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Research on Compatibility of Three Models Based on Combination Prediction of Pinus tabulaeformis Area

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Abstract: The study of the area model of Pinus tabulaeformis forest provides an important reference for improving the management of Pinus tabulaeformis and revealing the growth law of Pinus tabulaeformis. According to the classification method proposed by Munro, stand growth and harvest prediction models are divided into three categories: full stand model, single wood model and diameter distribution model. Based on the fixed sample data of Shangluo Pinus tabulaeformis, the spatial instead of time method is used to process the data, and the weight coefficient of each model in the combined prediction model is calculated by using the optimal weighting method. The single wood model, the whole forest model and the diameter distribution model are combined by the combined prediction method to integrate the fault area prediction of Pinus tabulaeformis forest. The results show that the combined prediction method is more accurate than the single model (single wood model, whole forest model and diameter distribution model). At the same time, the method can improve the compatibility of the forest break area prediction model, ensure the consistency of the forest break area prediction, and provide a new direction for the research of forest resource monitoring and investigation.

Keywords: Pinus tabulaeformis; Space instead of time; stand area; compatibility; combination prediction method

1. Introduction

Chinese pine (Pinus tabulaeformis) is widely distributed in my country. As one of the most common afforestation tree species in the north, it is an extremely important timber species and ecological tree species. The planting area of Chinese pine in Shangluo area is wide, its material is good and resistant to decay, it is suitable for making furniture, wood fiber industrial materials, etc., and is used as the main timber species. It is precisely because of the importance of the Pinus tabulaeformis forest in Shangluo area that if reasonable modeling techniques and methods can be used to establish a stand growth and harvest model for the Pinus tabulaeformis forest in Shangluo forest area, and accurately predict the dynamic changes in the growth of the Pinus tabulaeformis forest, it will be better. Understanding the growth and decline of Chinese pine forest resources has direct guiding significance for the operation and management of the Chinese pine forest, so as to better exert the economic and ecological benefits of the Chinese pine forest. The stand growth and harvest model are of great significance for studying the growth and change law of the stand and predicting the growth and yield of the stand. In recent years, various forestry workers at home and abroad have proposed a large number of growth models [1-6]. At present, the method of data acquisition in forest resource monitoring and investigation is mainly continuous and regular investigation. No new data processing method for forest growth research has been reported yet. The drastic changes in environmental conditions have a great impact on the data of continuous inventory, and the continuous inventory takes a long time and the cycle is long, which is not convenient for studying and formulating long-term forest growth and harvesting plans in a short period of time. The method of replacing time with space is to unfold something in a certain period of time according to the form of space, and regard them as the trajectory of the development of that kind of thing during that period of time, so as to overcome the problem of large research time span. In ecology It is widely used [12-16]. Based on the tree data of the Pinus tabulaeformis plantations in each plot of the first phase of the survey, the plots with the same stand age are classified into one category, and the forests in the plots with different stand ages are randomly corresponded with the method of using space instead of time to simulate In order to continuously check the tree data obtained in chronological order, to fit the growth situation of the forest trees during this period of time, and then predict the growth of the forest trees and reveal the growth law of the Chinese pine. Fitting the annual growth prediction model of forest trees by this method can effectively solve the problems of time-consuming and long period of continuous inventory, and because the forest trees surveyed are surveyed in the same time period and at the same place, the survey sample plots are avoided the impact on the growth of trees due to drastic changes in the environment.

Different growth and harvest models can be classified due to the different purposes of use of the growth and harvest models, the structural characteristics of the models, and the different objects reflected by the models. Domestic and foreign scholars have comprehensively compared various classification methods, and believe that the classification method proposed by Munro is the most suitable [17]. The classification method proposed by Munro mainly divides the growth and harvest models into full stand model, single tree model and diameter distribution model. These three types of models have their own characteristics. The whole stand model can directly predict the growth and yield of the stand, but it cannot show the detailed structure of the stand in the plot; the single tree growth model can reflect the detailed information of a single tree in the plot, and predict the growth process and volume of a single tree Yield. The diameter distribution model can reflect the diameter structure of the trees in the forest stand, the number density of the trees in each diameter class, and the growth and yield of the diameter class. However, both the single tree model and the diameter distribution model have shortcomings such as the accumulation of forest factor errors caused by prediction errors [18-19]. Theoretically speaking, these three models can predict the forest stand area, and the results of the forest stand area obtained by these three models should be consistent. However, in reality, due to the errors of the model and the model itself, the results of forest stand area estimated by different models may be different. In recent years, scholars have paid great attention to the issue of compatibility between models. The disaggregation method can match the single tree level and the stand factor obtained from the stand level model as much as possible, thereby improving the compatibility between the models, but this method only reduces the difference between the single tree level and the stand level prediction results, did not substantially help to improve the compatibility of the two types of models [20]. Bates et al. first proposed a combined forecasting method and effectively applied it to simulated forecasting [21]. As the name suggests, combination forecasting is a method of combining various models according to a certain weight ratio by comprehensively considering the characteristics of each model. The advantage of combined forecasting is that it can combine a model with a large prediction error and a model with a small prediction error to obtain a combined model. As long as the single model data information contained in the combined model is relatively independent, the model can be reduced Forecast error, improve forecast accuracy [22]. The essence of the combined prediction method to improve the prediction accuracy is to use the information characteristics of the single model to diversify the prediction error of the single model through weight distribution, and to improve the prediction accuracy of the combined model [23]. Common weight coefficient estimation methods include standard deviation method, variance covariance method, error square sum method, optimal weighting method, etc. [24-27]. The weight coefficient estimation method has been widely used in the study of forest stand growth and harvest models. By comparing the above four weight coefficient estimation methods, Zhang, X.Q.; et al. analyzed that the optimal weighting method has the best effect on improving the prediction accuracy of the combined forecasting model [28]. Afterwards, Zhang, X.Q. and Wang, S.J. used the optimal weighting method to determine the weight coefficients. On the basis of ensuring the consistency of the prediction results of the forest cross-sectional area, the combined prediction method solved the single tree level and the forest stand level based on the forest cross-sectional area and stand volume. The compatibility of the model has improved the prediction accuracy of the model [29-30]. However, there are only studies on the compatibility of single tree and stand level models, and there are few reports on the compatibility of three different levels of models.

To this end, this paper uses the survey data of Pinus tabulaeformis plantations in different stand age plots in Shangluo area, takes the stand area as the research object, uses the method of space instead of time to process the data, uses the combination forecasting method to combine the three models, and uses the most The optimal weighting method removes the prediction error of the single model, ensures the consistency of the prediction results of the forest stand area, solves the compatibility problem between the three models, improves the prediction accuracy of the model, and provides new ideas for the research methods of forest resource monitoring and investigation.

2. Materials and Methods

2.1. Overview of the study area and data processing

Shangluo City is located in the eastern section of the Qinling Mountains. Its geographic location is between 108°34′20″ ~111°1′25″ east longitude, 33°2′30″ ~34°24′40″ north latitude, and an altitude of 500m~1100m. Since the implementation of the "Twelfth Five-Year Plan" project, Shangluo has built 92.7 thousand hectares of natural forest protection projects, including 10,700 hectares of artificial afforestation, 10,400 hectares of closed hills for afforestation, 6,800 hectares of aerial seeding afforestation, and an annual forest management and protection area of 160,700 hectares. , The implementation of the project of returning farmland to forests covers an area of 15,300 hectares, and the proportion of Chinese pine accounted for about half. Shangluo area belongs to the monsoon climate zone, with an average annual temperature of 7.5℃~13.9℃, an average annual precipitation of 700 mm~830mm, and an average annual sunshine duration of 1848h-2056h. The soil in Shangluo area has obvious vertical belt distribution characteristics. The soil includes yellow brown earth, brown earth, clay, etc. The soil in the sample plot is mainly brown soil. The vegetation on the shade and semi-shady slopes of Shangluo is mainly composed of Pinus tabulaeformis plantations that have been air-sown for about 30 years, and are mostly distributed in moist and fertile areas. The research data comes from the Shangluo plot data surveyed from July to August 2018. The sample plots are mainly located in the southern Baiyu Temple of Shangluo and the ancient towns and villages. The landforms are all high mountains, with an altitude of 900 m to 1100 m. The slopes include sunny and shaded slopes. Most of the slopes are uphill, with a slope of 15 degrees, all of which are Pinus tabulaeformis. Plantation. A total of 78 sample plots were set up in this study, each with an area of 600m2. The forest stand factors of each sample plot were investigated, and the geographical location, soil type, soil layer thickness, altitude, vegetation type, total vegetation coverage, stand origin, single tree diameter at breast height, tree height, and average stand were determined. Age, tree species composition, forest layer, canopy closure, azimuth, position index, slope, slope position, aspect, slope shape, renewal grade, etc. Among them are 12 Pinus tabulaeformis plantations with an age of 10 years, 19 Pinus tabulaeformis plantations with an age of 20 years, 30 Pinus tabulaeformis plantations with an age of 30 years, and 9 Pinus tabulaeformis plantations with an age of 40 years. Pine plantation, 8 pieces of Pinus tabulaeformis plantation with an age of 50 years. The distribution map of the Pinus tabulaeformis plot is shown in Figure 1. Using these plot data, 78 plots with an age interval of 10 years can be formed, 39 plots are selected for modeling, and 39 plots are used for model testing. The statistical results of the variable factors of the Pinus tabulaeformis stands and the distribution of the modeling plots and test plots are shown in Table 1 and Table 2, respectively.

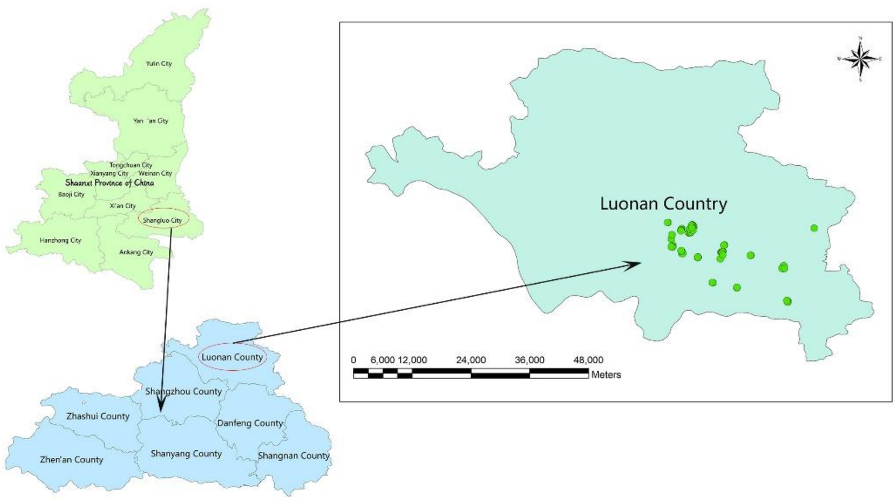


Figure 1. Distribution of Chinese pine plots.

Table 1. Statistical table of variable factors of Pinus tabulaeformis stand

Stand variable	Modeling plot				Test plot			
	Min	**Max**	**Mean**	**Sd**	**Min**	**Max**	**Mean**	**Sd**
Age(a)	10	50	30.5	6.79	10	50	30.25	8.45
Dominant height*(m)	3.82	17.45	7.65	4.26	4.15	17.32	7.83	3.56
No of trees (plant/hm^2)	283.65	2341.36	1502.3	623.15	259.32	2131.49	1325.26	426.59
Quadratic mean diameter(cm)	5.88	17.33	10.76	6.28	5.70	16.93	10.52	8.26
Arithmetic mean diameter(cm)	5.63	17.12	10.25	2.56	5.56	16.23	10.12	6.21
Standard deviation of diameter(cm)	0.0031	0.1032	0.0086	0.0073	0.38	3.26	1.95	1.06
Minmum diameter in a plot(cm)	5	10.6	3.03	0.93	5	11.2	5.81	2.03
Stand basal area(m^2/hm)	0.82	35.03	11.11	7.16	0.75	29.68	11.01	3.06
Diameter at breast(cm)	5	35.2	10.62	4.13	5	30.4	10.72	5.29
Stand volume(m^3/hm)	63.78	168.23	121.15	23.16	60.96	135.29	119.22	20.13

*Dominant height is average height of the dominant trees.

Table 2. Comparison of the estimators of stand basal area model using four different methods

Age	plots Number	
	Modeling plot	**Test plot**
10	6	6
20	10	9
30	15	15
40	5	4
50	4	5
Total	39	39

2.2. Research methods

Traditional forest surveys are carried out in stages at certain intervals, and the dynamic changes of forest stands can be estimated through retested sample plot data [7-11]. The study of forest stand growth models is helpful for us to have a deeper understanding of forest stand growth and change process and volume harvesting potential and other information, so as to effectively guide forest resource monitoring and rational forest management. This paper uses the method of space instead of time to process the data, replacing the traditional sample plot data of Pinus tabulaeformis plantation with different stand ages in place of the traditional retest plot data at certain intervals [12]. In order to improve the prediction accuracy, the variable growth rate method is introduced for research [31]. This method considering the influence of the annual growth of single tree diameter due to the changes in the cross-sectional area and dominant forest factors during the growth period [32]. In this study, we first established individual tree level, stand level, and diameter distribution level models to predict the stand area, and then obtained the combined prediction value of the stand area through the combination forecasting method [33].

2.2.1. Establishment of single tree level model and stand area model

According to the variable growth rate method, the following growth models were established. They are the high stand advantage model (H), stand number density model (N), stand average diameter model (D_m), stand section area model (B), stand standard deviation model (D_{sd}), and stand minimum diameter. Model (D_{min}), single tree diameter growth model (d_i), survival rate model (P).

At (t+1) years:

$$H_{t+1} = Exp[(A_t/A_{t+1})\ln(H_t) + (1 - A_t/A_{t+1})(\kappa_1 + \kappa_2/A_t + \kappa_3 H_t)] \quad (1)$$

$$N_{t+1} = Exp\left\{\left[\left(\frac{A_t}{A_{t+1}}\right)\ln(N_t) + \left(1 - \frac{A_t}{A_{t+1}}\right)\left[\left(\gamma_1 + \frac{\gamma_2}{A_t} + \gamma_3 \ln(N_t)\right)\right]\right]\right\} \quad (2)$$

$$D_{m_{t+1}} = Exp\{[(A_t/A_{t+1})\ln(D_{m_t}) + (1 - A_t/A_{t+1})[(\delta_1 + \delta_2/A_t + \delta_3/\ln(N_t) + \delta_4 \ln(N_t))]\} \quad (3)$$

$$B_{t+1} = Exp\left\{\left(\frac{A_t}{A_{t+1}}\right)\ln(B_t) + \left(1 - \frac{A_t}{A_{t+1}}\right)\left[\varphi_1 + \varphi_2 H_t + \frac{\varphi_3}{\ln(N_t)}\right]\right\} \quad (4)$$

$$D_{sd_{t+1}} = Exp\{[(A_t/A_{t+1})\ln(D_{sd_t}) + (1 - A_t/A_{t+1})[(\beta_1 + \beta_2 \ln(H_t) + \beta_3 \ln(N_t))]\} \quad (5)$$

$$D_{min_{t+1}} = Exp\{[(A_t/A_{t+1})\ln(D_{min_t}) + (1 - A_t/A_{t+1})[(\alpha_1 + \alpha_2/A_t + \alpha_2/\ln(N_t))]\} \quad (6)$$

$$d_{i,t+1} = d_{i,t} + Exp[\mu_1 + \mu_2 A_t/A_{t+1} + \mu_3 Ba_t + \mu_4/Rs_t + \mu_5/\ln(d_{i,t})] \quad (7)$$

$$P_{i,t+1} = \{1 + Exp[\lambda_1 + \lambda_2 A_t + \lambda_3 d_t/\ln D_{m_t} + \lambda_4 \ln(N_t)]\}^{-1} \quad (8)$$

At (t+q) years:

$$H_{t+q} = Exp[(A_{t+q-1}/A_{t+q})\ln(H_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})(\kappa_1 + \kappa_2/A_{t+q-1} + \kappa_3 H_{t+q-1})] \quad (9)$$

$$N_{t+q} = Exp\{[(A_{t+q-1}/A_{t+q})\ln(N_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[(\gamma_1 + \gamma_2/A_{t+q-1} + \gamma_3 \ln(N_{t+q-1}))]\} \quad (10)$$

$$D_{m_{t+q}} = Exp\{[(A_{t+q-1}/A_{t+q})\ln(D_{m_{t+q-1}}) + (1 - A_{t+q-1}/A_{t+q})[(\delta_1 + \delta_2/A_{t+q-1} + \delta_3/\ln(N_{t+q-1}) + \delta_4 \ln(N_{t+q-1}))]\} \quad (11)$$

$$B_{t+q} = Exp\{(A_{t+q-1}/A_{t+q})\ln(B_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\varphi_1 + \varphi_2 H_{t+q-1} + \varphi_3/\ln(N_{t+q-1})]\} \quad (12)$$

$$D_{sd_{t+q}} = Exp\{[(A_{t+q-1}/A_{t+q})\ln(D_{sd_{t+q-1}}) + (1 - A_{t+q-1}/A_{t+q})[(\beta_1 + \beta_2 \ln(H_{t+q-1}) + \beta_3 \ln(N_{t+q-1}))]\} \quad (13)$$

$$D_{min_{t+q}} = Exp\left\{\left[\left(\frac{A_{t+q-1}}{A_{t+q}}\right)\ln(D_{min_{t+q-1}}) + \left(1 - \frac{A_{t+q-1}}{A_{t+q}}\right)\left[\left(\alpha_1 + \frac{\alpha_2}{A_{t+q-1}} + \frac{\alpha_3}{\ln(N_{t+q-1})}\right)\right]\right]\right\} \quad (14)$$

$$d_{i,t+q} = d_{i,t+q-1} + Exp[\mu_1 + \mu_2 A_{t+q-1}/A_{t+q} + \mu_3 Ba_{t+q-1} + \mu_4/Rs_{t+q-1} + \mu_5/\ln(d_{i,t+q-1})] \quad (15)$$

$$P_{i,t+q} = \{1 + Exp[\lambda_1 + \lambda_2 A_{t+q-1} + \lambda_3 d_{t+q-1}/\ln D_{m_{t+q-1}} + \lambda_4 \ln(N_{t+q-1})]\}^{-1} \quad (16)$$

In the formula: $\kappa_1, \kappa_2 \dots \lambda_4$ are the parameters to be estimated; D_{m_t} is the arithmetic mean diameter of the stand in year t; D_{sd_t} is the standard deviation of the plot diameter in year t; D_{min_t} is the smallest diameter of the plot in year t; Rs_i is the relative planting distance index, $Rs_i = (\sqrt{10000/N_i})/H_i$; A_t is the average age of the forest stand in year t; H_t is the average height of the dominant tree in the forest stand in year t; N_t is the plant number density of the forest stand in year t; B_t is the stand breast height section area of the forest stand in year t; $d_{i,t}$ is the year t When is the diameter at breast height of the i^{th} tree; $P_{i,t}$ is the survival probability of the i^{th} tree in the t year.

Formula (12) can be used to calculate (t+q) the estimated forest stand area at the annual stand level; Formula (15) and formula (16) can be used to calculate the single tree level (t+q) The predicted value of the forest standing cross-sectional area at the time of the year.

There is a certain correlation between the stand factors in the above growth equations, so the seemingly uncorrelated simultaneous estimation method (SUR) is used to estimate the parameters of each equation to eliminate system errors and ensure the validity of parameter estimates [34]. The single tree survival rate equation is estimated separately.

2.2.2. Diameter distribution model

The distribution functions commonly used to describe the distribution of forest stand diameters include Weibull distribution, Beta distribution, Lognormal distribution, SB distribution, etc. [35-39]. Compared with other distribution models, the Weibull distribution function can better fit curves of different shapes, the fitting effect is better, and the parameter estimation method is simpler [40-42]. Therefore, the Weibull distribution function was selected to fit the diameter distribution of Shangluo Pinus tabulaeformis forest. The Weibull density distribution function formula is:

$$\mathrm{f(x,a,b,c)} = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} Exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] (a \le x \le \infty) \tag{17}$$

In the formula: x represents the diameter of a single tree in the plot; a represents the location parameter; b represents the scale parameter; c represents the shape parameter.

Maximum likelihood estimation method, moment method estimation, percentile method, etc. are more common parameter estimation methods, among which moment method estimation is the most widely used in Weibull distribution parameter estimation [43]. Because the parameter values in the Weibull distribution function are related to the moments of the diameter distribution to a certain extent, the method of moments is the best way to estimate the parameter values [44]. Therefore, this study chooses the method of moment estimation as the parameter estimation method of Weibull distribution function. When the location parameter a of the Weibull distribution is often considered to be equal to the starting diameter order of the stand diameter, the prediction error of the model is the smallest [45]. In the study of stand diameter growth model, the predicted value of stand square average diameter ($\widehat{D}_g$) may be close to or smaller than the predicted value of arithmetic average diameter ($\widehat{D}_m$). The parameters of Weibull distribution are more sensitive to the predicted values of the two forest stand diameters, which will lead to inaccurate parameter estimates, and even the parameter values cannot be estimated [46]. Through research, it is found that the estimation effect of replacing $\widehat{D}_g$ with diameter variance($\widehat{D}_{var}$)is better [47].

This is because

$$\mathrm{E(x)} = \int_a^\infty x f(x)dx = \int_a^\infty \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} Exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] = a + b\lambda_1 \tag{18}$$

$$\mathrm{E(x^2)} = \int_a^\infty x^2 \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} Exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] dx = b^2\lambda_2 + 2ab\lambda_1 + a^2 \tag{19}$$

$$\mathrm{Var(x)} = \mathrm{E}(x^2) - E^2(X) \tag{20}$$

The method of moments estimation formula is:

$$\mathrm{b} = (\widehat{D}\mathrm{m} - \mathrm{a})/\lambda_1 \tag{21}$$

$$\widehat{D}var - b^2\left(\lambda_2 - {\lambda_1}^2\right) = 0 \tag{22}$$

In the formula: $\lambda_1 = \lambda(1 + 1/c)$; $\lambda_2 = \lambda(1 + 2/c)$

Substituting the parameter values estimated by the method of moments into the following formulas (23) and (24), the estimated value of the forest stand cross-sectional area under the level of diameter distribution can be obtained.

$$\widehat{D}g^2 = b^2\lambda_2 + 2ab\lambda_1 + a^2 \tag{23}$$

$$\widehat{B} = (\pi/40000)\widehat{N} * \widehat{D}g^2 \tag{24}$$

2.2.3. Combination forecasting

The combined prediction formula of forest stand area is:

$$B^C = \omega_1 B^I + \omega_2 B^S + \omega_3 B^D \tag{25}$$

Where: B^C stands for the combined predicted value of the stand sectional area; B^I stands for the estimated stand sectional area under the single tree level; B^S stands for the predicted stand sectional area under the stand level; B^D stands for the stand sectional area under the diameter distribution level Estimated value of; ω_1, ω_2, ω_3 are weight coefficients, and $\omega_1 + \omega_2 + \omega_3 = 1$.

Common weight coefficient estimation methods in combined forecasting models include standard deviation method, variance covariance method, error square sum method, optimal weighting method, etc. [24-27]. Zhang, X.Q.; et al. compared and analyzed these four weight coefficient estimation methods, and concluded that the optimal weighting method has the best effect on improving the prediction accuracy of the combined forecasting model [28]. Therefore, the optimal weighting method is selected for calculation.

The core of the optimal weighting method is to construct a minimized objective function under constraint conditions according to the optimal criterion, and obtain the weight coefficient value of the combined forecasting model [24]. The optimal weighting method can remove the biased influence of the single prediction in the combined forecasting model, so that the forecasting effect of the combined forecasting model is unbiased [48]. According to the principle of least square method in the optimal criterion, this research constructs a minimized objective function to obtain the weight coefficient value in the combined forecasting model.

The value of the weight coefficient in the objective function $\min\sum_{t=1}^{n}[B_C - (\omega_1 B_C^I + \omega_2 B_C^S + \omega_3 B_C^D)]^2$ satisfies $\omega_1 + \omega_2 + \omega_3 = 1$. B_C in the formula represents the cross-sectional area of breast height per hectare of the forest stand in the Cth plot. A matrix can be used to simplify the calculation of the secondary programming model. Let $W = (\omega_1, \omega_2, \omega_3)^T$, $R = (1,1,1)^T$, $\lambda_i = (\lambda_{i1}, \lambda_{i2}, \dots \lambda_{in})$. In the equation represents transpose; W represents the column vector of combined prediction weighting coefficients; R represents the n-dimensional column vector with all 1 element; λ_i represents the prediction error vector of the ith prediction model; n is the number of plots; i=1, 2, 3, namely single tree level model, stand level model and distribution level model.

Let $\delta = (\lambda_1, \lambda_2, \lambda_3)$, then $\delta^T\delta = \begin{pmatrix} \lambda_1^T \\ \lambda_2^T \\ \lambda_3^T \end{pmatrix} (\lambda_1 \quad \lambda_2 \quad \lambda_3) = \begin{pmatrix} \lambda_1^T\lambda_1 & \lambda_1^T\lambda_2 & \lambda_1^T\lambda_3 \\ \lambda_2^T\lambda_1 & \lambda_2^T\lambda_2 & \lambda_2^T\lambda_3 \\ \lambda_3^T\lambda_1 & \lambda_3^T\lambda_2 & \lambda_3^T\lambda_3 \end{pmatrix} = E$, and then calculate the weight coefficients of 3 different level models, and derive the weight vector:

$$W = \frac{E^{-1}R}{R^T E^{-1} R} \quad (26)$$

In the formula, E^{-1} is the inverse matrix of the residual matrix, and R is a column vector whose 3-dimensional elements are all 1.

2.2.4. Model evaluation

Through the statistical mean error (MD), mean absolute error (MAD), root mean square error (RMSE) and coefficient of determination (R^2) to establish the stand sectional area model, the single tree breast height sectional area model and the single tree survival probability model, etc. Carry out fitting effect evaluation [43]. If the MD, MAD, and RMSE of the model are smaller and R^2 is larger, the model or the prediction method is better. The calculation formulas of these kinds of fitting effect evaluation statistics are:

$$\mathrm{MD} = \sum(\kappa_i - \kappa_i')/n \quad (27)$$

$$\mathrm{MAD} = \sum|\kappa_i - \kappa_i'|/n \quad (28)$$

$$\mathrm{RMSE} = \sqrt{\sum(\kappa_i - \kappa_i')^2/(n-m)} \quad (29)$$

$$R^2 = EF = 1 - \sum(\kappa_i - \kappa_i')^2/\sum(\kappa_i - \bar{\kappa}_i)^2 \quad (30)$$

Where: κ_i, κ_i', $\bar{\kappa}_i$ are the actual value, predicted value, and average value of each stand variable factor respectively; n is the number of samples; m is the number of parameters. This study uses SAS software to complete parameter estimation and model checking [49].

3. Results

The estimated values of the model parameters and the evaluation statistics of the fitting effect are shown in Table 3 after the model parameters are tested. According to the standard error of the parameters in the table, it can be seen that the estimated values of the parameters of various growth models are significant at the 0.05 level, and the fitting effect is good. The coefficients of stand dominance and stand number density are all positive in the stand area growth model, which indicates that the stand dominance is high, and the plant number density is positively correlated with the stand area growth, that is, the stand area increases with the stand advantage. And the increase of plant number density increases. This result can be explained by theoretical knowledge: the site quality of forest land is often expressed by the high stand advantage. The larger the site index, the greater the number of trees and the larger the area of breast height. It can be seen from Table 3 that the RMSE of the high stand advantage model is 1.1134 and R^2 is 0.7794; the RMSE of the stand section area model is 1.7236 and R^2 is 0.9258; the RMSE of the stand arithmetic mean diameter model is 0.7012, R^2 is 0.8926, and the tree diameter The RMSE of the standard deviation model is 0.5513, R^2 is 0.9012, the RMSE of the stand minimum diameter model is 0.6353, R^2 is 0.6012, the RMSE of the stand number density model is 188.2600, R^2 is 0.9026, and the RMSE of the single tree diameter model is 1.0023, R^2 It is 0.8234, the RMSE of the single tree survival rate model is 0.1347, and the R^2 is 813.5549. After Kolmogorov-Smirnov normality test, the residuals of various growth models obey normal distribution [50].

Table 3. Parameter estimates and model evaluations

Attribute	Parameter	Estimate	Std.	R^2	RMSE
Dominant height	κ_1	4.8383	0.2119	0.7794	1.1134
	κ_2	-20.2162	1.2369		
	κ_3	-0.0895	0.0095		
Stand basal area	φ_1	7.5269	0.2015	0.9258	1.7236
	φ_2	0.0404	0.0063		
	φ_3	-26.2365	1.3695		
Arithmetic-mean diameter	δ_1	3.4125	0.0226	0.8926	0.7012
	δ_2	-130856	0.3667		
	δ_3	1.7728	0.4562		
	δ_4	0.0523	0.0081		
Diameter standard deviation	β_1	1.2658	0.0955	0.9012	0.5513
	β_2	0.5526	0.0212		
	β_3	-0.0145	0.0321		
Minimum diameter	α_1	1.26925	0.1111	0.6012	0.6353
	α_2	-11.0216	0.7015		
	α_3	6.0236	0.8068		
Stand density	γ_1	2.0632	0.1805	0.9026	188.2600
	γ_2	20.1369	0.7034		
	γ_3	0.8216	0.0312		
Diameter	μ_1	17.0236	0.9562	0.8234	1.0023
	μ_2	-18.1232	0.9293		
	μ_3	-0.0621	0.0046		
	μ_4	0.1623	0.0325		
	μ_5	-1.5212	0.0125		
Tree survival	λ_1	-11.0269	1.1129	813.5549	0.1347
	λ_2	0.0625	0.0096	（Log*）	
	λ_3	-0.3629	0.0238		

λ_4	0.9914	0.2013

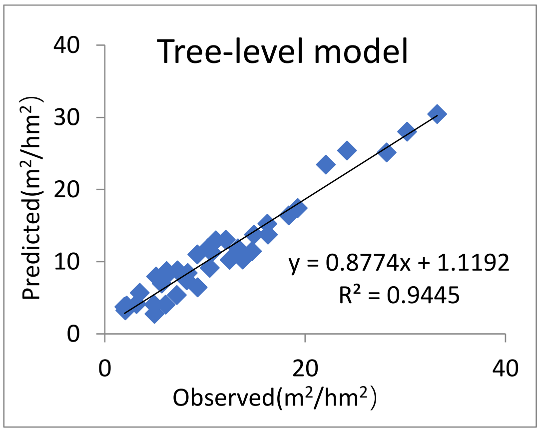
*The units of Log in this table are Log likelihood.

According to the optimal weighting method, the three weight coefficients ω_1, ω_2, and ω_3 are 0.1527, 0.4428, and 0.4045 respectively. Then the combined predicted value of the forest stand section area can be obtained using formula (25). See Table 4 for the evaluation statistics of the four different forest stand sectional area prediction models. In the modeling data, the MAD of the stand-off area model at the single tree level is 0.3012, the RMSE is 1.7125, and R^2 is 0.9238; the MAD of the stand-off area model at the stand level is -0.2089, the RMSE is 1.6623, and the R^2 is 0.9318; diameter The MAD of the stand-off area model under the distribution level is -0.2531, the RMSE is 1.7235, and R^2 is 0.9201; the MAD of the stand-off area model under the combined prediction level is 0.0083, the RMSE is 1.0123, and the R^2 is 0.9458. It can be seen from the results that the combined forecasting model has the best prediction effect, and the conclusion calculated by the test data is consistent with the conclusion of the modeling data. The reason is that the optimal weighting method uses the minimum sum of error squares as the objective function to determine three weight coefficients to remove the influence of a single prediction model on the overall error, thereby dispersing the errors of the individual models in the combined prediction model to reduce the overall error.

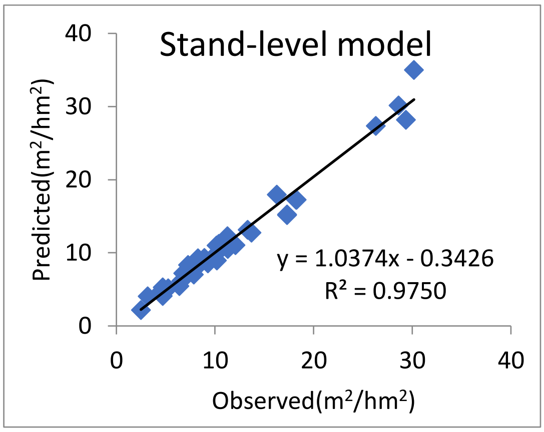
Table 4. Comparison of the estimators of stand basal area model using four different methods

Attribute	Tree-level model	Stand-level model	Distribution model	Forecast combination
MAD	0.3012	0.3245	-0.2089	-0.2356
RMSE	1.7125	1.5623	1.6623	1.5837
R^2(EF)	0.9238	0.9305	0.9318	0.9428

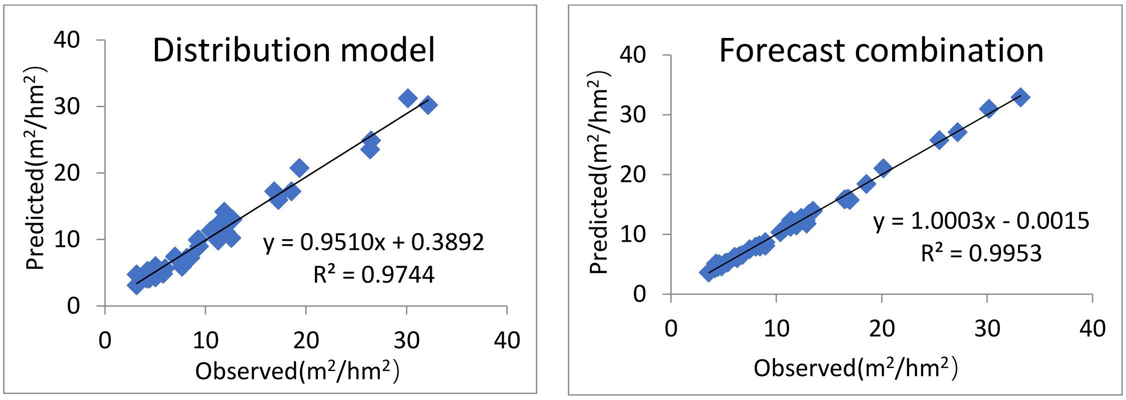
The linear relationship between the predicted value and actual value of the four forest stand-off area models is shown in Figure 2. It can be seen from the figure that the correlation coefficient R^2 of the relationship between the predicted value of the forest stand sectional area and the actual value is the largest when the combined forecasting method is used, and the correlation coefficient R^2 is the smallest under the single tree level. Therefore, the combined forecasting model has the best forecasting effect compared with other horizontal models. This method makes the prediction results of forest stand area at different levels tend to be consistent, and improves the compatibility of the predictions of the three models. The prediction effect of the data processed by the space instead of time method at the single tree level is worse than the prediction effect at the stand level and the diameter distribution level.



(a)



(b)



(**c**) (**d**)

Figure 2. The linear relationship between the predicted value and observed value of the four forest stand-off area models, they are listed as: (a) Tree-level model; (b) Stand-level model; (c) Distribution model; (d) Forecast combination.

4. Discussion

The combined forecasting method can fully consider the influence of the errors brought by each single model, and use the characteristics of the single model to disperse the errors, thereby improving the prediction accuracy. The forest standing variables predicted by this method can better reflect the growth law of forest trees, and it is a good method to improve the prediction accuracy of the model. This study uses the method of space instead of time to process the data, and uses the combination forecasting method to combine the stand sectional area models at different levels to ensure the consistency of the stand sectional area prediction, and solve the compatibility problem of the three types of models, and improve at the same time The fitting effect of the forest stand cross-sectional area is verified, and the applicability of the space instead of time method is verified. The model uses three common fitting effect evaluation statistics for evaluation and verification. After comparison and analysis, it can be seen that the combined forecasting and estimation method is obviously better than the other three forest stand-off area forecasting methods, and the prediction accuracy of the model is higher. This is the same as Zhang X.Q. and Wang S.J.'s research conclusions based on the compatibility of combined forecasting single tree and stand level models. It also proves that the data obtained by using space instead of time method is suitable for stand growth prediction research [29-30].

The better the prediction effect of the single model, the higher the accuracy of the combined prediction model [51]. The selection of weight coefficients plays a decisive role in the fitting effect of the combined forecasting model. The combined forecasting method can be applied to any other stand factors including stand area [52]. The combination forecasting model obtained by the optimal weighting method to select the weight coefficients has the best forecasting effect. Compared with the error sum-of-squares method and the variance-covariance method, the advantage of the optimal weighting method is to remove the error of the single model prediction and improve the prediction accuracy of the model [48]. In fact, the weight coefficient of the model is not fixed, but the value obtained according to these three coefficient calculation methods is fixed, which violates the general law of forestry science research. The higher the prediction accuracy of the single model in different levels of model methods, the larger the weight coefficient of the single model in the combined prediction model. As the data structure of the sample site changes, the weight coefficients of the model will change accordingly. If a fixed weight coefficient is used, the true condition of the data structure of the plot cannot be represented, and the prediction results of the combined prediction model obtained by fitting may not match the actual situation. Therefore, the next step should be to in-depth study of the non-negative variable weight combination forecasting model, combining the actual situation of the data structure of the sample site for fitting, so that the model forecast becomes more accurate and effective.

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