Research Article

Dimensionality, Fibonacci Numbers & Quantum Physics

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Abstract: The importance and near ubiquity of the Golden Ratio in disciplines like chemistry and biology is well known, but only recently has it come to light in areas pertaining to the Quantum domain. By using a modified tool-kit of hyper-complex numbers (known as Dimensional Gate Operators) and numerical analysis, we uncover a connection between hyper-dimensional objects, the Fibonacci sequence and that of quasiparticles and excitations. The results show that dimensionality increases in step with the Fibonacci sequence.

Keywords: Fibonacci numbers; the Golden Ratio; the Golden Mean; dimensionality; quasiparticles; anions; non-Abelian; Fibonacci; excitations; Dimensional Gate Operators.

1. Introduction

The Fibonacci sequence and the Golden Ratio (1.618) enjoys widespread acknowledgement in a variety of disciplines including; Physics, Biology, Astrophysics, Chemistry, and Mathematics. But its presence in Quantum Physics is less well known. It turns out, the Golden Ratio can be found in relation to several quantum effects including; the emergence of E8 symmetry in the quantum phase transitioning of cobalt niobate, in speculative predications of solar neutrino mixing and that of Fractal Cosmic Strings [1, 2, 3]. The Golden Ratio can be found in the 2-dimensional quasi-particle called the 'anyon'. When non-Abelian anyons are combined together, the dimension of their associated Hilbert Space increases in step with the Fibonacci sequence. These Fibonacci anyons are expected to have some major relevance to the science of Quantum Computing [4].

By now we should begin to see a pattern, especially in regards to quasiparticles and extra dimensions [1, 4]. This pattern was not very obvious to me until I started my own investigation into hypercomplex numbers and extra-spatial dimensions. It was only as I became drawn to the same conclusions that I began to notice it. This is an important result for the Dimensional Gate Operator (DGO) version of the Standard Model, as it reinforces its assumptions and shows that it is a theory capable of predictions and tangible results. This opens the door for future research into the DGO view of the Standard Model, which could help shift perceptions in future and move Quantum Physics forward in novel and rewarding directions.

2. Materials and Methods

Once we have accepted the DGO viewpoint that imaginary numbers are simply ordinary numbers which obey different rule sets, we can begin to apply these methods to hyper-complex numbers, like the quaternions, Octonions and Sedenions [5, 6]. In fact, we are not limited to hypercomplex numbers of Order 2n as is traditionally the case; since √-1 is now just ±1, we can contract the dimensions of the octonions to the heptonions, hexonions, pentonions, trionions and dionions [6]. Or indeed expand them outwards to the

Sedenions via the nononions, decanions and so forth. These are the 'onions' and it is an apt name, because they are similarly arranged to the layers of an onion.

Table 1. Dionions and Trionions are the two yellow sectors. Quaternions are the yellow plus the orange identity and the Pentonions are the yellow and red sectors taken together.

Notice that the quaternions were skipped over in the previous list, because they already exist formally and don't need construction. Another curious feature of the quaternions is that they include the identity elements 1, i, j, and k. These elements are usually dropped from the octonions and sedenions (and all of the other onions), making them 7x7 and 15x15 matrices respectively. Therefore, when constructing an overview of the hypercomplex dimensionality, we are arrive at the structure in Table 1.

We begin with the dionions and then expand out to the trionions. These are the two yellow sectors in Table 1. To get to the quaternions from the trionions, we simply include the identity (orange sector). But to go from the quaternions to the pentonions, we need to drop this and expand the non-identity elements out twice into the red. This makes the pentonion matrix or universe a 6-dimensional matrix or universe, in total.

1	*i*	*j*	*k*	*E*	*I*
i	-1	*k*	-*j*	*I*	-*E*
j	-*k*	-1	*i*	*j*	*K*
k	*j*	-*i*	-1	*K*	-*j*
E	-*I*	-*j*	-*K*	-1	*i*
I	*E*	-*K*	*j*	-*i*	-1

Table 2. The 6 matrices of the DGO Standard Model. Notice that the 5D sector only contains one flavour of G and H. The three 4D matrices can be described by one matrix (see Table 3).

In 'The Higgs & the Graviton', we looked at six tables describing the leptons, quarks and their intermediate particles [7]. 3 of the 6 tables are dimension 4. This means that the six tables can be reduced down to just four (Table 3).

In Quantum Electro Dynamics (QED), Quantum Chromo Dynamics (QCD) etc. the spins of the various particles are described by Group Theory. The electromagnetic force, for example is described by U(1). Where as the Weak force and QCD are described by

5D	
G	H
G	H

(AU) 4D					
g	u	g	s	g	b
g	d	g	c	g	t

(A JA) 4D					
g	u	W/Z	s	W/Z	b
g	d	W/Z	c	W/Z	t

(IVD) 4D					
W/Z	u	W/Z	s	W/Z	b
W/Z	d	W/Z	c	W/Z	t

3D					
W/Z	e	W/Z	μ	W/Z	τ
W/Z	νe	W/Z	νμ	W/Z	ντ

2D					
γ	e	γ	μ	γ	τ
γ	gt	γ	gt	γ	gt

SU(2) and SU(3), respectively. These matrices can be multiplied by each other to create higher order symmetries of other particles. U(1) is mathematically speaking a 1-dimensional operation. SU(2) is said to be 4-dimensional (3 and the identity) and SU(3) is eight dimensional. These dimensions are not degrees of freedom, however. They are values ascribed to properties like; motion, magnetic moment, angular momentum and charge.

So how do these matrix groups compare with the dimensionality of tables 2 and 3? In DGO, the QED Tables are 2D, but this isn't really true, since we can always represent these forms in 3D easily enough. Note, however, that in place of a secondary set of particles, we simply have abstract figures which stand for the annihilation operators. Technically, the 'y' (or photons) on this line are creation operators. This means that this 3D table is in effect 2D. The next table deals with the interaction between the Weak Force and the leptons and is 3D. This also spans into the 4D sector of the quarks. This force is equivalent to the SU(2) group, in terms of placement and because it has (3+1) dimensions.

The gluons and quarks are SU(3), which is 8-dimensional. However, we were able to reduce this down to four-dimensions, easily enough.[8] Finally, we could say that the higher 5-Dimensions, conform to: SU(3)xSU(2)xU(1), meaning that they contain all of the lower matrix groups. This is also shown to be true in DGO, where the higher order groups contain all of the forces and elements of the lower order groups implicitly.[7]

While, we can show correspondence between our DGO particles and Group Theory, they are not identical, because they are dealing with such different properties. The purpose of the DGO Standard Model is to model the particles and show interesting relationships between the particles that other methods may have missed. Its purpose is not to compete with the Standard Model and Quantum Mechanics in describing quantities like spin, charge, or magnetic moment, as these are already adequately explained.

5D					
G			H		
G			H		
(Δ, !Δ) 4D					
g	u	W/Z	s	W/Z	b
g	d	W/Z	c	W/Z	t
3D					
W/Z	*e*	W/Z	*μ*	W/Z	*τ*
W/Z	*νe*	W/Z	*νμ*	W/Z	*ντ*
2D					
y	*e*	y	*μ*	y	*τ*
y	$a^\dagger$	y	$a^\dagger$	y	$a^\dagger$

Table 3. The reduced set of tables.

3. Results

Since the DGO Model does not follow the Group Theory formula, we are free to investigate its structure based upon its own merits. If we start with the DGO quaternions, we see that they are constructed either with 3 Real axes and 1 imaginary axis (3R + 1i) or 1 Real and 3 imaginary axes (1R + 3i). This makes for 4 Real and 4 Imaginary axes, or 8 dimensions in total.

$$3R + 1i + 1R + 3i = 4R + 4i = 8 \tag{1}$$

However, since we have 3 Real axes and 3 Imaginary axes, we can build both of our above combinations from just the two groups; (1, 1, 1) and (i, i, i). This can be done either as:

$$(1, 1, 1,) + (i) \tag{2}$$

or

$$(i, i, i) + (1) \tag{3}$$

We have a universe of 6, so this is equivalent to 6 choose 4. Therefore, we will set our combination number to r = 6. Without repetition, this gives us a list of 15 entries. And after removing duplicates, we are left with these 3 entries:

$$\begin{matrix} [[1, 1, 1, i] \\ [1, 1, i, i] \\ [1, i, i, i]] \end{matrix} \tag{4}$$

We can do the same for the dionions, trionions, and pentonions. However, notice when we get to the Pentonions, the Real and Imaginary numbers shoot from 4 up to 6.

	Real	Imaginary	Total	Combination
Dionions	2	2	4	2
Trionions	3	3	6	4
Quaternions	4	4	8	6
Pentonions	6	6	12	10

The reason for this is found in Table 1, where the dimensions jump from 4 to 6, as we move from the Quaternions to the Pentonions (See Table 4).

Table 4. The reduced set of tables.

If we look at the 'Combination' column, we see the pattern: 2, 4, 6, 10. This is the Fibonacci Sequence starting at F(2), or simply F(n)*2. Or, the 'double Fibonacci numbers', as I like to call them.

1	1	1	i	i
i	1	1	1	i
i	i	1	1	1
i	i	i	1	i
1	1	1	1	1
1	1	1	1	i
1	1	1	i	i
1	1	i	i	i
1	i	i	i	i
i	i	i	i	i

We can show this by working out the answers for each of the Onion Universes and plotting them all together:

Table 5. The combinations of Onion Universes plotted together to form the Fibonacci sequence

If we sum the first 4 terms, we get 22. This is an interesting number. It is equal to the number of letters in the Hebrew alphabet. It was this fact that made be overly eager to add 22 columns to Table 2. [7] Originally, there were 2 flavours of graviton and Higgs in Table 2 and 3, to give a total of 22 columns. But given the lack of any experimental evidence for a heavier Higgs particle, I am permanently removing them from the model. [9]

Dimensionality expands under the Fibonacci sequence:

0, 2, 2, 4, 6, 10, 16, 26, 42, 68, 110, 178, 288, 466, 754, 1220, 1974, 3194, 5168, 8362, 13530, 21892, 35422, 57314…

Notice that there are two 2s at the beginning of the sequence. This first set of twos

5D	
G	H
G	H

(Δ, !Δ) 4D					
g	u	W/Z	s	W/Z	b
g	d	W/Z	c	W/Z	t

3D					
W/Z	*e*	W/Z	*μ*	W/Z	*τ*
W/Z	*νe*	W/Z	*νμ*	W/Z	*ντ*

(^U) 3D					
y	*e*	y	*μ*	y	*τ*
y	$a^{\dagger}$	y	$a^{\dagger}$	y	$a^{\dagger}$

2D (Quasiparticle)					
y	*e*	y	*μ*	y	*τ*
y	$a^{\dagger}$	y	$a^{\dagger}$	y	$a^{\dagger}$

could describe another set of QED leptons. One candidate for these are the 2-dimensional quasi-particle called the ‘anyon’. This means that we can technically expand out our set of tables to include a second set of 2-dimensional quasiparticles, which we will label the anyons, for now, but it could be any type of quasiparticle or exciton, for that matter.

Table 6. The original 4 tables expanded to include the anyons. The final count has 26 columns, another good reason why there is no need for a heavier H and G.

To make matters even weirder, it is theorised when non-Abelian anyons are combined together, the dimension of their associated Hilbert Space increases in step with the

Fibonacci sequence. [4] These Fibonacci anyons are expected to have some major relevance to the science of Quantum Computing, should they ever be discovered. [10] My understanding is that DGO research confirms the existence of these particles. It offers them a place in the DGO hierarchy and confirms the status and importance of Fibonacci numbers in relation to dimensionality, all at the same time.

3.1 Non-Onions

The first mapping of the particles in the Standard Model to the quaternions gave this rather neat correspondence.

Table 7. Leptons and quarks as quaternion matrix

Mapping them to the Pentonions, we begin to see the Fibonacci expansion into 6 dimensions rather than just 5.

Table 8. All particles in the DGO Standard Model mapped to the Pentonions in dimension 6.

After the particles are mirrored into the Octonions, the option to expand them into a 9x9 matrix is offered by the consecutive triangular numbers; 36 and 45. A 9x9 matrix is clearly indicative of the non-onions or nonions. [11]

But this means that the actual matrix will be 10x10, just as the Fibonacci sequence also predicts. So, the correct expansion runs the gamut of the Sedenions from e1 to e9 (see inset matrix in Table 10), proving that this is still an nonion matrix.

This is also the final step, as far as our 5D DGO Standard Model is concerned. However, we can keep going with our Octonion Model. When we do, our next expansion fills

g	*d*	*c*	*t*
u	*y*	*ve*	*μ*
s	*e*	*y*	*ντ*
b	*νμ*	*τ*	*y*

the entirety of Sedenion space, which also just happens to be the next number in our Fibonacci set: 16.

And at this point we are done. But it is clear that the dimensions themselves can continue to expand outwards indefinitely. Following on from 16, we have 26 (the numbers

g	*d*	*1W1Z*	*c*	*WZ*	*t*
u	*y*	*ve*	*μ*	H^1	G_1
1W1Z	*e*	*y*	*ντ*	G_4	H^2
s	*νμ*	*τ*	*y*	H^3	G_5
WZ	H^1	G_6	H^3	G_3	H^4
b	G_2	H^2	G_7	H^4	G_8

of letters in the English alphabet, as well as the number of columns in Table 6) and the number 42, which as Douglas Adams has pointed out is the 'Meaning of Life', because it appears everywhere…

… And it just so happens to appear here too.

Table 9. Expansion of the DGO-SM into the Nononions.

Table 10. The black square shows the particles from e1 to e9, which form the Nonions.

g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8	--	--
u	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3	--	--
1W	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*	--	--
s	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*	--	--
WZ	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*	--	--
b	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W*	--	--
G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*	--	--
H^3	G_2	*b*	*WZ*	*s*	*1W*	*u*	g	--	--
--	--	--	--	--	--	--	--	--	--
--	--	--	--	--	--	--	--	--	--

	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9
e0	g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8	g	*d*
e1	*u*	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3	*u*	*y*
e2	*1W*	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*	*1W*	*e*
e3	*s*	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*	*s*	*νμ*
e4	*WZ*	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*	*WZ*	H^1
e5	*b*	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W*	*b*	G_6
e6	G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*	G_2	H^2
e7	H^3	G_2	*b*	*WZ*	*s*	*1W*	*u*	g	H^3	G_2
e8	g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8	g	*d*
e9	*u*	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3	*u*	*y*

	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12	e13	e14	e15
e0	g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8	g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8
e1	*u*	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3	*u*	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3
e2	*1W*	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*	*1W*	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*
e3	*s*	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*	*s*	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*
e4	*WZ*	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*	*WZ*	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*
e5	*b*	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W*	*b*	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W*
e6	G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*	G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*
e7	H^3	G_2	*b*	*WZ*	*s*	*1W*	*u*	g	H^3	G_2	*b*	*WZ*	*s*	*1W*	*u*	g
e8	g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8	g	*d*	*1W*	*c*	*WZ*	*t*	H^3	G_8
e9	*u*	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3	*u*	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3
e10	*1W*	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*	*1W*	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*
e11	*s*	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*	*s*	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*
e12	*WZ*	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*	*WZ*	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*
e13	*b*	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W*	*b*	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W*
e14	G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*	G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*
e15	H^3	G_2	*b*	*WZ*	*s*	*1W*	*u*	g	H^3	G_2	*b*	*WZ*	*s*	*1W*	*u*	g

Table 11. The complete Sedenion matrix of Matter-Antimatter, Dark Matter-Antimatter and Helicity. Notice the lack of any indication of different flavours of H and G.

4. Discussion

It is easy to fall into the mistaken idea that higher dimensions are somehow closed off from the lower dimensions. This conception comes from Science Fiction tropes, where parallel dimensions exist separate and independent of one another. The lack of access to higher dimensions in theoretical physics is usually explained by compact Lobachevskian spaces curled up very small, or infinitely large. But since dimensionality is now known to follow the logic of the Fibonacci curve, I would speculatively propose that the curve of the function operates as a kind of horizon, beyond which it is not possible to see. But, this is merely an idea. A more concrete analysis — assuming one is even possible — will have to wait until a later date.

In the 'The Higgs Boson & the Graviton' [7], I made the following statement:

> *Is it accurate to say that there are 3 generations of Higgs particles, just as there are 3 flavours of quark and lepton? Not likely. The Higgs particle breaks the pattern, since it is a boson. Therefore, we have no reason to suggest that there are 3 generations of it. But that does not mean there are no generations of it. After careful considerations of the numbers, I believe that there may be two flavours of these particles, at most.*

At the time, I made this statement I was unaware that a second and third generation of the Higgs had already been entirely ruled out by experiment [9]. Had I paid more attention to the data in Table 11, for example, I would have seen that there is obviously no extra flavours of Higgs and Graviton, as the triple symmetry of the quarks and leptons does not appear in H/G sector.

The reason for my error was in attributing too much significance to the summation of the Fibonacci dimensions; 2, 4, 6, 10. The number 22 is still important for other reasons, however. For instance, it was the deciding factor that made me realise the importance of Dark Matter (DM) and its place in the Universe. Since then, I have factored DM into the Sedenion and the Rhombic Dodecahedron Standard Model. [11, 12] In doing so, I have made these two models infinitely richer and more symmetric. A full explanation of why the number 22 might relate to DM will be presented in an upcoming paper.

5. Conclusions

The DGO Standard Model and its related Sedenion structure reveals an elaborate and perplexing relationship to the Fibonacci sequence. It shows that dimensionality of quantum systems expands via the Golden Ratio, something which has been proposed with the Fibonacci anyons, under similar but different circumstances. The existence of these anyons appears to be predicted in some way by the DGO Standard Model Fibonacci sequence. As was the expansion of the Quaternionic Standard Model out to the Nonions and eventually the Sedenions.

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