Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 November 2020 doi:10.20944/preprints202011.0619.v1

Article

How to erase quantum monogamy?

Ghenadie Mardari[1,*]

[1] Open Worlds Research, Baltimore, Maryland, USA
* Correspondence: gmardari@gmail.com.

Abstract: The phenomenon of quantum erasure exposed a remarkable ambiguity in the interpretation of quantum entanglement. On the one hand, the data is compatible with the possibility of arrow-of-time violations. On the other hand, it is also possible that temporal non-locality is an artifact of post-selection. Twenty years later, this problem can be solved with a quantum monogamy experiment, in which four entangled quanta are measured at the same time. If Bell violations can be recovered from a "monogamous" quantum system, then the arrow of time is obeyed at the quantum level.

Keywords: quantum entanglement; delayed-choice experiments; quantum erasure; quantum monogamy; experiment proposal.

1. Introduction

Quantum entanglement allows for the manifestation of surprising phenomena, by studying the coincidences between pairs of detection events. Famous examples include non-local interference, ghost imaging and quantum teleportation, while more recently this has become an essential ingredient of quantum communication technologies [1-5]. Originally, quantum entanglement was seen as an affront to classical conceptions of space [6] (think "spooky action at a distance"). Yet, Wigner's delayed-choice experiment expanded this discussion to the dimension of time [7-8]. If quanta can be forced to change distribution patterns long after being recorded, does it mean that measurement correlations transcend the boundaries of time? Quantum erasure appeared to give a positive answer to this question [9-11]. If a signal beam is passed through an interferometer with distinguishable paths, the idler beam can be used to obtain path knowledge, making interference impossible. However, it is also possible to detect the idler in a way that "erases" path knowledge. This results in the display of interference fringes, even if the idler is detected long after the signal (and the signal event is already recorded).

While quantum erasure was compatible with novel interpretations, it did not fully close the door on classical approaches. On the one hand, the data was not uniquely compatible with measurement-based changes in individual quantum properties. In other words, it was not clear whether switching the method of detection of the idler actually controlled the *behavior* of the signal quanta, rather than the rules of their *post-selection*. If two interference patterns are superimposed out of phase, fringe visibility is "washed out", rather than "erased". On the other hand, theoretical considerations did not seem decisive either. Alternative joint quantum measurements obey the superposition principle, which means that every pairwise distribution corresponds to a component from a wide spectrum of possible outcomes. Therefore, it is not possible to conclude, on theoretical grounds alone, that two alternative measurements target the same subset of quanta (after post-selection). In order to settle this debate, the best strategy is to design an experiment that can falsify one of the two interpretations conclusively.

The problem with two-quantum entanglement is that every joint measurement requires a different experiment. Yet, sharp observations result in partial measurements, when the input projections have multi-mode profiles. Hence, changing the rules of observation can change the conditional probability of collapse. This is why it is difficult to verify the physical nature of temporal non-locality: is it real, or is it just an "as if" phenomenon? A tempting solution would be to use four-quantum entanglement, which allows for the observation of four variables in the same experiment [12-14]. Would it not be nice to see directly which pattern in the signal overlaps with which state of the idler? Unfortunately, this is a naive expectation, because two-quantum correlations do not persist in the four-quantum regime. For example, four-quantum coincidences cannot contain pairwise correlations in violation of the CHSH inequality, even though detecting just two quanta out of four is sufficient for such observations. This is known as "quantum monogamy" [15-17]. Because of the tendency of entangled quanta to display non-locality at the level of pairs, but not at the level of triplets or quadruplets, hypotheses about "actual mechanisms" for temporal non-locality appear to remain confined to the level of counterfactual analysis.

Notwithstanding, there is a way out of this conundrum, because the main concern in this debate is not whether quantum distributions agree with predictions, but whether clues about underlying mechanisms are available. What exactly is happening in the physical realm, when the predictions of quantum theory are confirmed? Unlike two-quantum records, four quantum data-plots can answer "how" questions. The crucial detail is that non-commuting quantum variables are superposed, meaning that every variable is defined as a spectral component of the input wave-function. Thus, it is not possible for individual quanta to generate events in every conceivable measurement setting, even with ideal detectors. The experiments are designed to isolate relevant components from complex spectra. As a consequence, the expected rates of two-quantum coincidence are smaller than the rates of single detection, but also larger than the rates of four-quantum coincidence. This makes it possible to verify: does quantum monogamy happen simply because four quanta are detected (rather than two), or does it happen because a smaller subset of the data plot is post-selected? In other words, does the relationship between two variables change because other parameters are observed (whether or not coincident events are registered), or does it only change for a narrow slice of the data record, in which entangled quanta are simultaneously detectable in sets of four? The answer to this question is beyond the scope of quantum theory, as it concerns undetectable events. Yet, the experimental data can reveal it beyond reasonable doubt. Therefore, it is essential to carry out monogamy erasure experiments, because they can finally elucidate the underlying principles for the analysis of quantum behavior, at least with regard to temporal non-locality.

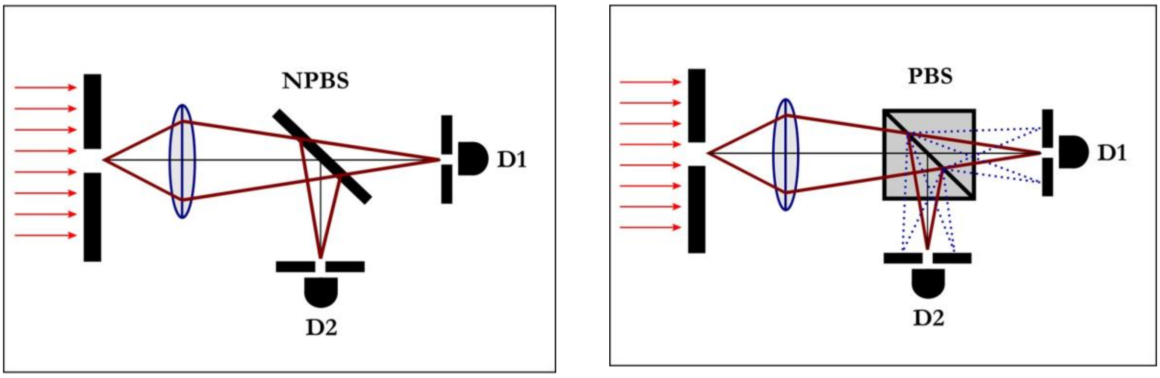
2. Background considerations

The main difference between quantum phenomena and classical phenomena is in the nature of superposition. For example, a classical depolarized optical beam is presumed to contain many "wave-packets", each of them in a sharp state of polarization (e.g., vertical, horizontal, and any value in-between). The effects of wave superposition are expected to manifest collectively, when all of these entities are able to propagate together. In contrast, quantum systems are expected to express the effects of superposition at the individual level. Every single photon is in a state of superposition for all the possible states of polarization at the same time. And yet, despite this important difference, every polarization vector is defined as a *component* of a wide spectrum, both in classical and quantum projections. A single quantum is expected to be in all the component states in superposition before the measurement, but it can only "collapse" to one component in a sharp observation. Moreover, the probability of expressing any given component is determined by the magnitude of the corresponding vector in the input state. This leads to an important question about the scope of "non-local" quantum measurements: do they aim to determine full input profiles, or merely to isolate individual components? In other others words, are we dealing with partial measurements or complete measurements, when EPR-type phenomena are predicted and verified?

A major concern for the interpretation of quantum behavior is the risk of importing classical principles and concepts, without proper justification. For the purpose of this discussion, the relevant issue is how to define the act of measurement. What is the difference between a classical observation, and a quantum observation? Is it merely a difference of degree (large numbers of simultaneous events *vs* one-at-a-time), or is there a qualitative difference? And, if there is a qualitative difference, does it just happen spontaneously because one "enters the quantum realm", or is the measurement scheme actually different in a substantial way? The get to the answer, consider the following. If it is

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 November 2020 doi:10.20944/preprints202011.0619.v1

possible to measure a classical system in many different ways, all of those potential outcomes are treated as alternative to each other (we can have *either* outcome A, *or* outcome B, *or* any other outcome). In contrast, if several quantum measurements over non-commuting spin ½ variables are possible, then they are treated as superposed with each other (*i.e.*, the system is in the vector sum of outcome A *and* outcome B, *and* any other possible outcome). In light of the correspondence principle, large numbers of quantum events should approximate classical behavior. Yet, there is no logical continuity between conjunction and disjunction. We cannot get one pattern from the other, merely by increasing the number of iterations. Therefore, the difference between classical and quantum measurements cannot be simply a matter of degree. There must be a qualitative difference between them, and this difference should be obvious in the corresponding scheme of detection.



(**a**) (**b**)

Figure 1. Spatial filtering of polarization components in optical arrangements. Input modes of polarization can suffer differential phase-shifts during propagation through birefringent media. If the axis of polarization is neither parallel to the fast axis nor to the slow axis of a polarizing beam-splitter, it will suffer a phase shift relative to the rest of the output projection. As a result, a single spatial mode can be transformed into a projection with several spatial modes, if it is not polarized. These modes can be resolved in the focal plane of a lens, in which case input polarization information is converted into output spatial information. **(a)** Optical modes are not perturbed by ideal non-polarizing beam-splitters (*e.g.*, a half-silvered mirror). Component states of polarization cannot be spatially resolved. **(b)** A single optical mode is transformed into multiple optical modes after passing through a polarizing beam-splitter. Individual modes can be isolated in the focal plane with a pinhole (or, even better, with the tip of a single-mode optical fiber). The input state of polarization is erased by the birefringent medium, but the relative magnitude of the corresponding spatial mode can be measured and used for post-processing.

In order to clarify this conclusion, consider a perfectly depolarized classical optical beam. In ideal conditions, the beam is likely to be split 50-50 by a polarizing beam-splitter, with negligible losses. By design, there are numerous input states of polarization, but only two output states. This means that a classical measurement of this sort is very coarse. It erases all the information about individual components, and only reveals the output of a transformation: how are the input components redistributed between the chosen orthogonal modes of polarization (*e.g.*, vertical and horizontal)? In contrast, quantum measurements are not meant to enforce transformations. They are designed to reveal the internal structure (*i.e.*, the spectrum) of input beams, in the form of a distribution. Similar to spatial distributions, where different measurements reveal the proportion of quanta arriving at different coordinates in the cross-section of a beam, polarization measurements expose the magnitude of individual components for any axis of interest. In practice, this is achieved by combining polarizing beam-splitters with spatial filters, as shown in Figure 1. Birefringent crystals have only two kinds of paths (slow and fast), but there are numerous incident states of polarization. Therefore, different polarization modes suffer different phase-shifts during propagation through this medium. Some components are slowed down and still exit the fast output, others are slowed down less and still exit the slow output. As a result, a single optical mode (with sharp focal point after passing through a lens)

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 24 November 2020 doi:10.20944/preprints202011.0619.v1

is converted into a multi-mode projection (with numerous focal points), such that every output spatial mode is correlated with a polarization mode. This makes it possible to isolate component states of polarization, either with a pinhole or with the tip of a single-mode optical fiber, carefully positioned in the focal plane of the lens. In short, quantum measurements are qualitatively different from classical measurements, because they target a few superposed components rather than complete input profiles. This is why, for example, quantum projections are amenable to analysis with non-Kolmogorov probability models (including negative probability values) [18-19], but classical projections are not.

In hindsight, the partial nature of spin ½ measurements could have been acknowledged from the inception of modern quantum theory. After all, it follows directly from the basic features of the superposition principle. If several measurement outcomes in superposition can be derived from the same wave-function, then every individual outcome should be treated as a component of a wider spectrum (like the color "red" in the rainbow spectrum). By definition, to be in superposition is to be a part of something. Nonetheless, the road to this realization was quite tortuous, and only recently it has become a topic for public discussion. A major obstacle for this development was the widespread (though unproven) belief that quantum theory predicts Bell violations for complete measurements. Thus, for a long time it wasn't clear which predictions of quantum theory are "non-local", and which of them are "loopholes" for the non-locality paradigm. An interesting example is the 1991 *PRL* article by Santos [20], in which he showed that single and joint quantum probabilities for complete output distributions require the following expressions, in realistic experiments:

$$p(u_1a_1) = \langle\psi|U_1A_1|\psi\rangle = \frac{\Omega}{8\pi}, \tag{1}$$

$$p(u_1a_1, u_2b_2) = \langle\psi|U_1A_1U_2B_2|\psi\rangle = \left(\frac{\Omega}{8\pi}\right)^2 a(\theta,\varphi)[1 + F(\theta,\varphi)cos2(a-b)]. \tag{2}$$

These rules do not entail Bell violations, unlike the following partial measurement probabilities:

$$p(a_1) = \langle\psi|A_1|\psi\rangle = \frac{1}{2}, \tag{3}$$

$$p(a_1,b_2) = \langle\psi|A_1B_2|\psi\rangle = \frac{1}{4}[1 + cos2(a-b)] \tag{4}$$

Indeed, the rules (3) and (4) are the ones that were originally used to derive Bell-type inequalities. Hence, we have a demonstration that Bell violations are only expected for partial measurements in quantum theory. However, Santos only meant this argument to work as a loophole, specific to experiments with atomic cascades [21]. Presumably, the expectation was that loophole-free experiments are still possible, in which quantum theory predicts Bell violations for complete measurements. However, numerous loophole-free experiments have been published since then, and they still exhibit the same structural relationships between measurements. For example, loophole-free experiments with photons [22-25] use the same combinations of beam-splitters and spatial filters (shown above) as the classical demonstrations [26-28]. Does it mean that loophole-free experiments are not loophole-free? No, the simple explanation is that quantum theory actually makes those predictions for partial measurements. In a recent breakthrough essay, Cetto and collaborators presented a rigorous analysis of the bipartite correlation function [29], showing that joint spin measurements are necessarily partial in Bell-type experiments. In particular, every coefficient of the CHSH inequality corresponds to a different probability sub-space:

$$\begin{aligned} C_Q(a,b) + C_Q(a,b') + C_Q(a',b) - C_Q(a',b') \\ = \sum_k \int_{\Lambda_k} \alpha_k\beta_k\rho d\lambda + \sum_l \int_{\Lambda_l} \alpha_l\beta'_l\rho d\lambda + \sum_m \int_{\Lambda_m} \alpha'_m\beta_m\rho d\lambda \\ - \sum_n \int_{\Lambda_n} \alpha'_n\beta'_n\rho d\lambda\,, \end{aligned} \tag{5}$$

 doi:10.20944/preprints202011.0619.v1

"where $\{\Lambda_k\}$, $\{\Lambda_l\}$, $\{\Lambda_m\}$,$\{\Lambda_n\}$ represent the four different partitionings of Λ corresponding to the four different pairs of vectors" [21]. In other words, these types of measurements are by default conditional, because every joint observation is expected to extract a different slice from the output projection. It is not a loophole for realistic quantum experiments to obtain Bell-violations with partial measurements, if this is a necessary feature of quantum theory.

This conclusion is highly relevant for the interpretation of quantum erasure, especially with regard to hypotheses about the arrow of time. A notable feature of the hallmark experiments that confirmed this phenomenon [10-11] is the ambiguous nature of observable patterns. "Erased" distributions emerge neatly as the sum of two interference patterns out-of-phase. Thus, it isn't clear if quantum erasure entails a physical transformation in the behavior of measured systems, or if it is a side-effect of changes in the rules of post-selection. A tempting solution is to blame this ambiguity on experimental imperfections, hoping for unambiguous results from "loophole-free" experiments. However, as shown above, the source of this confusion is not in the limitations of the means of measurement. The formalism of quantum theory is already ambiguous, because it is compatible with both types of interpretations: the ones in which the arrow-of-time is obeyed, and the ones in which it is not. Accordingly, it is not possible to settle existing debates about temporal non-locality with more refined confirmations of theoretical predictions. Instead, the solution is to devise experiments that can actually expose the underlying mechanism for such observations. The advantage of four-quantum entanglement, given the discovery of quantum monogamy, is precisely this ability to go "behind the scenes" and to finally verify the nature of quantum behavior in the temporal dimension.

3. Proposal

As far as EPR-type systems are concerned, quantum theory has been tested and confirmed beyond reasonable doubt. Verifying another phenomenon, even one as strange as quantum monogamy, would not be a revolutionary advancement. Likewise, performing a test of quantum erasure over quantum monogamy would be interesting, but not fundamentally relevant, in and of itself. Four measurements can reveal information about four variables, in which case monogamy should be observed. Or, four measurement can provide redundant information about two variables, in which case monogamy would be erased. The reasonable expectation is that quantum theory would be confirmed, but the interpretive ambiguities would remain, because of the partial nature of these measurements, as explained above. The real value of four-quantum entanglement is in the ability to go beyond mere predictions, and to look at the details that show *how* these results are obtained. The basic interpretive question, debated since the discovery of EPR states, is whether quantum behavior changes in one device, just because measurement settings are changed in another. In the case of monogamy, the coefficients of correlation between two streams of quanta are expected to change just because two (or even one) other streams are detected in parallel. Notably, these two dimensions (simultaneous detection vs coincidence post-selection) can be separated from each other in a four-quantum entanglement protocol.

#	A	B	C	D
1.	1	1	1	1
2.	X	0	0	X
3.	X	X	1	1
4.	1	X	X	0
5.	0	0	X	X
6.	0	0	0	0
7.	1	1	X	X
8.	X	1	0	1

Figure 2. Coincidence pattern in a four-quantum experiment. If a set of four entangled beams is measured in the same way, the rates of coincidence are expected to be constant for any number of events (2, 3 or 4). If each beam is measured for a different variable, as shown, then two-quantum coincidences are expected to outnumber four-quantum coincidences. With increasing measurement sharpness, the probability of collapse for more than one event diminishes. In this case, (A,B) coincidences can be seen in iterations 1, 5, 6 and 7, while (A,B,C,D) coincidences can only be found in iterations 1 and 6. This feature of quantum entanglement is the reason for interpretive ambiguities, as described in the text.

Let us consider an experimental set-up, in which four entangled quanta are used to study four different variables (*e.g.*, polarization states). For a suitable model, consider the original four-quantum demonstration of Pan and collaborators [14]. As it is known, the most frequent result in a set-up like this is the observation of single events (*i.e.*, only one detector out of four is triggered). If single-event iterations are ignored, because they do not exhibit coincidences, the pattern of observations is likely to be as shown in *Fig. 2*. The largest ratio of coincidences will involve two quanta out of four, while other detectors have missing events. A smaller ratio will contain three coincident events, and by far the smallest ratio will contain four coincident events. This is a natural hierarchy, given that quadruple events include double coincidences, but the reverse is not true. The essence of quantum monogamy is that Bell violations are possible, if two quanta are detected (and the other two are ignored). These violations disappear if four quanta are detected at the same time. However, the definition of four-quantum detection is limited to the *observable* aspect. In other words, quantum theory is *only* able to make predictions for the subset of iterations that generate quadruple coincidences. In contrast, non-local interpretations apply to all the quanta, not just the ones that are detected. The stated reason to believe in temporal non-locality is that one and the same quantum is likely to change the way it "collapses", depending on how another quantum is observed. This model can be tested by looking at all the iterations that generate coincident events for two variables in two experimental arrangements: (1) two quanta are detected and two are discarded *versus* (2) four quanta are detectable but only two streams are used to post-select coincident events. If experiment (1) produces coefficients of correlation in violation of the CHSH inequality, but experiment (2) does not, then temporal non-locality would be confirmed. This would be an example of physical change just because the other two quanta are detected (whether or not they are counted). Alternatively, if both experiments produce coefficients in violation of the CHSH inequality, then temporal non-locality would become interpretively superfluous. It would not add anything to the discussion about the nature of quantum behavior. All the features of quantum entanglement would be explained sufficiently by models with temporal locality.

5. Conclusion

It may seem surprising that quantum behavior is still debated today, given that quantum mechanics is the most accurately verified theory in the history of physics. Yet, there are good reasons for this. At the current stage of development of the quantum formalism, several interpretations are simultaneously valid, because of the partial nature of quantum measurements in relevant experiments. The real challenge is to explain what happens at the individual level, when a measurement choice is realized. A single quantum is in the superposition of all the states at the same time, before it is observed, but the chosen measurement is only sampling a slice of the full spectrum of outcomes (because the corresponding projections are also superposed with each other). In order to violate a Bell-type inequality, a quantum measured by Alice must generate different observations, depending on the measurement choice made by Bob for its entangled partner, at some future moment. Does the quantum really change its properties, or does it merely fail to produce a coincidence event in every scenario? It is not enough to confirm the predictions of quantum theory, in order to answer this question. The challenge is to go "behind the scenes" and to find out how quantum predictions are actually confirmed. It is a major breakthrough that four-quantum protocols are making it possible to find such answers and settle the decades-old debates on this topic. Monogamy erasure quantum experiments are likely to have profound effects on the interpretation of quantum mechanics and a strong impact on its future progress.

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

References

1. R. A. Bertlman and A. Zeilinger (eds), *Quantum [Un]speakables: From Bell to Quantum Information* (Springer, 2002).
2. A. Zeilinger, "Experiment and the foundations of quantum physics", *Rev. Mod. Phys.* **71**, S288 (1999).
3. Y. Shih, "Two-photon entanglement and quantum reality", *Advances in Atomic, Molecular, and Optical Physics* **41**, 1 (1999).
4. Y. Shih, "The physics of 2 is not 1+1", *The Western Ontario Series in Philosophy of Science*, **73**, 157 (2009).
5. G. Jaeger, *Quantum Information: An Overview* (Springer, 2007).
6. A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Phys. Rev.* **47,** 777 (1935)
7. J. A. Wheeler and W. H. Zurek (eds), *Quantum Theory and Measurement* (Princeton, 1983).
8. X. Ma, J. Kofler and A. Zeilinger, "Delayed-choice gedanken experiments and their realizations", *Rev. Mod. Phys.* **88**, 015005 (2016).
9. M. O. Scully and K. Drühl, "Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics", *Phys. Rev. A* **25**, 2208 (1982).
10. Yoon-Ho Kim, R. Yu, S. P. Kulik, Y. Shih and M. Scully, "A Delayed Choice Quantum Eraser", *Phys. Rev. Lett.* **84**, 1 (2000).
11. S. P. Walborn, M. O. Terra Cunha, S. Pádua and C. H. Monken, "Double-Slit Quantum Eraser", *Phys. Rev. A* **65**, 033818 (2002).
12. M. Eibl, S. Gaertner, M. Bourennane, C. Kurtsiefer, M. Żukowski and H. Weinfurter, "Experimental Observation of Four-Photon Entanglement from Parametric Down-Conversion", *Phys. Rev. Lett.* **90**, 200403 (2003).
13. J. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger, "Experimental Demonstration of Four-Photon Entanglement and High-Fidelity Teleportation", Phys. Rev. Lett. **86**, 4435 (2001).
14. J. Pan, Z. Chen, C. Lu, H. Weinfurter, A. Zeilinger and M. Żukowski, "Multiphoton entanglement and interferometry", *Rev. Mod. Phys.* **84**, 777 (2012).
15. V. Coffman, J. Kundu and W. K. Wootters, "Distributed entanglement", *Phys. Rev. A* **61**, 052306 (2000).
16. M. Koashi and A. Winter, "Monogamy of quantum entanglement and other correlations", *Phys. Rev. A* **69**, 022309 (2004).

 doi:10.20944/preprints202011.0619.v1

17. T. J. Osborne and F. Verstraete, “General Monogamy Inequality for Bipartite Qubit Entanglement”, *Phys. Rev. Lett.* **96**, 220503 (2006).
18. A. Khrennikov, *Interpretations of Probability* (VSP, 1999).
19. R. P. Feynman, "Negative Probability", *in* F. D. Peat and B. Hiley (*eds*.), *Quantum Implications: Essays in Honour of David Bohm*, p. 235 (Routledge, 1987).
20. E. Santos, “Does quantum mechanics violate the Bell inequalities?” *Phys. Rev. Lett.* **66**, 1388 (1991).
21. E. Santos, “Critical analysis of the empirical tests of local hidden-variable theories”, *Phys. Rev. A* **46**, 3646 (1992).
22. B. G. Christensen *et al*., “Detection-Loophole-Free Test of Quantum Nonlocality, and Applications”, *Phys. Rev. Lett.* **111**, 130406 (2013).
23. M. Giustina *et al.*, “Bell violation using entangled photons without the fair-sampling assumption”, *Nature* **497**, 227 (2013).
24. M. Giustina *et al*., “Significant-loophole-free test of Bell's theorem with entangled photons”, *Phys. Rev. Lett.* **115**, 250401 (2015).
25. L. K. Shalm *et al*., “A strong loophole-free test of local realism”, *Phys. Rev. Lett.* **115**, 250402 (2015).
26. A. Aspect, P. Grangier, and G. Roger, “Experimental tests of realistic local theories via Bell’s theorem”, *Phys. Rev. Lett.* **47**, 460 (1981).
27. A. Aspect, P. Grangier, and G. Roger, “Experimental realization of Einstein-Podolsky-Rosen-Bohm gedankenexperiment: a new violation of Bell’s inequalities”, *Phys. Rev. Lett.* **49**, 91 (1982).
28. A. Aspect, J. Dalibard, and G. Roger, “Experimental test of Bell’s inequalities using time-varying analyzers”, *Phys. Rev. Lett.* **49**, 1804 (1982).
29. A. M. Cetto, A. Valdés-Hernández and L. de la Peña, “On the spin projection operator and the probabilistic meaning of the bipartite correlation function”, *Found. Phys.* **50**, 27 (2020).