CAN QUANTUM ENTANGLEMENT BE MODULATED BY GRAVITATIONAL WAVE?

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ABSTRACT

In 1935 Einstein, Podolsky, and Rosen (EPR) studied general entangled states in the two photon experiment and pointed out the contradiction between local realism and the completeness of quantum mechanism. Most of the EPR experiments in recent years are based on the detection of polarization correlations of optical photons between spatially separated photon channels, some of which are split and directed to two spatially separated Michelson interferometers. Later, the two arms of Michelson interferometers are replaced by dual-channel Fabry-Perot (F-P) interferometry enabling precise analysis of the energy-time entanglement between a pair of photons. On the other hand, F-P type detectors on gravitational radiation have caught dozens of gravitational wave (GW) events successfully. This paper proposes a combined experiment of EPR and GW, exhibiting whether the coincident rate of EPR is modulated by GW induced change of cavity length or not. Such an experiment could test the coupling of quantum mechanics and general relativity for the first time, and be a useful tool to explore the nature of quantum gravity.

Subject headings: Quantum Entanglement, Gravitational Wave

1. INTRODUCTION

As the premise of local realism taken for granted by most physicists, Einstein, Podolsky, and Rosen suggested(Einstein et al. 1935): “Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system." In contrast, by the theory of quantum mechanics their physical properties should be related in such a way that any measurement made on one particle should instantly convey information about any future measurement result of the other particle. Such correlations that Einstein skeptically saw as spooky, action at a distance.

An important development was due to John Bell (1964) who continued the EPR line of reasoning, and demonstrated an upper limit (Bell’s inequality) regarding the strength of correlations that can be produced in any theory obeying local realism. It showed that quantum theory predicts violations of this limit for certain entangled systems(Bell 1964, 1966).

Since the original Bell’s inequality assumes perfect correlations of single state, which are never perfect, it cannot be tested experimentally. A generalization of the original inequality to the imperfect case leads to the Clauser, Horne, Shimony, and Holt (CHSH) inequality in 1969, in which the joint probability of two spatially separated subsystems A and B can test the local hidden variable theories deviating from quantum mechanics(Clauser et al. 1969). The usual form of the CHSH inequality is

$$S = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2 \qquad (1)$$

where terms $E(a,b)$ are the quantum correlations of the particle pairs, with a and a' are detector settings on side A; b and b' are detector settings on side B respectively.

The source is assumed to produce photon pairs, one pair at a time to opposite directions. Emerging signals from each channel are detected and coincidences counted by the coincidence monitor. As entanglement and local realism predict different values on S, the experiment gives an indication of the validity of two theories.

After the realization that the polarization entangled state of photons emitted in atomic cascades can be used to test Bell’s inequalities, the first experiment was performed by Freedman & Clauser (1972).

Up to now, there has been a large number of such experiments. The largest violation of a Bell-type inequality have for a long time been by the experiments of Aspect et al. (1981, 1982). Aside from two early experiments, all agreed with the predictions of quantum mechanics and violated inequalities derived from assumptions of Bell and CHSH.

The first attempt of causal separation, the absence of communication between the two measurement sites, was achieved by Aspect et al. (1982). Weihs et al. (1998) improved such an experiment with a distance on the order of a few hundred meters and adding a random settings retrieved from a quantum system. Scheidl et al. (2010) improved on this further by extending locations of measurement sites to a distance of 144km. As summarized by Zeilinger (1999) and Aspect (2002) these experiments exclude the most appealing local realistic theories and thus represent strong evidence in favor of abandoning the local realism premise.

Beside spins and polarizations, Franson (1989) proposed Bell inequality by time and energy, who derives a new violation of Bell’s inequality that is dependent upon interference between the probability amplitude for a pair of photon to have been emitted at various times by an excited atom, in which the coincidence counts in the two detectors can be either totally correlated or anticorrelated, depending on the relative sittings of the two phase shifters. The results show that the quantum-mechanical uncertainty associated with the usual wave-packet description of a particle is inconsistent with any local hidden variable theory.

Following the idea of Franson (1989), a test of Bell’s inequality for energy and time in a simple and easily analysable configuration emerges, where photon pairs split and directed to two spatially separated Michelson interferometers(Brendel et al. 1992).

In such an experiment, the nonlocal character of the fourth-order interference can be applied to test Bell’s inequality in-

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volving energy and time instead of polarization correlations, with a resultant violation of Bell's inequality for energy and time by several standard deviations(Brendel et al. 1992).

2. THE EXPERIMENT WITH F-P CAVITY

Furthermore, the photon coincidences between two spatially separated Michelson interferometers replaced by dual-channel F-P interferometry have been reported (Sun 2015). Such a F-P cavity is usually made of two mirrors aligned to the optical axis: the first one (input mirror) is partially reflecting and the second one (end mirror) is almost completely reflective.

The entanglement of such a dual-channel F-P interferometer can be represented by the transmission coincidence rate which is calculated as follows when normalized to the coincidence rate if the two F-P interferometers are removed(Sun 2015),

$$P_{rates} = \frac{T^8}{1+R^8-2R^4\cos(2k_xL_x+2k_yL_y)} \tag{2}$$

where R is the reflectivity and T the transmissivity of the mirrors satisfying, $R^2+T^2=1$, which are assumed to be the same in the F-P interferometer of equal cavity length. Notice that the roles of kL_x and kL_y are similar to those of the orientations ϕ_1 and ϕ_2 of the polarizers in the Bell experiment.

With the proper normalization constant $C_T = [T^4/(1-R^4)]^2$, the CHSH inequality for the transmission coincidence rates read(Sun 2015),

$$S_{FP} = |\frac{1}{C_T}\{P_{rat}(L_x,L_y)+P_{rat}(L'_x,L_y)+ \\ +P_{rat}(L_x,L'_y)-P_{rat}(L'_x,L'_y)\}| \leq 2 \tag{3}$$

where the counting coincidence is: 1 for count and 0 for no count, differing from the polarization test case that uses 1 for horizontal polarization and -1 for the vertical polarization.

The fully quantum mechanical prediction of Equation (2) clearly violates the CHSH inequality of Equation (3). E.g., when T = 0.6, set $2k_xL_x = 2N\pi$, $2k_yL_y = 2N\pi$, $2k_xL'_x = 2N\pi+\pi/8$, $2k_yL'_y = 2N\pi+\pi/8$, and with N of a positive integer, obtains $S_{FP} = 2.1042$ in Equation (3).

With the normalization constant C_T , the CHSH inequality for the transmission coincidence rates of such a F-P interferometer, Equation (2) of Sun (2015) becomes,

$$\bar{P}_{rates} = \frac{(1-R^4)^2}{(1-R^4)^2+2R^4(1-\cos\phi)} \tag{4}$$

A proper tuning of the microscopic length gives, e.g., $\phi_{norm} = 2k_xL_x+2k_yL_y+\theta$, where k is the positive integer number, and θ can be, 0, or, $\pi/8$ as Sun (2015).

In such a circumstance, count rates within the selected coincidences between the output arms of the interferometers can be recorded and stored by a computer, so that a violation of Bell's inequality can be investigated by such a F-P interferometer.

3. THE JOINT EPR-GW EXPERIMENT

On the other hand, GWs are disturbances in the curvature of spacetime, generated by accelerated masses, like astronomical compact binaries, propagating as waves outward from their source at the speed of light. The GW induced spacetime fluctuation causes change of arm length, so that the end mass of x-axis and y-axis oscillate longitudinally with displacements(Hawking & Israel 1987),

$$\delta L_x(t) = \frac{1}{2}Lh_+(t) \\ \delta L_y(t) = -\frac{1}{2}Lh_+(t) \tag{5}$$

where $L = L_x = L_y$ is the length of each arm. Then the difference $\delta l(t) = \delta L_x(t) - \delta L_y(t)$ can be written as,

$$\delta l(t) = h(t)L \tag{6}$$

In the case of F-P cavity, the bouncing light beam will build up during its B trips, which corresponds to a total phase delay of,

$$\Delta(t) = \frac{2B\delta l(t)}{\lambda} \tag{7}$$

Consequently, the phase ϕ of Equation (4) becomes $\phi(t) = \phi_{norm}+\Delta(t)$, As the amplitude of phase delay of Equation (7) is very small, $|\Delta(t)| \ll 1$, it corresponds to a phase fluctuation to Equation (4), which can be represented, $\cos\phi = 1-\Delta^2/2+O(\delta^3)$.

Interestingly, such a GW induced phase fluctuation can results in measurable variation of the coincidence rate of Equation (4), when the reflectivity of the mirrors of F-P cavity, $R = 1-\delta$, is high enough,

$$\bar{P}_{rates}(L_x,L_y) = \frac{P_{rates}(L_x,L_y)}{C_T} = \frac{1}{1+(\frac{\Delta(t)}{4\delta})^2} \tag{8}$$

In the case that the gravitational wave induced coincident rate of Equation (8) is negligible, we have $\Delta(t)/(4\delta) \ll 1$, and a coincident rate of unity is expected. In contrast, as gravitational wave varies the phase to an amplitude of $\Delta(t)/(4\delta) \approx 1$, a coincident rate of 0.5 is expected in Equation (8). This means the GW can cause 50% change of the coincident rate if δ is small enough, which corresponds to an extremely high reflectivity of the mirrors of F-P cavity, $R = 1-\delta$.

The coincident rate of 0.5 in Equation (8) can be reached by following combination of parameters, e.g., number of trips, $B = 2\times10^4$; the length of F-P cavity, $L = 4km$; amplitude of incident GW, $h = 5\times10^{-21}$; and wave length of laser, $\lambda = 900nm$, which correspond to a phase change of $\Delta(t) = 1\times10^{-5}$ as shown in Equation (7). And such a GW induced phase change in the F-P cavity can be measured when the reflectivity of cavity mirrors is of $R = 1-\delta$ where $\delta = 3\times10^{-6}$. In other words, the joint effect of EPR-GW can be measured with a high reflectivity of the cavity mirrors of $R = 0.999997$, which corresponds to a very small mirror transmission, $T = (1-R^2)^{1/2}$ and thus an extremely high fitness of cavity.

On the other hand, to exclude the mutual influence between the two observations spatially separated by 400m across within the realm of Einstein locality, the individual measurements had to be shorter than 1.3 ms(Weihs et al. 1998). They achieved the goal with the duration of an individual measurement kept far below the 1.3μs limit through high speed physical random number generators and fast electro-optic modulators(Weihs et al. 1998).

For a ground base GW detector, the merger of a binary of black hole or neutron star corresponds to a frequency shift, like a chirp, from a few tens of Hz to a hundred Hz.

To be detected by a joint GW-EPR experiment, the corresponding time scale of such a chirp should be much longer than that of coherence time of the light and much shorter than the time of entanglement of a photon pair. This requires the duration of an individual measurement to be kept in the range $1\mu s$ - $1ms$, which is not difficult to realize. E.g., sources which can be used in EPR experiment have been increasing in quality and brightness, e.g., an entangled-photon pair source with count rate of over 1×10^6 per second and fidelity of 97.7% has been reported(Altepeter et al. 2005).

In fact, the joint GW-EPR experiment corresponds to varying arms, then how to get the terms with $'$ as shown in Equation (3)? In one "circle" of oscillation of $h(t)$, one can define the state of a minimum and a maximum length variation in x-axis as, $\delta L_x(t_0) \to 0>$, and $\delta L_x(t_1) \to 1>$ respectively, corresponding to $\Delta(t)/(4\delta) \ll 1$ and $\Delta(t)/(4\delta) \approx 1$ in Equation (8) respectively; so are values on y-axis.

In such a case, if the GW induced change of coincident rate of Equation (8) can be detected as varying with the "circle" of the chirp, the coupling of between the coincident rate and GW induced change of cavity length is supported. This is similar to many modern experiments directed at detecting quantum entanglement rather than ruling out local hidden variable theories.

Furthermore, with definition of the minimum and the maximum states of variation of cavity length, e.g., $0>$, and $1>$ above, the CHSH inequality can be obtained,

$$S_{EG} = |\frac{1}{C_T}\{P_{rat}(0>,0>)+P_{rat}(1>,0>)$$

$$+P_{rat}(0>,1>)-P_{rat}(1>,1>)\}| \quad (9)$$

if $S_{EG}>2$ can be confirmed, then the coupling of quantum mechanics (nonlocality) and GR (locality) can be tested in one experiment for the first time.

4. DISCUSSION

The dual-channel F-P interferometry(Sun 2015) is to test CHSH inequality by constant length of arms and fixed angle difference between coincident terms as shown under Equation (3).

Whereas, a joint EPR-GW experiment differs from the dual-channel F-P interferometry(Sun 2015) on three aspects. First, the misalignment angle between the two F-P cavities changes from π to $\pi/2$; second, the overall set up of the experiment especially the suspension of the mirrors must satisfy the requirement of GW detection; third, the CHSH inequality is realized by the phase change originating in a varying fluctuation of cavity length due to GW.

To make the influence of GW induced phase change measurable in the coincidence rates (up to 50%), the phase resolution of the cavity must be extremely high, which requires an extremely high reflectivity of cavity mirrors and thus the fitness of cavity.

The goal of a joint EPR-GW experiment can be achieved by two steps. The first is to test whether the coincident rate is modulated by the chirp like frequency shift of the GW, $h(t)$, or not. The second one is to further test quantum mechanics (nonlocality) as shown in Equation (9).

Such a joint experiment of EPR and GW can be realized by either changing the suspension of the mirrors of EPR experiment, or by changing the source of laser and detection of a GW interferometer and adding two single-photon counting modulas SPCM to each end of the arm. It can work together with a LIGO like GW detector. Suppose an incident GW comes, which is detected by the normal GW detectors. Then it should vary the arm length of the joint EPR-GW detector also (with certain time different due to their different locations on the Earth), and hence modulates the coincident rate of the EPR-GW detector.

Such a joint EPR-GW experiment can also be applied to GW detector in space by adding two single-photon counting modulas SPCM to each end of the arm similar to Fig.1.

The joint EPR-GW experiment may help physicists to achieve a long-sought goal: a quantum theory of gravity that can merge general relativity and quantum mechanics(Maldacena & Susskind 2013; Papadodimas & Raju 2013), the two grand theories of the universe that tend not to get along.

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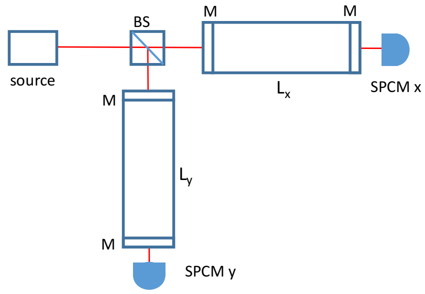

FIG. 1.— Schematic of a F-P interferometer combining test of EPR and GW. Abbreviation in the graph: mirror,M; SPCM , single-photon counting modulas. The beam splitter BS is semi-transparent for the horizontally polarized light. Such an energy-time entangled photon pair traveling separately along the two arms at right angle each other. The transmissions and the reflections in both arms are monitored by SPCMs. The two large boxes are individual F-P interferometers. The GW induced variation of cavity lengths will modulate the coincident rates of monitors, SPCMs, which tests EPR under GW.