tensor $g_{\mu\nu}$ for the maximally symmetric spaces which is the simplest form for it to adopt. It is given by

$$R_{\mu\nu\zeta\pi} = K\left(g_{\mu\zeta}g_{\nu\pi} - g_{\mu\pi}g_{\nu\zeta}\right) \tag{A8}$$

$$g^{\mu\pi}R_{\mu\nu\zeta\pi} = Kg^{\mu\pi}\left(g_{\mu\zeta}g_{\nu\pi} - g_{\mu\pi}g_{\nu\zeta}\right) = K\left(g^{\mu\pi}g_{\mu\zeta}g_{\nu\pi} - g^{\mu\pi}g_{\mu\pi}g_{\nu\zeta}\right) \tag{A9}$$

$$R_{\nu\zeta} = K\left(\delta^{\pi}_{\zeta}g_{\nu\pi} - \delta^{\zeta}_{\zeta}g_{\nu\zeta}\right) = K\left[g_{\nu\zeta} - \left(\delta^1_1 + \delta^2_2 + \delta^3_3\right)g_{\nu\zeta}\right] \tag{A10}$$

$$R_{\nu\zeta} = K\left[g_{\nu\zeta} - 3g_{\nu\zeta}\right] = K\left(-2g_{\nu\zeta}\right) \tag{A11}$$

Then, Ricci scalar or curvature scalar from above Eq. (A11) can be had by contraction with inverse metric tensor $g^{\nu\zeta}$

$$g^{\nu\zeta}R_{\nu\zeta} = -2g^{\nu\zeta}g_{\nu\zeta}K \tag{A12}$$

$$R = -2\delta^{\zeta}_{\zeta}K = -2\left(\delta^1_1 + \delta^2_2 + \delta^3_3\right)K = -2\left(1+1+1\right)K = -6K \tag{A13}$$

The metric of an isotropic 3-space must depend on rotational invariants given by

$$\begin{aligned} &\vec{x}\vec{x} = r^2 \\ &d\vec{x}d\vec{x},\ \vec{x}d\vec{x} \end{aligned} \tag{A14}$$

and in spherical polar coordinates (r, θ, ϕ), it should take the form

$$d\sigma^2 = C\left(r\right)\left(\vec{x}d\vec{x}\right)^2 + D\left(r\right)\left(d\vec{x}d\vec{x}\right)^2 \tag{A15}$$

$$d\sigma^2 = C\left(r\right)r^2dr^2 + D\left(r\right)\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\theta^2\right) \tag{A16}$$

Redefining the radial coordinate $\bar{r}^2 = r^2D\left(r\right)$ and dropping the bars on the variables, we can write the above Eq. (A16) in the form

$$d\sigma^2 = B\left(r\right)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\theta^2 \tag{A17}$$

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where B(r) is an arbitrary function of r. solving the metric in Eq. (A17)

$$\begin{aligned} g_{11} &= B(r) \\ g_{22} &= r^2 \\ g_{33} &= r^2 \sin^2\theta \end{aligned} \tag{A18}$$

Non-vanishing Christoffel symbols we find, are

$$\begin{aligned} \Gamma^1_{11} &= \Gamma^r_{rr} = \frac{1}{2B(r)}\frac{dB(r)}{dr} \\ \Gamma^1_{22} &= \Gamma^r_{\theta\theta} = -\frac{r}{B(r)} \\ \Gamma^1_{33} &= \Gamma^r_{\phi\phi} = -\frac{r\sin^2\theta}{B(r)} \\ \Gamma^2_{13} &= \Gamma^\theta_{r\phi} = \frac{1}{r} \\ \Gamma^3_{13} &= \Gamma^\phi_{r\phi} = \frac{1}{r} \\ \Gamma^2_{33} &= \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta \\ \Gamma^2_{32} &= \Gamma^\phi_{\phi\theta} = \cot\theta \end{aligned} \tag{A19}$$

Now from the Ricci tensor

$$R_{\mu\nu} = \partial_\nu \Gamma^\rho_{\mu\rho} - \partial_\rho \Gamma^\rho_{\mu\nu} + \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\sigma\nu} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho} \tag{A20}$$

We calculate non-vanishing components

$$\begin{aligned} R_{11} &= R_{rr} = -\frac{1}{rB}\frac{dB}{dr} \\ R_{22} &= R_{\theta\theta} = -1 + \frac{1}{B} - \frac{r}{2B^2}\frac{dB}{dr} \\ R_{33} &= R_{\phi\phi} = R_{\theta\theta}\sin^2\theta = \left(-1 + \frac{1}{B} - \frac{r}{2B^2}\frac{dB}{dr}\right)\sin^2\theta \end{aligned} \tag{A21}$$

and Ricci scalar is

$$R = -2\delta^\zeta_\zeta K \tag{A22}$$

$$\begin{aligned} \frac{1}{rB}\frac{dB}{dr} &= 2KB(r) \\ 1 + \frac{r}{rB^2}\frac{dB}{dr} - \frac{1}{B} &= 2Kr^2 \end{aligned} \tag{A23}$$

Integrating 1st part of Eq. (A23), we obtain

$$B(r) = \frac{1}{A - Kr^2} \tag{A24}$$

where A being a constant of integration can be found by substituting Eq. (A24) into 2nd part of Eq. (A23), we get

$$\begin{aligned} 1 - A + Kr^2 &= Kr^2 \\ A &= 1 \end{aligned} \tag{A25}$$

so we obtain the metric

$$d\sigma^2 = \frac{dr^2}{1 - Kr^2}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\theta^2 \tag{A26}$$

Eq. (A26) incorporates a hidden symmetry characterized by homogeneity and isotropy and represents the line element of a maximally symmetric 3-space. Due to arbitrary origin of radial coordinate system we considered and due to symmetry of space we can take all the points of this space equivalent and the origin of this coordinate system can be chosen arbitrarily at any point which means that there exists no center in this space. Therefore the maximally symmetric space is infinite and open. Further the line element is equivalent perfectly to the metric of a 3-sphere embedded in a four dimensional Euclidean space which has spherical symmetry as well.

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Appendix C Spectrum of the Black Body

A blackbody can absorb hypothetically radiation of all wavelengths falling on it and reflecting nothing at all.

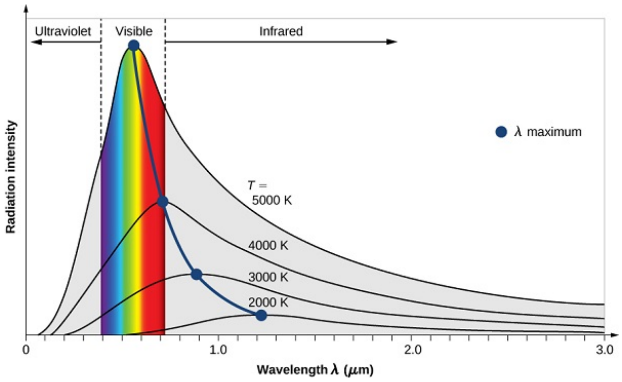

Figure A1. radiation distribution of blackbody at different wavelengths

In the early universe when matter and radiation decoupled from each other, the so-called decoupling, the primordial radiation given off gives a snapshot of the universe at that time and is known as cosmic microwave background radiation (CMBR) observed accidentally in the 60s. The recent observations conducted on cosmic microwave background radiation reveals the fact that this is the perfect black body radiation with a temperature of 2.7255 Kelvin on average. We know that the wavelength distribution of a black body is given by

$$u\left(\lambda, T\right) d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right) d\lambda \tag{A27}$$

Where $u\left(\lambda, T\right) d\lambda$ is the energy per unit volume of the radiation with wavelength between λ and $\lambda + d\lambda$ emitted by a blackbody at temperature T. We consider now a black body radiation from the big bang when the universe first became transparent to photons after 400000 years after big bang to this time about 4000000000 years. The wavelength of the primordial photons λ is Doppler shifted to λ' due to expansion of universe ,certainly $\lambda' > \lambda$. Let $f\left(\lambda', T'\right) d\lambda'$ be the current per unit volume of the residual big bang radiation as measured from the earth. since the shell of charged particles that emitted the radiation is moving away from the Earth at extremely relativistic speed so we should use the relativistic Doppler shift for light from a receding source to relate λ' to λ that is

$$\lambda' = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}\lambda = B\lambda \tag{A28}$$

Where we put $B = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}$, and v is the speed of recession of the charged shell. Since $v < c$, Clearly $\lambda' > \lambda$ by a factor

$$\frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} \tag{A29}$$

Eq. (A29) can be interpreted by generalization that all the distances have grown since first radiation emitted. In order to have a relation between currently observed spectrum $f\left(\lambda', T'\right) d\lambda'$ and original black body radiation distribution

$$u\left(\lambda, T\right) d\lambda \tag{A30}$$

we put from Eq. (A28) $\lambda = \frac{\lambda'}{B}$ into Eq. (A27)

$$u(\lambda, T)\, d\lambda = \frac{8\pi hc}{\left(\frac{\lambda'}{B}\right)^5} \left(\frac{1}{e^{\frac{hc}{\frac{\lambda'}{B} k_B T}} - 1} \right) \frac{d\lambda'}{B} \tag{A31}$$

$$\frac{u(\lambda, T)\, d\lambda}{B^4} = \frac{8\pi hc}{\lambda'^5} \left(\frac{1}{e^{\frac{hc}{\lambda' k_B T'}} - 1} \right) d\lambda' \tag{A32}$$

Where $T' = \frac{T}{B}$ and RHS of Eq. (A32) can be identified with current black body spectrum $f(\lambda', T')\, d\lambda'$ which has standard functional form of a blackbody spectrum with wavelength λ' and temperature T'. Eq. (A30) becomes

$$\frac{u(\lambda, T)\, d\lambda}{B^4} = f(\lambda', T')\, d\lambda' \tag{A33}$$

Eq. (A33) says that the radiation from a receding blackbody has same spectral distribution as yet but its temperature T' and energy

$$u(\lambda, T)\, d\lambda \tag{A34}$$

dropped by factors of B and B^4 respectively.

Appendix D Big Bang Theory of Creation

A theory coming forth on the base of the standard cosmological model describes that our universe had had a beginning and had erupted from an extremely dense, pointlike singularity about 14 billion years ago. At the singularity state, all basic interactions of nature had coalesced symmetrically where all the matter-energy melted down into an indistinguishable quark-gluon primordial soup. Historically the name of this theory as big bang is due to Fred Hoyle (1915-2001), one of the inventors and staunch proponents of steady state theory who coined the term accidentally with showing abhorrence towards it. The steady state theory, once a rival theory of the big bang lends support to an eternally evolving universe without a beginning and an end. It is speculated that during the Planck time of the order of $10^{-43}s$ all the forces of nature namely electroweak nuclear, strong nuclear, electromagnetic and gravitational were so merged into one another such that they were indistinguishable bearing perfect symmetry. From the beginning of time, $t = 0s$ to Planck time $t_p \sim 10^{-43}s$ within the time span of very first second is known as the Trans-Planckian era whose physics is yet incomplete and is open hitherto to investigation. It is being conjectured that during the time ranging from $10^{-43}s$ to $10^{-35}s$, the gravitational force freed itself from the rest of interactions, and during this period there exist the particles that supersymmetry predicts and are known as quarks, leptons, their antiparticles, and some certain massive particles. After the time interval that begins with $10^{-35}s$ to some shortly later time $10^{-32}s$, the universe expanded exponentially and gradually cooled down where the strong and electroweak forces get separated from the rest. As the universe continues to cool after the big bang, around the time $10^{-10}s$, the electroweak force splits into weak force and electromagnetic force and within few minutes after it, protons and neutrons start to condense out of the cooling quark-gluon plasmic soup. During the first half of creation, the universe can be viewed as a thermonuclear bomb fusing protons and neutrons into deuterium and then helium producing most of the helium nuclei that exist now. After the big bang until about 400000 years radiation-dominated era prevailed. Vibrant photonic radiation halted itself to become a clumped matter rather even forming single atom hydrogen or helium due to photon-atom collisions which would result in ionization instantly in the case if any atom happened to form, therefore no chance occurs for the formation of atoms and the universe remains opaque to electromagnetic radiation due to incessant Compton scattering experienced by photons with free electrons that abound in. On further cooling electrons could bind to protons forming helium nuclei with the reduction in the number of charged particles, absorption or scattering of photons consequently the universe suddenly became transparent to photons and radiation dominated era diminishes and neutral matter domination begins in the form of atoms, molecules, gas clouds, stars and in the end galaxies-the universe today. This is the whole saga of the big bang theory of creation.

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