The Analysis and the Measurement of Poverty: An Interval-Based Composite Indicator Approach

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Abstract

The analysis and measurement of poverty is a crucial and unsolved issue in the field of social science. This work aims to measure poverty as a multidimensional notion using a new composite indicator. However, subjective choices as different weighting schemes on the indicator's construction could affect their interpretation and policy. It is necessary to consider the possible weighting configurations randomly to overcome this problem, and it is proposed in this work as interval-based composite indicators based on the results. This work aims to obtain robust and reliable measures based on a relevant conceptual model of poverty we have identified, considering various factors as weightings. Methodologically speaking, it is proposed an original procedure for measuring poverty in which it is computed a different composite indicator for each simulated weighting scheme of the identified factors. The weighting scheme in the Monte-Carlo simulation randomly creates an interval-based composite indicator based on the results. The different intervals are compared using different criteria (upper bound, center, and lower bound), and various rankings help analyze extreme scenarios and policy hypotheses. Critical situations are identified in Sicilia, Calabria, Campania and Puglia. The results demonstrate a relevant and consistent indicator measurement and the shadow sector's relevant impact on the final measures.

Keywords: Poverty, composite indicators, interval data, interval-based composite indicators, symbolic data
JEL codes: C02, C15, C43, I3, I32

1. Introduction: Measuring poverty

Poverty measurement and, in general, poverty is a fundamental theme in Social Science literature. Poverty is an actual refutation of human rights because it determines the impossibility of covering relevant expenses. Simultaneously, the relationship with well-being is more complicated because of both concepts' multidimensional structures ([1]). Therefore, the measurement and monitoring of poverty are nowadays fundamental. At a macro level, poverty and inequality can impact modern societies in the long run. In literature, poverty and well-being are usually associated, and they are fundamental concepts to understand. In particular, poverty gravely affects a person's well-being (household, children, migrants). Structural poverty can lead to an erosion of the basements on which the societies are born. At the same time, poverty can impact people's lives and be a problem for the institutions (it is necessary to think about the effects of poverty on children).
Poverty can eradicate institutions, and institutions can hurt poverty by creating obstacles to access to income. In this regard exists a specific causal loop that can become deleterious (see [2]). The United 2030 Agenda and the Sustainable Development Goals (SDGs) have created structured monitoring systems adopted in different countries ([3]). In this context, it is necessary to investigate the phenomenon, its determinants, and the possibilities of policy intervention. Simultaneously, identifying poverty states is very relevant and needs adequate methodologies ([4]). In literature, another crucial problem is identifying the areas in which the poor live and design adequate intervention policies.
It is essential to measure the phenomenon and consider approaches to evaluate poverty size and then localize poverty situations using spatial data to design interventions. It is necessary to understand, in that regard, which the poor are and their geographical location. One of the most important aims is to design and evaluate adequate anti-poverty policies.

At this point, the adoption of adequate quantitative methodologies is necessary. These methodologies can detect and monitor poverty over space and space. Modern data richness calls for approaches that consider synthesizing a group of indicators for the selected statistical units. Nowadays, in data richness, we can consider the situation we have in many different data cases. We can manage these data in order to measure the construct adequately. In this context, there are many different problems. The synthesis of indicators comprehends different phases ([5] and also [6]).
One relevant problem is to manage the different numbers of indicators (2018 [3]). In this sense, we need an approach to synthesize the different indicators. In that respect, income is considered the most significant predictor of economic status and well-being ([7]). At the same time, income is not a unique indicator to measure poverty. Therefore, exist at the same relevant approaches as the construction of composite indicators, which can be considered in order to measure poverty correctly ([8] [9]). So, in this case, it is necessary to consider different characteristics that can adequately consider other relevant aspects that can be useful to measure poverty. In this way, there is also the use of many different approaches to measuring poverty using different methodologies (using population census, administrative data, household surveys see [10]). A composite indicator arises in this context: synthesize and combine different indicators useful to measure poverty. Composite indicators are generated by constructing a linear weighted function of a combined normalized sub-indicator ([11], [12]). One relevant problem usually is that the different approaches can lead to different results, so it is usually necessary to provide sensitivity analysis and robustness checks of different approaches. These methodologies are addressed to analyze the results' sensitivity to the different methodological choices ([11]). Our proposal aims different: the composite indicator's uncertainty is directly measured by considering the critical factors impacting the indicator variability. In this respect, the possible changes to the indicator results are simultaneously considered considering the different identified factors. So it is identified the crucial varying factors (for instance, the composite indicator). By considering a Monte-Carlo simulation, it is finally obtained the interval-based composite indicator considering all the different results simultaneously. A Monte-Carlo simulation is necessary, where there are many sources of uncertainty as usual in composite indicator construction to internalize the different effects of the different assumptions (i.e., different weights). In this sense, the interval comprehends all the results of the different composite indicators simulated.
A different approach has been proposed in Polish literature. Simulation studies, in this case, are allowing the alignment of linear ordering (ranking) of the set of objects via composite indicators values, procedures (the procedure takes into account weights of variables, selected normalization methods, and selected constructions of aggregation measures), from the perspective of determining the correctness (quality) of aggregated measures, were conducted in [13] and [14].
So, in the end, in this work, it is obtained not a unique measure but an interval, and are compared no single values but the entire intervals. Then, of course, it is possible to interpret the different intervals adequately. The specific aim of the study is to provide a composite interval indicator that measures both a measure that synthesizes different indicators to measure poverty and takes into account the variability of the different results due to different choices in the construction of the same composite indicator. The use of aggregate measures based on interval data can be found in the works [15] and [16].
The second section starts by considering the different ways to measure poverty; the composite indicators are how to approach one of the most frequent ways to measure poverty. The third section is departed from the concept of composite indicators to describe the interval-based composite indicator's approach. The fourth section describes the data we have used, and finally, in the last section, the obtained in terms of interval-based composite indicators of poverty in Italy.

2. Measuring Multidimensional Poverty: a Literature Review

One relevant approach in measuring poverty is to use and consider groups of indicators to synthesize these indicators. Following [3], this synthesis problem is fundamental to social indicators literature. Therefore, the same authors consider a multivariate approach to measuring multidimensional poverty and well-being analysis. At the same time, they consider the different indicators' synthesis to monitor the different outcomes obtained. From this perspective, the authors' main contribution is that the general level of well-being that the different persons can reach can be linked with the level of substitutability of the different dimensions obtained ([3]).
It is widely accepted today that poverty, for its nature, is a multidimensional concept ([17] and also [18], [19] [20]). See about the multidimensional measurement of poverty [21]. The fundamental point is that poverty is

a multidimensional concept. The multidimensional poverty is analyzed as a concept and the measurement methodologies by [22]. If poverty is a multidimensional concept, it is necessary to analyze them by considering adequate methodologies. Multidimensional poverty calls naturally for the use of good indicators ([21]). Simultaneously, the problem to consider is that by considering different approaches and different methodologies. So it is necessary to consider the different subjectivity of the choices (the choices of weightings in composite indicators see [6]), which can lead to different results.

So in this context, relevant sets of indicators need to be considered. The different indicators, which explicitly characterize poverty as a concept, need to consider the essential dimensions like income needs and capabilities. It is essential, too, at the same time, to use other indicators related to the framework and the living situation ([23]).

Another different approach is the one followed in [24], here the authors consider a composite indicator to measure multidimensional child poverty. In this case, the multidimensional approach considers the complexity of the poverty phenomenon by considering different aspects that can be combined to provide a unique measure. Thus, this measure can be considered a synthesis. All these measures are significant because they allow us to consider a measurement used to evaluate policies and programs explicitly.

Also, [25] raises the problem of considering a multidimensional and longitudinal perspective on measuring poverty. In this sense, it is the idea to measure the concept of poverty. The novelty introduced by the author is considering the dimension of time. At the same time, the author concludes that the weighting of the "social capital" and the weight for health can have a higher impact over time.

There are also some elements of uncertainty on collecting the correct variables which can be considered. For instance, it is necessary to collect the income as an essential variable on measuring poverty as a specific part of the surveys considered ([7]). Therefore, one of the most relevant approaches to measure poverty is composite indicators. The composite indicators depart from the use of different indicators to provide a synthesis of the same indicators. It is considered the construction of the composite indicators in the following section.

At the same time, uncertainty and vagueness of the concepts could be significant. In that regard, poverty measurement needs to consider the fuzzy logic (see in this context [26]). This approach to measuring multidimensional poverty is also considered by [27], which uses fuzzy sets at the same time [28] and [29]. These approaches show that exist many different dimensions to be carefully checked and considered on building composite indicators.

Composite indicators are a relevant and consistent way to measure poverty. The usual approach is described in different works. The different indicators need to be synthesized, and it is necessary to consider different phases. (see in this respect [3], [5], [6]). The different methodology selects the indicators and then uses them to synthesize the underlying concept and measure the latent variable. Following [3], the synthesis of different indicators allows monitoring specific outcomes of the considered statistical units.

In [21] are reviewed the different quantitative methodologies used to construct indicators on the multidimensional poverty context. The suggestion in [21] is that author is that the use of multivariate methodologies (principal component analysis), for example, can be an advantage for the choice of the specific weights used.

Different approaches in the measurement of the multidimensional poverty measurement are also in [30]. An alternative approach to analyzing the measurement of multidimensional poverty is by [31]. Various approaches were proposed in this context. For instance, [32] consider the composite indicator approach to measure poverty. Their approach is based on the penalty of the geographical areas characterized by single "unbalanced" statistical units. To approximate the different variables in composite poverty indicators, we can usually follow the procedure, which leads to constructing a composite indicator ([6]). The different phases can be considered selecting the different indicators used, the aggregation method's choice, and the considered scaling of the different indicators used. In the end, we can obtain the considered latent variable as a component indicator. From the selection of the different indicators, we can scale the different indicators using various methodologies. Usually, we consider an aggregation function and a weighting scheme relevant to defining the single indicator's relevance or importance on the composite indicator created. Finally, it is possible to compare the different results obtained by the composite indicator constructed, and finally, it is possible to obtain a rank of the different values.

Here, the critical point is that the composite indicators' different components can hide relevant policy messages (see [33]). In this respect exists many different critical points. Several choices, for example, the weighting of the composite indicator, are based on subjective choices. Nevertheless, simultaneously, different choices can have a specific impact on the different results in terms of ranking. For this reason, robustness analysis and

sensitivity analysis are usually followed by various analyses in which different approaches are considered and compared to evaluate the impact of each approach on the final results.
This analysis is usually performed by considering the different impacts on the rankings ([6]). The approach we will consider is different. It is based on the interval data by taking into account simultaneously many different random measurements in which we try to cover all the meaningful options. In this case, sensitivity analysis could be fundamental to assess the different approaches and assumptions ([11]). The approach we will present in the next section allows us to "endogenize" the sensitivity analysis on the structure of the composite indicator computed.

3. Methodology

3.1 Methodology: interval-based Composite Indicators

In that context, composite indicators' construction can be based on subjective choices ([34], [6]). These subjective choices (for instance, the composite indicator's weighting scheme see [6]) can lead to different results. The recent literature aims to construct composite indicators, which can avoid the subjectivity of considering an assumption or a different one. So the target is to measure the different impacts of the suitable choices for the construction of the indicator (see [35]).
Uncertainty techniques can be considered in this respect because they can measure uncertainty on constructing the composite indicator (for instance, using probabilistic rankings). See for a discussion [6], [11], and [6]. So, the idea is to consider some robustness checks and sensitivity analysis by considering different assumptions to evaluate its robustness. This work aims to internalize this robustness by considering the interval of possible results, which can be obtained by varying the composite indicator's assumptions (see [36] [37] [38] and [39]). In that way, it is essential to define the "model" for the composite indicators initially. Then, it is essential to declare the different factors that lead to the composite indicator variability. From the model, it is possible to identify the different internal sources of variability on the construction of the composite indicator, which leads to the uncertainty of the outcome.
At this point, it is possible to consider several replications of the composite indicator considered by taking into account different combinations of the assumptions given. At every stage, a different combination of assumptions is sampled, and a different outcome is computed. Then, they explicitly consider an interval of all the possible obtained composite indicators by considering different combinations of assumptions on the composite indicator. Finally, the different results are collected, and they can be represented utilizing interval data ([40], [41]). Thus, interval data can be used to represents uncertainty ([42]) and, in general, composite phenomena (for instance, in [43], [44], [45] statistical units characterized by different statistical features).
Our approach innovates the use of the interval proposing an approach which "internalize" the uncertainty analysis proposed in the literature ([11]), allowing us to directly measure the variability of the different assumptions used on the construction of the composite indicators using random weights in the Monte Carlo Simulation but also simulating different indicator structures. In this respect, the subjectivity of the weightings can be solved. It is also possible to obtain more consistent public policies that also consider the variability of the different composite indicator choices. Furthermore, using the intervals, it is possible better design policies because it is possible better measure the uncertainty related to a different situation, for instance, measured by a composite indicator. The results of this work are beneficial for all who are interested in the construction and the use of composite indicators so: analysts, policy analysts, economic and social researchers, and of course, policy-makers. The entire approach is described and visualized in table 1 and figure 1.

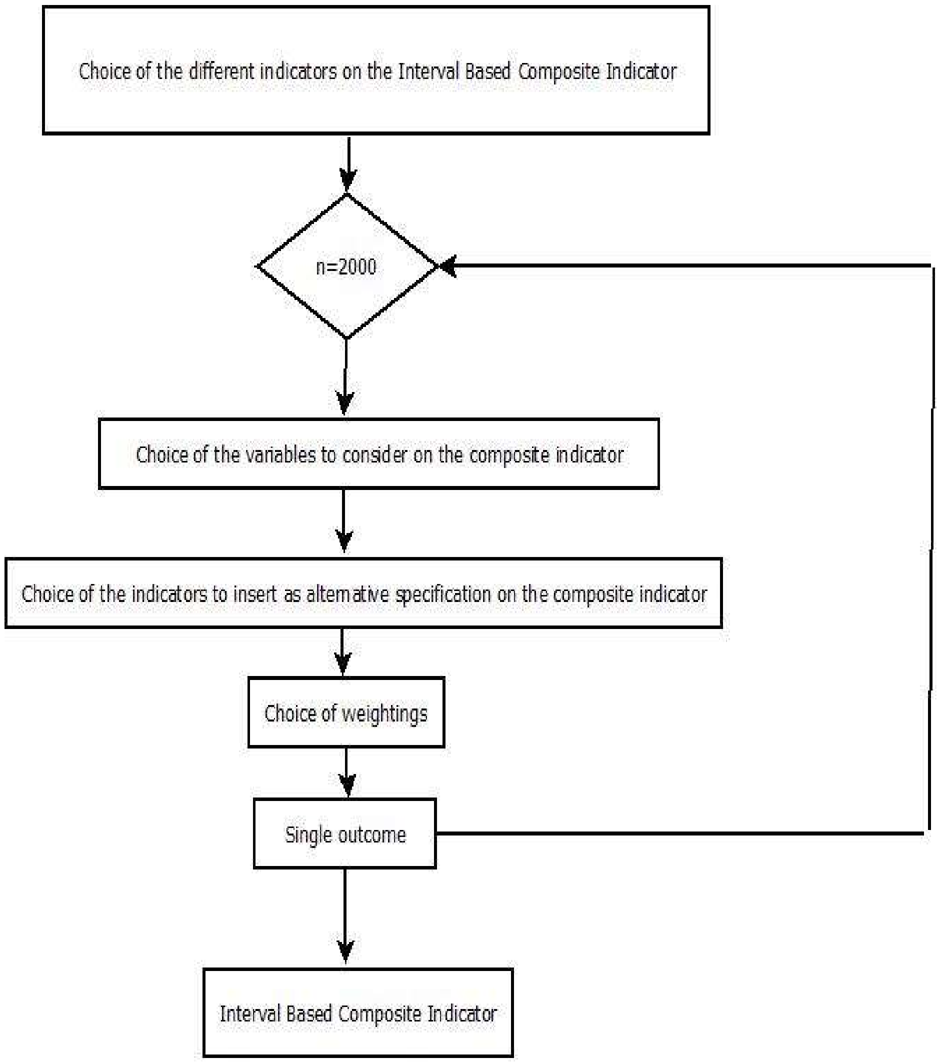

Figure 1. Flow chart of the procedure.

It is essential to note that our final result is interval data and not scalar data. In this logic, the interval allows to measure the uncertainty explicitly and permits to obtain a unique measure of the composite indicator ([46]). Moreover, the interval data own a specific algebra that allows different computations between intervals ([47] and [46]) and statistical analyses (Gioia Lauro 2005 [48] and Lauro Palumbo 2000 [49]).
So in this respect, it is started to considering n number of different composite indicators with $n = 1, \dots N$ (they contribute to the creation of the interval), computed by random combinations of factors ([50] and [51]). Then it is built each interval based composite indicator by having:

$$I[X]^c = \left[\underline{X^c}, \overline{X^c}\right]. \tag{1}$$

Where c is the considered, measured phenomenon to measure with the indicator X for $c = 1 \dots C$
From the composite interval, indicator obtained it is possible to compute the center

$$X^c_{center} = \frac{1}{2}\left(\underline{X^c} + \overline{X^c}\right). \tag{2}$$

Furthermore, the range or the width obtained:

$$X^c_{range} = \overline{X^c} - \underline{X^c}, \tag{3}$$

and finally, the radius

$$X^c_{radius} = \frac{1}{2}\left(\overline{X^c} - \underline{X^c}\right). \tag{4}$$

The range and the width represent the variability for the considered interval composite indicators considered ([47]). The parameters on which it is conducted the ranking analysis computed for the different intervals are the center, the minimum, the maximum, and the range ([52] and [53]). In order to measure the uncertainty, it is possible to consider the difference between the upper and the lower bound of the computed interval (see also [54])

Finally, it is possible to analyze at the same time the prototype (an average interval) using interval arithmetic. The interval arithmetic and the capacity to handle these composite indicators as intervals allow different advantages. First, they represent a more robust version of a classical composite indicator (based on a single value) and consider the internal variability. This one is determined by the various composite indicators' different performances on the same conceptual "model" ([6]). Finally, they can be used and considered in comparisons as a scalar (it is possible to use, for instance, the center) or genuinely as intervals (considering center, minima, and maxima). In this case, it is possible to use analytical approaches such as interval arithmetic to evaluate, for instance, a prototype (the statistical average of the different interval-based composite indicators). These interval-based composite indicators can contain a higher quantity of information so that the decision could be based on a more precise evaluation.

3.2 Methodology and Data

In the first step, we have to define the composite indicator model. The model is given by taking into account the following choices:

1). The essential variables to be considered on the composite indicator
2). The significant number on the total to be considered
3). The relevant aggregation function
4). The weights applied on the composite indicator

We consider the following variables for our composite interval indicator:

1. Families who live under a level of absolute poverty
2. An index of economic difficulty
3. Social exclusion
4. Material deprivation
5. Low labor intensity
6. Income

All these data come from the ASVIS database, and it is considered a unique source. The date for each variable is 31/12/2016. The different indicators on their original name and their name are defined in table 1. Each indicator is considered statistical units in the Italian regions (for the year 2016).

Table 1. Indicators considered.

Indicators considered and their reference date on the ASVIS database
Percentage of families living below the threshold of absolute poverty (31/12/2016)-Sotpovas
Index of great Economic difficulty (31/12/2016)-Diffeco
Percentage of population living in poverty or social exclusion (31/12/2016)-poves
index of severe material deprivation (31/12/2016)-depriv
Percentage of individuals in low working-intensity households (31/12/2016)-Basintlav
Percentage of people who live in households with an equivalent disposable income, less than 60% of median income (31/12/2016)-reddmed

In table 2, we compute the descriptives for each variable to evaluate the presence of some outliers on the data

Table 2. Descriptive Statistics of the indicators considered.

	sotpovas	diffeco	poves	depriv	basintlav	reddmed
Min.	3.580	3.90	16.10	5.00	6.10	8.90
1st Qu.	5.755	7.25	20.10	6.75	8.35	13.75
Median	10.370	8.80	24.40	9.40	9.90	16.00
Mean	12.434	11.29	30.28	11.39	12.74	20.84
3rd Qu.	16.320	14.80	39.00	14.55	16.70	27.55
Max.	34.940	21.60	55.60	26.10	26.70	41.80

We choose the dataset related to 2016 in order to ensure the reliability of the data considered. Data reliability is a significant issue (see, for instance, [55]). In this sense, these six variables are the most relevant we can consider for our model. In that way, these variables are considered the most significant in the framework we are explicitly considering. So in this sense, it is possible to proceed with the data analysis to evaluate our initial indicators and the structure of the indicators used as components or factors of the interval-based composite indicator. All variables are de stimulants in the study, but this is not usually the case in other studies. The stimulants and destimulants ([56]) are factors that also have a positive or negative effect on the considered phenomenon were introduced in [57]. These definitions can also be found in [58]. Other authors ([59] and [60]) use the terms 'positive polarity' and 'negative polarity' instead of the concept of stimulant and destimulant. They are considered some descriptive analysis of our data. In this respect, we are exploring our variables by observing if some situations request special attention (for instance, significant outliers). In this vein, it is possible to compute the descriptive statistics for the variables and examine the critical structure of the data we can observe. Then it is possible to consider the correlation matrices of the variables. In particular, the correlation matrix can be usefully considered and visualized as a network with a specific threshold. These are relevant in practice because we can think of different specific weighting schemes not to weigh many variables that show a high correlation. In extreme cases, the approach can be made not to use these indicators.

So in this respect, it is necessary to evaluate our choices primarily, and we are considered the correlation networks of the different variables to avoid select variables showing more relevant correlation problems eventually.

The network of variables is analyzed by considering several thresholds in order to show the data structure. Therefore, it is possible at this point to define at this point, our model of composite interval indicator by considering these specific factors (for the terminology in the composite indicators, see [6]):

1. The indicator choice
2. The number of the indicator choice on the total number of indicators considered (in this respect, we can explore alternative configurations of the composite indicator)
3. The different weightings

At the same time, we normalize each indicator by providing standardization for each of them, and we aggregate the different indicators by obtaining the outcome. The algorithm is described in figure 1. In the figure, it is described the construction of the interval-based composite indicator. First, it is necessary to choose the variables to be considered entirely for constructing the composite indicator (a set of feasible indicators to consider for the construction of the indicator). Then, to consider the uncertainty related to the construction of the composite indicator, it is considered a set of possible different random specifications. In this sense, they simulated 2000 different composite indicators by choosing a different combination of the variables considered and weights. So it is obtained a set of different composite indicators, and then the final interval. In the end, the intervals are estimated using 2000 simulations defined "a priori" as sufficient to estimate the intervals for each region.

It is also considered a choice of the relevant number of variables on the composite indicator. They are used for different indicators on the total consideration to evaluate different measurement approaches in constructing the poverty measure. In this respect, different results are obtained due to the variability of the different measures (there can be a not strong association between the different indicators so that some regions can perform better in some indicators than in others).

These characteristics can vary during the process of construction of the interval-based composite indicator. However, other elements on the construction of the composite indicator do not vary. For instance, it does not vary the standardization of the different variables. At the same time, it is not considered any outlier detection and missing imputation (in our case, there are no missing data detected on the analysis).

At this point, it is defined the computation of different parameters for the interval composite indicators: a measure for the minimum, a measure for the maximum, and also two-measure, which can be differently interpreted as a center and a radius. It is possible to note that the composite indicator's outcome comprehends a ranking for the minimum, the maximum, the center, and the radius. Therefore, the composite indicator can be interpreted as continuous. Furthermore, interval arithmetic makes it possible to compute the different prototypes (the interval average, which can be helpful as a benchmark).

At this point, it is possible to compute a different composite indicator by considering the random selection of a particular combination from the feasible initial indicators chosen. In this sense, our Monte Carlo simulation considers four factors out of 6 with a random weight (we obtain different composite indicators by considering both the components and their weights). Several 2,000 unique composite indicators are obtained based on the method mentioned above, and finally, an interval is quantified. We computed the interval data representing the different poverty measurements using these results by considering the defined model. In order to avoid outliers and provide a robust version of the interval, it is considered the quantile 0.10 to be the minimum and the quantile 0.90 to be the maximum. The different rankings are obtained by taking into account the different characteristics of the interval data: the minimum, the center, the maximum, and also the range. In the end, it is obtained a different ranking that can take into account the alternative scenarios.

The interpretation of the center (or mid-point) and the range (or width) is essential. In this respect, it is possible to interpret the center as the "result" of the composite interval indicator, which is comparable to the most probable scenario (in this way, compared to the classical composite indicator analysis, the center could be used). In order to compare the center, the same composite indicator is computed using the equal weights scenario (table 3). The interval range simultaneously is essential because it shows a critical difference in the results between different composite indicators. It is also possible to observe some scenarios producing relevant results when significant differences exist between the different indicators used to construct the composite indicator.

4. Results

In order to analyze the results accurately, it is essential to interpret the different composite interval indicators computed. Therefore, the findings related to the original interval computed with minimum and maximum defined are shown and visualized in Table 3, where there also is the computation of the center and the radius.

Table 3. Interval Based Composite Indicators: minimum, center and maximum (ranked for center), center and range (ranked for range), a classical composite indicator using the equal weights scenario

R	Region	Mi	Ce	Ma	R	Region	Ce	Ra	R	Region	E
1	Sicilia	1.30	1.72	2.14	1	Calabria	1.35	1.16	1	Sicilia	1.76
2	Campania	1.28	1.59	1.89	2	Sardegna	0.73	1.06	2	Campania	1.6
3	Calabria	0.77	1.35	1.93	3	Sicilia	1.72	0.84	3	Calabria	1.31
4	Puglia	0.59	0.86	1.14	4	Molise	0.32	0.79	4	Puglia	0.87
5	Sardegna	0.20	0.73	1.25	5	Basilicata	0.71	0.66	5	Basilicata	0.72
6	Basilicata	0.39	0.71	1.04	6	Abruzzo	0.10	0.63	6	Sardegna	0.72
7	Molise	-0.07	0.32	0.71	7	Campania	1.59	0.62	7	Molise	0.38
8	Abruzzo	-0.22	0.10	0.41	8	Puglia	0.86	0.55	8	Abruzzo	0.08
9	Lazio	-0.37	-0.21	-0.05	9	Piemonte	-0.39	0.54	9	Lazio	-0.22
10	Piemonte	-0.67	-0.39	-0.12	10	Friuli-VG	-0.83	0.51	10	Piemonte	-0.42
11	Liguria	-0.56	-0.46	-0.36	11	Lazio	-0.21	0.32	11	Liguria	-0.48
12	Umbria	-0.60	-0.46	-0.33	12	Valle d'Aosta	-0.66	0.27	12	Umbria	-0.48
13	Marche	-0.68	-0.55	-0.42	13	Umbria	-0.46	0.27	13	Marche	-0.54
14	Valle d'Aosta	-0.80	-0.66	-0.52	14	Marche	-0.55	0.26	14	Valle d'Aosta	-0.66
15	Lombardia	-0.87	-0.77	-0.66	15	Veneto	-1.02	0.25	15	Lombardia	-0.78
16	Friuli-VG	-1.08	-0.83	-0.57	16	Toscana	-0.97	0.24	16	Friuli-VG	-0.84
17	Toscana	-1.09	-0.97	-0.85	17	Lombardia	-0.77	0.21	17	Toscana	-0.98
18	Emilia-Romagna	-1.12	-1.02	-0.92	18	Liguria	-0.46	0.20	18	Veneto	-1.01
19	Veneto	-1.15	-1.02	-0.90	19	Emilia-Romagna	-1.02	0.20	19	Emilia-Romagna	-1.03

After conducting the different comparisons between the countries, it is possible to observe that the data indicates that the center's intervals rankings obtained give similar results regardless of the equal weightings scenario (table 3). In particular, we can see that the first ranks tend to be similar. This result means that the results tend to be robust. An important observation could be that we can observe differences between the ranks computed by Sardegna and Basilicata (in this sense, the ranks are inverted). At the same time, it can be noted that the results for the range allow essential reflections on the variability of the results. In that respect, the interval range is substantial because it shows how the results vary considering different weightings or assumptions on the composite indicator's construction. In particular, it can be observed that there is an evident variation between the results due, for example, to the presence of the shadow sector on the different first ranked regions. Calabria, Sardinia, and Sicily show a higher range than other regions, which can be explained by the shadow sector's presence (see [61] and [62]).

By analyzing the center's table of values, the minimum, and the maximum, we can observe that Sicily has the center's highest value, followed by the Campania and the Calabria. It can then be possible to observe a specific separation of the following regions: Calabria, Puglia, Sardegna, and Basilicata. The regions that perform well are Italy's northern regions, such as Veneto, Emilia-Romagna, Toscana, Friuli Venezia Giulia, and Lombardia. Assuming that the minimum for each region is considered, the conclusion does not change. It is possible to find the exact ranking between the interval center and the other descriptors of the interval as the minimum and the maximum. There are some relevant changes on Basilicata's ranking, which perform at a minimum better than Sardegna. The lowest observations' rankings tend to be the same for the center, considering the worst regions. Based on the results, in this respect, the conclusions can be considered robust. Robust means that we can observe jointly that the first interval tends to have higher values than the other one for the first ranks.

Calabria and Sardegna are ranked first and second, respectively, which indicates that they both perform slightly differently on the maximum ranking. Emilia-Romagna loses a position than Veneto by considering the lowest-performing poverty regions, but the situation remains stable overall considering the highest performing regions in poverty.

The results are consistent with the range of the intervals observed. The interval range is computed considering the difference between the maximum and the minimum and measuring the variation level. Interestingly, Calabria, Sardegna, Sicilia, and Molise show the highest range between the minimum and the maximum computed. On the other hand, Toscana, Lombardia, Liguria, and Emilia-Romagna show the lowest results obtained. The key findings are that the results depend on the shadow sector's presence; some variables are better or worse depending on whether the shadow sector is present.

The variance considers all the different components of the composite indicator and shows important values for regions with a high range. In particular, when there is a higher variance between the original variables, in this case, it is possible to obtain the meaningful radii, which in this case can be interpreted with a different performance on the indicator by using specific groups of variables than other groups of variables. In this regard, the shadow market is an essential factor. The index of great economic difficulty is slightly higher and the other variables.

The results are essential in that they allow for the identification and measurement of poverty in Italy. Simultaneously, some regions with very high interval values are observed to have a very high center, which can be interpreted as a reason for paying particular attention to these situations (Sicilia Campania and Calabria and also Puglia performing better). On the one hand, however, some different statistical variables make it possible to obtain significant differences with the single composite indicators' results between the different regions. So, the interval variability can be determined by considering the different variables that characterize the indicators, allowing different performances of the underlying composite indicators.

In this respect, it is possible to determine the range of the interval-based composite by the variance of the different factors it is constructed from.

5. Conclusions

In this work, the aim was to measure poverty in Italy consistently using interval data that allows using a Monte-Carlo Simulation on the different assumptions on which the composite indicator is constructed to explore the different results. The interval-based composite indicators show that the highest values of the social phenomenon studied, poverty, are obtained in Sicily, Campania, Calabria, and Puglia. At the same time, Calabria and Sardegna have a high value for the computed range (the difference between maxima and minima). These results improve the existing knowledge (see [63] and [64]) considering the equal weighting scenario, allowing to evaluate not only the result for a single scenario but allow to evaluate how the results vary considering different scenarios using different assumptions. In this sense, they have evaluated quantitatively the sensitivity of the different results considering different scenarios. Instantaneously, it is possible to observe the different impacts of the variables on the final composite indicator, which can be observed on the range of interval-based indicators. It is possible to measure poverty using an interval-based composite indicator, which combines and considers several different assumptions. This is a relevant innovation; using a Monte-Carlo Simulation, we can construct composite indicators considering various assumptions, such as weightings, can now be considered. In this case, a significant finding is also found that the range of the composite indicator (determined by the different performances on the variables considered for each region on the indicator) can allow the discovery of critical underlying and latent phenomena which can be discovered using this approach. In general, by considering the methodological findings and conclusions, these composite indicators obtain consistent results and consider many different assumptions to be less sensitive to subjective assessments. In particular, they can consider many different factors of variation on a composite indicator (for example, weigthings or different structures of the composite indicator) and consider the different combinations of factors identified in constructing the composite indicator. In the end, the uncertainty of the composite indicator can be "endogenized" and usefully compared.

The interval-based composite indicator's center indicates the final value of the composite indicator, which may differ from the value identified on a single scalar composite indicator. The lower bound (the minimum) and the upper bound (the maximum) can also be considered critical indicators of extreme scenarios that can be

usefully compared. At the same time, this is an important finding and result for policy-makers: minima and maxima allow to design of economic policies because there can be uncertainty on the results obtained. Policies can be considered in this sense these extreme scenarios as policy targets. In this sense, the policy aims to improve the minima representing some relevant territorial weaknesses.
Simultaneously, the range has a vital interpretation: it can identify significant differences in the different single indicators, further explored through a multivariate analysis. On the theoretical field of the subject, a significant result is that using these interval-based composite indicators, we can observe that the result of the indicator which uses poverty is also associated with the presence and size of the shadow sector. In this sense, we can observe a relevant association between a poverty indicator (synthesizing different variables related to poverty) and the size of the shadow sector.
Also, in this sense, final results are more reliable than other composite indicators and can be used for policy purposes. On the other hand, limitations of this work are related to the fact that the number of the variables can be increased, and it is possible to consider a more complex structure of composite indicators, which consider many different blocks of variables. In this sense, a possible future development can be the construction of interval-based composite indicators based on a different structure. At the same time, future will be considered approaches to define and increase the number of simulations to perform.
Another relevant point is the exploration of different ways to measure the intervals. In this respect, an attractive possible future development is to consider the intervals for computation robust central tendency (trimmed mean, median, Winsorized mean, Tukey's biweight mean) instead of simple interval measures. These approaches on the computation of the intervals allow handling extreme results for the simulated composite indicators. Different approaches on the robustification of the results are also proposed in [65]. The authors work on the interval of ranks instead of the composite indicator's original values where the extreme scenarios' analysis and the decomposition of the intervals using bi-clustering procedures are proposed in [66].
Finally, by considering the theoretical point of view, it could be essential to explore the relationships between these indicators' results and the shadow sector.

References

1. D'Ambrosio, C. (Ed.). (2018). Handbook of Research on Economic and Social Well-being. Edward Elgar Publishing.
2. Sindzingre, A. (2007). The multidimensionality of poverty: An institutionalist perspective. In The many dimensions of poverty (pp. 52-74). Palgrave Macmillan, London.
3. Mauro V, Biggeri M, Maggino F. Measuring and monitoring poverty and well-being: A new approach for the synthesis of multidimensionality. Social Indicators Research. 2018 Jan;135(1):75-89.
4. Alkire S, Foster J. Counting and multidimensional poverty measurement. Journal of public economics. 2011 Aug 1;95(7-8):476-87.
5. Maggino F. La misurazione dei fenomeni sociali attraverso indicatori statistici. Aspetti metodologici. Working Paper, Università degli Studi di Firenze, www. eprints. unifi. it; 2009.
6. Nardo M, Saisana M, Saltelli A, Tarantola S, Hoffman H, Giovannini E. Handbook on constructing composite indicators: methodology and user guide. Organisation for Economic Cooperation and Development (OECD). Statistics Working Paper JT00188147, OECD, France. 2005.
7. Hansen K, Kneale D. Does how you measure income make a difference to measuring poverty? Evidence from the UK. Social indicators research. 2013 Feb;110(3):1119-40.
8. Marlier E, Atkinson AB. Indicators of poverty and social exclusion in a global context. Journal of Policy Analysis and Management. 2010 Mar;29(2):285-304.
9. Atkinson AB, Marlier E. Human development and indicators of poverty and social exclusion as part of the policy process. Indian Journal of Human Development. 2011 Jul;5(2):293-320.
10. Baker J, Schuler N. Analyzing urban poverty: a summary of methods and approaches. The World Bank; 2004 Sep 2.
11. Saisana M, Saltelli A, Tarantola S. Uncertainty and sensitivity analysis techniques as tools for the quality assessment of composite indicators. Journal of the Royal Statistical Society: Series A (Statistics in Society). 2005 Mar;168(2):307-23.
12. Aiello F, Attanasio M. Some issues in constructing composite indicators. In VIII international meeting on quantitative methods for applied sciences, Certosa di Pontignano 2006 Sep (pp. 11-13).
13. Grabiński, T. Wielowymiarowa analiza porównawcza w badaniach dynamiki zjawisk ekonomicznych [Multivariate comparative analysis in research over the dynamics of economic phenomena], Zeszyty

Naukowe Akademii Ekonomicznej w Krakowie, Special series: Monografie No. 61, ISSN: 0209-1674 (1984).
14. Bąk, A. Modelowanie symulacyjne wybranych algorytmów wielowymiarowej analizy porównawczej w języku C++ [Simulation modeling of selected algorithms of multivariate comparative analysis with C++ language], Wrocław: Wydawnictwo Akademii Ekonomicznej we Wrocławiu, ISBN: 8370114016 (1999)
15. Dehnel, G., Walesiak, M. (2019), A comparative analysis of economic efficiency of medium-sized manufacturing enterprises in districts of Wielkopolska province using the hybrid approach with metric and interval-valued data, STATISTICS IN TRANSITION new series", June, Vol. 20, No. 2, pp. 49-67. https://doi.org/10.21307/stattrans-2019-014
16. Walesiak, M., Dehnel, G. (2020), The Measurement of Social Cohesion at Province Level in Poland Using Metric and Interval-Valued Data, "Sustainability", 12(18), 7664, 1-19. https://doi.org/10.3390/su12187664
17. Anand S, Sen A. Concepts or human development and poverty! A multidimensional perspective. United Nations Development Programme, Poverty and human development: Human development papers. 1997:1-20.
18. Sen A. Poverty and famines: an essay on entitlement and deprivation. Oxford university press; 1982.
19. Sen A. The standard of living. Cambridge University Press; 1988 Dec 8.
20. Sen A. Inequality reexamined. Oxford University Press; 1992.
21. Asselin LM. Composite indicator of multidimensional poverty. Multidimensional Poverty Theory. 2002 Jun
22. Alkire S, Roche JM, Ballon P, Foster J, Santos ME, Seth S. Multidimensional poverty measurement and analysis. Oxford University Press, USA; 2015.
23. Lok-Dessallien R. Review of poverty concepts and indicators. UNDP Soc Dev Poverty Elimin Div Poverty Reduct Ser from http://www. undp. orgpovertypublicationspovReview pdf. 1999;21.
24. Abdu M, Delamonica E. Multidimensional child poverty: From complex weighting to simple representation. Social Indicators Research. 2018 Apr;136(3):881-905.
25. Kim SG. What have we called as "poverty"? A multidimensional and longitudinal perspective. Social Indicators Research. 2016 Oct;129(1):229-76.
26. Cerioli A, Zani S. rA fuzzy approach to the measurement of poverty, s in Income and Wealth Distribution, Inequality and Poverty, ed. by C. Dagum, and M. Zenga, pp. 272t284. Springer&Verlag, Berlin. 1990.
27. Lemmi AA, Betti G, editors. Fuzzy set approach to multidimensional poverty measurement. Springer Science & Business Media; 2006 Dec 6.
28. Costa M, De Angelis L. The multidimensional measurement of poverty: a fuzzy set approach. Statistica. 2008 Dec 31;68(3/4):303-19.
29. Mussard S, Noel PA. Multidimensional decomposition of poverty: a fuzzy set approach. 2005.
30. Kakwani, N., & Silber, J. (Eds.). Quantitative approaches to multidimensional poverty measurement. Springer. (2008)
31. Bourguignon, F., Chakravarty, S.R. The Measurement of Multidimensional Poverty. The Journal of Economic Inequality 1, 25–49 (2003). https://doi.org/10.1023/A:1023913831342
32. De Muro P, Mazziotta M, Pareto A. Composite indices of development and poverty: An application to MDGs. Social indicators research. 2011 Oct 1;104(1):1-8.
33. Nyiwul L, Selim TH. Poverty As Social Deprivation: A Survey. Review of Social Economy. 2006 Feb 1;64(2).
34. Becker W, Saisana M, Paruolo P, Vandecasteele I. Weights and importance in composite indicators: Closing the gap. Ecological indicators. 2017 Sep 1;80:12-22.
35. Paruolo P, Saisana M, Saltelli A. Ratings and rankings: voodoo or science?. Journal of the Royal Statistical Society: Series A (Statistics in Society). 2013 Jun;176(3):609-34.
36. Drago, C. (2017) Interval Based Composite Indicators. FEEM Working Paper No. 42.2017. Available at SSRN: https://ssrn.com/abstract=3038751 or http://dx.doi.org/10.2139/ssrn.3038751
37. Drago C. Il monitoraggio della domanda di reddito di cittadinanza in tempo reale facendo uso di Big Data: un'analisi basata su indicatori ad intervallo. V Convegno Nazionale dell'Associazione Italiana per gli Studi sulla Qualità della Vita-Fiesole (FI). 2018 Dec:13-5.
38. Gatto, A., & Drago, C. (2020). Measuring and modeling energy resilience. Ecological Economics, 172, 106527

39. Drago C. Gatto A. (2018) A robust approach to composite in dicators exploiting interval data: The Interval-Valued Global Gender Gap Index (IGGGI) Ipazia 4th Workshop on gender: Culture and gender issues 9 March 2018
40. Billard L, Diday E. From the statistics of data to the statistics of knowledge: symbolic data analysis. Journal of the American Statistical Association. 2003 Jun 1;98(462):470-87.
41. Billard L. Some analyses of interval data. Journal of computing and information technology. 2008 Dec 30;16(4):225-33.
42. Barclay M, Dixon-Woods M, Lyratzopoulos G. The problem with composite indicators. BMJ quality & safety. 2019 Apr 1;28(4):338-44.
43. Młodak A. On the construction of an aggregated measure of the development of interval data. Computational Statistics. 2014 Oct;29(5):895-929.
44. Fura B, Wojnar J, Kasprzyk B. Ranking and classification of EU countries regarding their levels of implementation of the Europe 2020 strategy. Journal of cleaner production. 2017 Nov 1;165:968-79.
45. Schang L, Hynninen Y, Morton A, Salo A. Developing robust composite measures of healthcare quality–Ranking intervals and dominance relations for Scottish Health Boards. Social Science & Medicine. 2016 Aug 1;162:59-67
46. Sunaga T. Theory of interval algebra and its application to numerical analysis. RAAG memoirs. 1958;2(29-46):209..
47. Moore RE. Methods and applications of interval analysis. Society for Industrial and Applied Mathematics; 1979 Jan 1.
48. Gioia F, Lauro CN. Basic statistical methods for interval data. Statistica applicata. 2005;17(1):75-104.
49. Lauro CN, Palumbo F. Principal component analysis of interval data: a symbolic data analysis approach. Computational statistics. 2000 Mar;15(1):73-87.
50. Saltelli A. (2016) Sensitivity Analysis: an Introduction. Presentation Summer School on Sensitivity Analysis SAMO 2016, Villa Orlandi, Anacapri, July 4-8, 2016
51. Saltelli A, Ratto M, Andres T, Campolongo F, Cariboni J, Gatelli D, Saisana M, Tarantola S. Global sensitivity analysis: the primer. John Wiley & Sons; 2008 Feb 28.
52. Mballo C, Diday E. Decision trees on interval valued variables. The electronic journal of symbolic data analysis. 2005;3(1):8-18.
53. Song P, Liang J, Qian Y. A two-grade approach to ranking interval data. Knowledge-Based Systems. 2012 Mar 1;27:234-44.
54. Grzegorzewski, P. (2018, September). Measures of Dispersion for Interval Data. In International Conference Series on Soft Methods in Probability and Statistics (pp. 91-98). Springer, Cham.
55. Kamanou, G., Ward, M., & Havinga, I. Chapter VI. Statistical Issues in Measuring Poverty from Non-Household Survey Sources, 2005
56. Kuc-Czarnecka, M., Piano, S. L., & Saltelli, A. (2020). Quantitative storytelling in the making of a composite indicator. Social Indicators Research, 1-28.
57. Hellwig, Z. (1972). Procedure of Evaluating High-Level Manpower Data and Typology of Countries by Means of the Taxonomic Method, [in:] Gostkowski Z. (ed.), Towards a system of Human Resources Indicators for Less Developed Countries, Papers Prepared for UNESCO Research Project, Ossolineum, The Polish Academy of Sciences Press, Wrocław, 115-134
58. Walesiak, M. (2018), The choice of normalization method and rankings of the set of objects based on composite indicator values, Statistics in Transition - new series, December, Vol. 19, No. 4, 693–710. https://doi.org/21307/stattrans-2018-036
59. Mazziotta, M., Pareto, A. (2016). On a generalized non-compensatory composite index for measuring socio-economic phenomena. Social indicators research, 127(3), 983-1003, https://doi.org/10.1007/s11205-015-0998-2
60. Mazziotta, M., Pareto, A. (2018). Measuring well-being over time: the adjusted Mazziotta–Pareto index versus other non-compensatory indices. Social Indicators Research, 136(3), 967-976, https://doi.org/10.1007/s11205-017-1577-5
61. Smith A. (2005) Lurking in the shadows – Italy's other economy, The Florentine June 9, 2005
62. Bovi, M, and Castellucci L. "What do we know about the size of the underground economy in Italy beyond the" common wisdom"? Some empirically tested propositions." (1999).
63. Stranges M. Poverty and social exclusion in the Italian regions: an attempt of measurement through a simple index. Dynamiques de Pauvretés Et Vulnérabilités en Démographie Et en Sciences Sociales: Actes de la Chaire Quetelet 2007. 2011:59.

64. Giuliano G, Raciti P, Tenaglia S. An Application of Multidimensional Poverty Indicator to Survey Data, 2020
65. Gatto A. Drago C. When Renewable Energy, Empowerment, and Entrepreneurship connect: Measuring Energy Policy Effectiveness in 230 Countries. Energy Research and Social Science Forthcoming (2021)
66. Drago, C., Decomposition of the Interval Based Composite Indicators by Means of Biclustering (September 2019). CLADAG 2019 12-th Scientific Meeting Classification and Data Analysis Group Cassino, September 11 – 13, 2019