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Article

Gravitational, Electromagnetic and Quantum Interaction: From String to Cloud Theory

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Abstract: Considerable efforts have been devoted to modifying gravity, which aim to elucidate the possible existence or nature of dark matter and dark energy, achieve a better description of observation data, verify theoretical restrictions in the strong curvature regime and to formulate quantum gravity. In addition, despite the enormous success of the quantum field theory, the framework requires the so-called renormalization techniques and breaks down at high energies. Recently, the Planck Legacy 2018 release has confirmed the presence of an enhanced lensing amplitude in the cosmic microwave background power spectra, which prefers a positively curved early Universe with a confidence level greater than 99%. This study considers the implied positive curvature of the early Universe as the curvature of 'the background or the 4D conformal bulk' and distinguishes it from the localized curvature that is induced into the bulk by the presence of comparably smaller celestial objects that are regarded as '4D relativistic cloud-worlds'. To consider the interaction between the bulk and cloud-worlds, this paper presents interaction field equations in terms of the brane-world modified gravity counting for the bulk conformal curvature and the boundary terms, which could remove the singularities and satisfy a conformal invariance theory. Similarly, the quantum fields and clouds are regarded as '4D relativistic branes' embedded in vacuum energy of a field strength reliant on the background curvature due to gravity. A visualization of the evolution of the 4D relativistic cloud-worlds over the conformal space-time of the 4D bulk is presented.

Keywords: Galaxy Formation; Conformal Spacetime; Brane-World Modified Gravity.

1. Introduction

After the formulation of Einstein's General Relativity (GR) utilizing 4D spacetime, Kaluza discovered in 1919 a potential field unification of gravitation and electromagnetism in 5D spacetime. To deal with the 5th dimension, Klein posited that it could be compactified. Nonetheless, those attempts and their expansions to more dimensions have not culminated in testable predictions nor the competence to elucidate observations yet. An alternative to compactification, Gogberashvili, Randall and Sundrum showed in 1999 that the weak force of gravity could be explained using a model of 4D spacetime embedded in a negatively curved and large 5th dimension; nevertheless, it required massive gravitons [1–3].

On the other hand, to achieve an effective action for quantum corrections, several theories have been formulated on the modification of Lagrangian fields and curvature terms. Such modifications appear to be inevitable, which include higher-order curvature terms as well as non-minimally coupled scalar fields [4–6]. While quantum anomalies require a non-local Lagrangian action, one of the major differences between GR and the quantum field theory (QFT) framework is that GR is background independent, which allows it to require fewer inputs while the latter requires a background metric that in turn impacts its predictions [7].

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Recently, the Planck Legacy 2018 (PL18) release has confirmed the existence of an enhanced lensing amplitude in the cosmic microwave background power spectra, which prefers a positively curved early Universe with a confidence level greater than 99% [8,9]. Based on this sign of the early curved background and its feasible evolution over the conformal time, it is obvious that the background independence theories such as GR ignore the variations in the background curvature and treat celestial objects in the early Universe of a preferred positive curvature on equal footing with their counterpart in the present Universe of a spatially flat spacetime background. A desirable theory should be able to predict both metrics of the celestial object and the background where involving the background could require a non-local Lagrangian action. It should reduce to GR in a spatially flat spacetime background.

This study considers the implied positive curvature of the early Universe as the curvature of 'the background or the 4D bulk as a manifestation of the vacuum energy', where the bulk curvature can evolve over the conformal time from the preferred early positive curvature to the present spatial flatness. The celestial objects are regarded as 'relativistic 4D cloud-worlds', which can induce a localized curvature into the bulk. Similarly, the quantum fields and clouds are regarded as '4D relativistic branes' embedded in the vacuum energy of a field strength reliant on the stress-energy of the cloud-word and the bulk, where the field strength of vacuum energy due to background curvature is incorporated to investigate the influence of gravity on quantum fields.

The paper is organised as follows. Section 2 discusses the mathematical derivations of the gravitational and electromagnetic interaction field equations. Section 3 visualizes the field equations while Section 4 discusses the gravitational, electromagnetic and Quantum interaction field equations. Section 5 reproduces quantum electrodynamics from the interaction field equations. Finally, Section 6 summarises the conclusions and suggests the future work.

2. Gravitational and Electromagnetic Interaction Field Equations

The PL18 release has preferred a positively curved early Universe, that is, is a sign of a primordial background curvature or a curved bulk. To incorporate the bulk curvature and its evolution over the conformal time, a modulus of spacetime deformation, E_D in terms of energy density, is introduced based on the theory of elasticity [10]. The modulus can be expressed in terms of the resistance of the bulk to the localized curvature induced by celestial objects using Einstein field equations or in terms of the field strength of the bulk using the Lagrangian formulation of the energy density existing in the bulk as a manifestation of the vacuum energy density as follows

$$E_D = \frac{T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}}{R_{\mu\nu}/\mathcal{R}} = \frac{-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{4\mu_0} \tag{1}$$

where the stress is signified by the stress-energy tensor $T_{\mu\nu}$ of trace T while the strain is signified by the Ricci curvature tensor $R_{\mu\nu}$ as the change in the curvature divided by the scalar of the bulk curvature $\mathcal{R}$ as the background or conformal curvature. $\mathcal{F}_{\lambda\rho}$ is the field strength tensor and μ_0 is vacuum permeability. By incorporating the bulk influence, the Einstein–Hilbert action can be extended to

$$S = E_D \int_C \left[\frac{R}{\mathcal{R}} + \frac{L}{\mathcal{L}}\right] \sqrt{-g}\, d^4\rho \tag{2}$$

where R is the Ricci scalar curvature representing the localized curvature induced into the bulk by a celestial object that is regarded as a 4D relativistic cloud-world of metric g_{uv} and Lagrangian density L. $\mathcal{R}$ represents the scalar curvature of the 4D bulk of metric $\tilde{g}_{\mu\nu}$ while $\mathcal{L}$ is the Lagrangian density of the bulk as an indication of its internal stresses and

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momenta reflecting its curvature. Since the bulk modulus, E_D, is constant with regards to the cloud-world action under constant vacuum energy density condition and by considering the bulk expansion over the conformal time owing to the Universe expansion and its implication on the field strength of the bulk, a dual-action concerning the conservation of energy on global (bulk) and local (cloud-world) scales can be introduced as follows

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}}{4\mu_0}\right]\sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} + \frac{L_{\mu\nu}g^{\mu\nu}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}}\right]\sqrt{-g}\, d^4\rho\, d^4\sigma \tag{3}$$

where the relationship of the conformal bulk metric $\tilde{g}_{\mu\nu}$ with the embedded cloud-world metric g_{uv} can be characterized by Weyl's conformal transformation as $\tilde{g}_{\mu\nu} = g_{\mu\nu}\Omega^2$, here Ω^2 is a conformal function [11]. The global-local action should hold for any variation as

$$\delta S = \int_B \left[\begin{matrix} \dfrac{-\delta\left(\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}\right)\sqrt{-\tilde{g}}}{4\mu_0} \\ -\dfrac{\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}\delta\sqrt{-\tilde{g}}}{4\mu_0}\end{matrix}\right] \int_C \left[\begin{matrix} \dfrac{\delta\left(R_{\mu\nu}g^{\mu\nu}\right)\sqrt{-g}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} - \dfrac{\delta\left(\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}\right)R_{\mu\nu}g^{\mu\nu}\sqrt{-g}}{\left(\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}\right)^2} + \dfrac{R_{\mu\nu}g^{\mu\nu}\delta\sqrt{-g}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} \\ + \dfrac{\delta\left(L_{\mu\nu}g^{\mu\nu}\right)\sqrt{-g}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}} - \dfrac{\delta\left(\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}\right)L_{\mu\nu}g^{\mu\nu}\sqrt{-g}}{\left(\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}\right)^2} + \dfrac{L_{\mu\nu}g^{\mu\nu}\delta\sqrt{-g}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}}\end{matrix}\right] d^4\rho\, d^4\sigma \tag{4}$$

By utilizing Jacobi's formula, $\delta\sqrt{-g} = -\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}/2$ [12]. Hence, the variation is

$$\delta S = \int_B \left[\begin{matrix} -\dfrac{\mathcal{F}_{\lambda\rho}\mathcal{F}^{\rho}_{\gamma}\delta\tilde{g}^{\lambda\gamma} + \mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\delta\mathcal{F}^{\rho}_{\gamma}}{2\mu_0} \\ +\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}\dfrac{\tilde{g}_{\mu\nu}\delta\tilde{g}^{\mu\nu}}{8\mu_0}\end{matrix}\right]\sqrt{-\tilde{g}} \int_C \left[\begin{matrix} \dfrac{R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}} - \dfrac{\mathcal{R}_{\mu\nu}\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R - \dfrac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{R}}R \\ + \dfrac{L_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta L_{\mu\nu}}{\mathcal{L}} - \dfrac{\mathcal{L}_{\mu\nu}\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{\mathcal{L}^2}L - \dfrac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{L}}L\end{matrix}\right]\sqrt{-g}\, d^4\rho\, d^4\sigma \tag{5}$$

where the electromagnetic and Lagrangian densities are considered on the boundaries. By considering the cloud-world's boundary term: $\int_c g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\, d^4\sigma/\mathcal{R}$, the variation in the Ricci curvature tensor δR_{uv} can be expressed in terms of the covariant derivative of the difference between two Levi-Civita connections, the Palatini identity: $\delta R_{\mu\nu} = \nabla_\rho\left(\delta\Gamma^{\rho}_{\nu\mu}\right) - \nabla_\nu\left(\delta\Gamma^{\rho}_{\rho\mu}\right)$, where the variation with respect to the inverse metric $g^{\mu\nu}$ can be obtained by using the metric compatibility of the covariant derivative, $\nabla_\rho g^{\mu\nu} = 0$ [12], as $g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\rho\left(g^{\mu\nu}\delta\Gamma^{\rho}_{\nu\mu} - g^{\mu\rho}\delta\Gamma^{\sigma}_{\sigma\mu}\right)$. The cloud-world's boundary term as a total derivative for any tensor density can be transformed based on Stokes' theorem as follows

$$\begin{aligned}\int_C \left[\frac{g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}}\right]\sqrt{-g}\, d^4\rho &= \frac{1}{\mathcal{R}}\int_C \left[\nabla_\rho\left(g^{\mu\nu}\delta\Gamma^{\rho}_{\nu\mu} - g^{\mu\rho}\delta\Gamma^{\sigma}_{\sigma\mu}\right)\right]\sqrt{-g}\, d^4\rho \\ &= \frac{1}{\mathcal{R}}\int_C \left[\nabla_\mu H^\mu\right]\sqrt{-g}\, d^4\rho = \frac{\epsilon}{\mathcal{R}}\int_{\partial C}[K]\sqrt{|q|}\, d^3\varrho\end{aligned} \tag{6}$$

where the bulk scalar curvature, $\mathcal{R}$, is left outside the integral transformation as it only acts as a scalar to the transformation. The same transformations are applied to the bulk and Lagrangian boundary terms and the electromagnetic tensor variation as it represents a boundary term. Thus, the transformed boundary action, S_b, along with a transformed global boundary term are

$$S_b = \int_{\partial B}\left[\frac{\epsilon\mathcal{F}_{\lambda\rho}J^\rho}{2}\right]\sqrt{-\tilde{q}}\left(\frac{\epsilon}{\mathcal{R}}\int_{\partial C}[K]\sqrt{|q|} - \frac{R\epsilon}{\mathcal{R}^2}\int_{\partial C}[\mathcal{K}]\sqrt{|\tilde{q}|} + \frac{\epsilon}{\mathcal{L}}\int_{\partial C}[l]\sqrt{|q|} - \frac{L\epsilon}{\mathcal{L}^2}\int_{\partial C}[\ell]\sqrt{|\tilde{q}|}\right) d^3\varrho\, d^3\varsigma \tag{7}$$

where K and $\mathcal{K}$ are the traces of the cloud-world and the bulk extrinsic curvatures, l and ℓ are the extrinsic traces of the Lagrangian density on the cloud-world and the bulk boundaries, q and $\tilde{q}$ are the determinants of their induced metrics respectively, and ϵ

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equals 1 when the normal $\hat{n}_u$ is a spacelike entity and equals -1 when it is a timelike entity. J^ρ is the four current. The boundary action should hold for any variation and by considering the transformed cloud-world's boundary term, the variation in the term yields

$$\frac{\epsilon}{\mathcal{R}}\int_{\partial C}\left[K_{\mu\nu}\delta q^{\mu\nu}+q^{\mu\nu}\delta K_{\mu\nu}+K\frac{\delta\sqrt{|q|}}{\sqrt{|q|}}\right]\sqrt{|q|}\,d^3\varrho \tag{8}$$

where $K=K_{\mu\nu}q^{\mu\nu}$. By utilising Jacobi's formula for the determinant differentiation; thus, $\delta\sqrt{|q|}=-\sqrt{|q|}\,q_{\mu\nu}\delta q^{\mu\nu}/2$ and by utilising the variation in the metric times the inverse metric, $q^{\mu\nu}q_{\mu\nu}=\delta^\mu_\nu$ as $q^{\mu\nu}=-q_{\mu\nu}\delta q^{\mu\nu}/\delta q_{\mu\nu}$; thus, the boundary term is

$$\frac{\epsilon}{\mathcal{R}}\int_{\partial C}\left[K_{\mu\nu}\delta q^{\mu\nu}-\frac{1}{2}K\left(q_{\mu\nu}\delta q^{\mu\nu}+2q_{\mu\nu}\frac{\delta K_{\mu\nu}}{\delta q_{\mu\nu}K}\delta q^{\mu\nu}\right)\right]\sqrt{|q|}\,d^3\varrho \tag{9}$$

here $\delta K_{\mu\nu}/\delta q_{\mu\nu}K=(\delta K_{\mu\nu}/K_{\mu\nu})(q_{\mu\nu}/\delta q_{\mu\nu})=\delta\ln K_{\mu\nu}/\delta\ln q_{\mu\nu}$ resembles the Ricci flow in a normalized form reflecting the conformal distortion in the boundary over the conformal time, which can be expressed as a positive function Ω^2 according to Weyl's conformal transformation as $\tilde{q}_{\mu\nu}=q_{\mu\nu}\Omega^2$ [13]. Thus, the boundary term is

$$\frac{\epsilon}{\mathcal{R}}\int_{\partial C}\left[K_{\mu\nu}\delta q^{\mu\nu}-\frac{1}{2}K\hat{q}_{\mu\nu}\delta q^{\mu\nu}\right]\sqrt{|q|}\,d^3\varrho \tag{10}$$

where $\hat{q}_{\mu\nu}=q_{\mu\nu}+2\tilde{q}_{\mu\nu}$ denoting the conformally transformed induced metric on the cloud-world boundary. The same is applied for the bulk and Lagrangian boundary terms. Thus, the variation in the whole action with renaming the dummy indices is

$$\begin{aligned}\delta S=&\left(-\int_B\left[\frac{1}{2\mu_0}\left(\mathcal{F}_{\mu\lambda}\mathcal{F}^\lambda_\nu-\frac{\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{4}\tilde{g}_{\mu\nu}\right)\right]\delta\tilde{g}^{\mu\nu}\sqrt{-\tilde{g}}\,d^4\sigma-\int_{\partial B}\left[\frac{\epsilon}{2}\delta f_\nu/\delta\tilde{q}^{\mu\nu}\right]\delta\tilde{q}^{\mu\nu}\sqrt{|\tilde{q}|}\,d^3\varsigma\right)\\&\left(\int_C\left[\frac{R_{\mu\nu}}{\mathcal{R}}-\frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R-\frac{R}{2\mathcal{R}}g_{\mu\nu}+\frac{L_{\mu\nu}}{\mathcal{L}}-\frac{\mathcal{L}_{\mu\nu}}{\mathcal{L}^2}L-\frac{L}{2\mathcal{L}}g_{\mu\nu}\right]\delta g^{\mu\nu}\sqrt{-g}\,d^4\rho\right.\\&+\int_{\partial C}\left[\frac{\epsilon}{\mathcal{R}}\left(K_{\mu\nu}-\frac{1}{2}K\hat{q}_{\mu\nu}\right)+\frac{\epsilon}{\mathcal{L}}\left(l_{\mu\nu}-\frac{1}{2}l\hat{q}_{\mu\nu}\right)\right]\delta q^{\mu\nu}\sqrt{|q|}\,d^3\varrho\\&\left.-\int_{\partial C}\left[\frac{R\epsilon}{\mathcal{R}^2}\left(\mathcal{K}_{\mu\nu}-\frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu}\right)+\frac{L\epsilon}{\mathcal{L}^2}\left(\ell_{\mu\nu}-\frac{1}{2}\ell\hat{q}_{\mu\nu}\right)\right]\delta q^{\mu\nu}\sqrt{|q|}\,d^3\varrho\right)\end{aligned} \tag{11}$$

By encapsulating the new Lagrangian terms on boundaries into an extended stress-energy tensor defined as $\hat{T}_{\mu\nu}\coloneqq(2L_{\mu\nu}-L\hat{g}_{\mu\nu})-(2l_{\mu\nu}-l\hat{q}_{\mu\nu})+(2\ell_{\mu\nu}-\ell\hat{q}_{\mu\nu})L/\mathcal{L}$ that counts for the energy density and flux of the cloud-world, $L_{\mu\nu}$, and electromagnetic energy flux from its boundary, $l_{\mu\nu}$, over the conformal time where $\hat{g}_{\mu\nu}=g_{\mu\nu}+2\tilde{g}_{\mu\nu}$ is the conformally transformed metric since $\tilde{g}_{\mu\nu}=\mathcal{L}_{\mu\nu}/\mathcal{L}=\mathcal{L}_{\mu\nu}/\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}$ while the Lagrangian density on bulk's boundary, $\ell_{\mu\nu}$, has $L/\mathcal{L}$ factor, i.e. it is only significant when the cloud-world has a high Lagrangian density such as black holes where the entire contribution belongs to the boundary terms when finding the black hole entropy [14,15]. The outcomes of the global action resembled an extended electromagnetic stress-energy tensor as $\mathcal{T}_{\mu\nu}\coloneqq(\mathcal{F}_{\mu\lambda}\mathcal{F}^\lambda_\nu-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}\tilde{g}_{\mu\nu}/4)/\mu_0)+\delta f_\nu/\delta\tilde{q}^{\mu\nu}$ denoting energy density exists in the bulk as the vacuum energy density in addition to the variation of the 4D Lorentz force density with regards bulk's boundary variation over conformal time. From Equations (1), (2) and (11), $\mathcal{T}_{\mu\nu}\coloneqq E_D=\mathcal{L}=\mathcal{R}c^4/8\pi G_t$ is proportional to the fourth power of the speed of light that in turn is directly proportional to the frequency, which can be in accordance with frequency cut-off predictions of vacuum energy density in the quantum field theory [16,17].

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By choosing ϵ as a timelike entity and applying the principle of stationary action yield

$$\frac{R_{\mu\nu}}{\mathcal{R}}-\frac{1}{2}\frac{R}{\mathcal{R}}g_{\mu\nu}-\frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R+\frac{R\left(\mathcal{K}_{\mu\nu}-\frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu}\right)-\mathcal{R}\left(K_{\mu\nu}-\frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}^2}=\frac{\hat{T}_{\mu\nu}}{\mathcal{T}_{\mu\nu}} \tag{12}$$

The interaction field equations can be interpreted as indicating that the cloud-world's induced curvature over the bulk's pre-existing curvature equals the ratio of the cloud-world's imposed energy density and its flux to the bulk's vacuum energy density and its flux through the expanding/contracting Universe. By utilizing Equations (1) and (11) that state $\mathcal{T}_{\mu\nu} \coloneqq E_D = \mathcal{R}c^4/8\pi G_t$, the field equations can be simplified to

$$R_{\mu\nu}-\frac{1}{2}R\hat{g}_{\mu\nu}+\frac{R\left(\mathcal{K}_{\mu\nu}-\frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu}\right)-\mathcal{R}\left(K_{\mu\nu}-\frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}}=\frac{8\pi G_t}{c^4}\hat{T}_{\mu\nu} \tag{13}$$

where $\hat{g}_{\mu\nu}=g_{\mu\nu}+2\tilde{g}_{\mu\nu}$ is the conformally transformed metric counting for both cloud-world, $g_{\mu\nu}$, and bulk, $\tilde{g}_{\mu\nu}=\mathcal{R}_{\mu\nu}/\mathcal{R}=\mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}$, metrics whereas Einstein spaces are a subclass of conformal spaces [11]. The evolution in G_t reflects the field strength evolution with the Universe expansion and it can accommodate the bulk curvature evolution over the conformal time against constant G for a special flat spacetime case. The new boundary term given by the extrinsic curvatures of the cloud-world, K, and the bulk, $\mathcal{K}$, is only significant at high energies where the difference between the induced, R, and existing, $\mathcal{R}$, curvatures is significant. The field equations could remove the singularities and satisfy a conformal invariance theory.

3. Evolution of 4D Relativistic Cloud-Worlds Travelling in the 4D Conformal Bulk

This section aims to visualize the evolution of the 4D relativistic cloud-words over the conformal space-time of the 4D bulk. Galaxy formation and evolution as a 4D relativistic cloud-world travelling throughout a 4D conformal bulk is considered. This scenario reveals the galaxy formation as a forced vortex due to the curved bulk background, which could resemble galaxy rotational curves. The mathematical derivations of the galaxy core are presented in [18], where the entire contribution comes from the boundary term when calculating the black hole entropy using the semiclassical approach [14,15]. Applying this concept and by rearranging the field equations for this setting as

$$R_{\mu\nu}-\frac{1}{2}R\hat{g}_{\mu\nu}=\frac{8\pi G_t}{c^4}\hat{T}_{\mu\nu}-\frac{R\left(\mathcal{K}_{\mu\nu}-\frac{1}{2}\mathcal{K}\hat{q}_{\mu\nu}\right)-\mathcal{R}\left(K_{\mu\nu}-\frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}}=0 \tag{14}$$

The derived conformally metric $\hat{g}_{\mu\nu}=g_{\mu\nu}+2\tilde{g}_{\mu\nu}=g_{\mu\nu}(1+2\Omega^2)$ in [18] is

$$ds^2=\left(1-\frac{r_s}{r}-\frac{\tau_p}{\tau}\right)\left(-c^2dt^2+S^2\left(\frac{dr^2}{1+\frac{r_s{}^2}{r^2}-2\frac{r_s}{r}}+\frac{r^2d\theta^2+r^2sin^2\theta d\phi^2}{1-\frac{r_s}{r}-\frac{\tau_p}{\tau}}\right)\right) \tag{15}$$

This metric reduces to the Schwarzschild metric in a flat background ($\tau\to\infty$), where τ is the background or bulk curvature radius, r_s is Schwarzschild radius and S^2 is a dimensionless spatial scale factor. In the case of PI18 release of a preferred early Universe positive curvature, the bulk gravitational potential can be expressed as $\tau_p=2G_pM_p/c^2$, where M_p is the early Universe plasma mass and τ_p is early Universe preferred curvature radius. The bulk potential decreases with the Universe's expansion and vanishes in the flat spacetime background ($\tau\to\infty$).

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Figure 1 shows the evolution of the 4D cloud-world through its travel and spin in the conformal space-time of the 4D bulk.

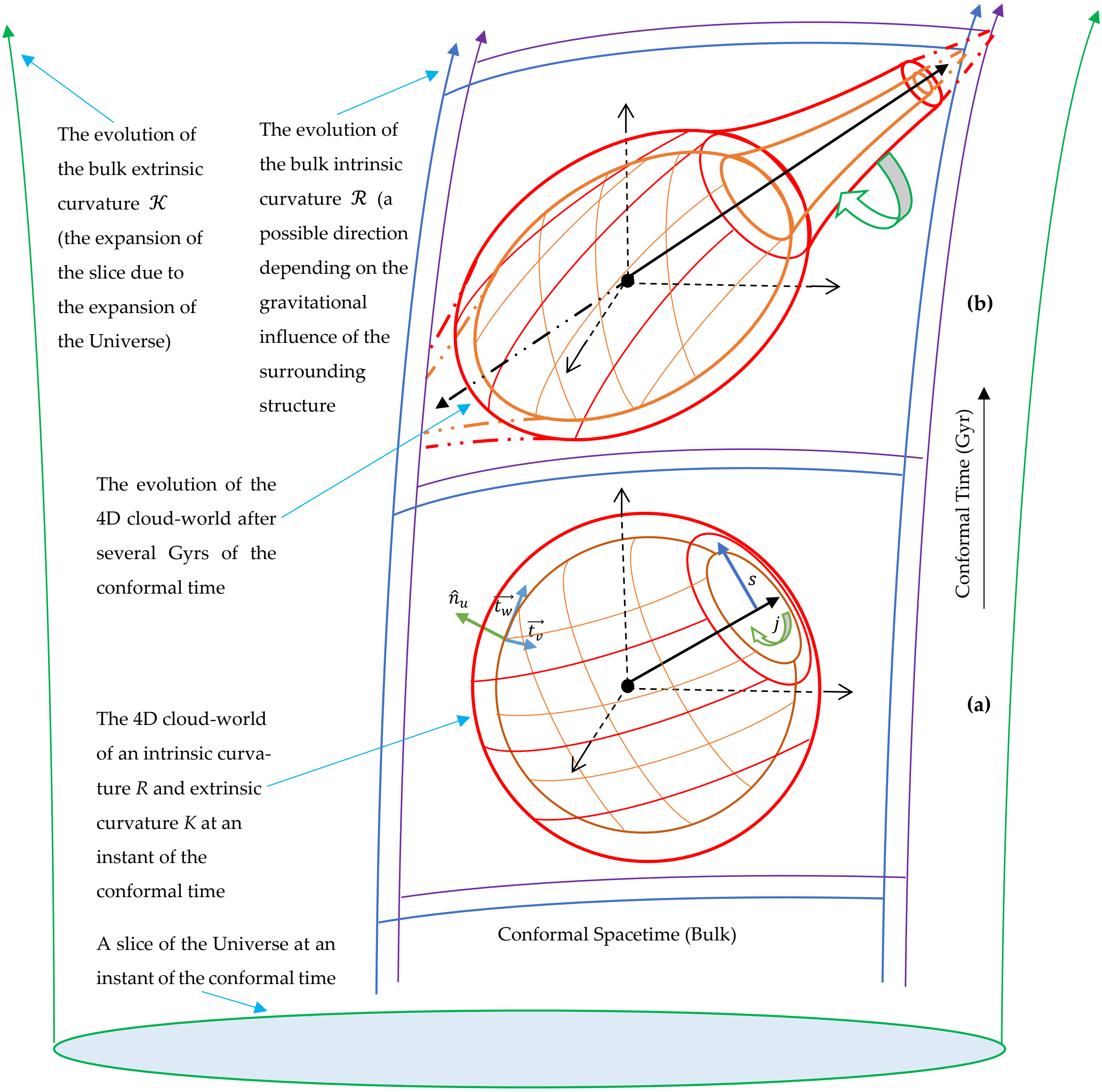

Figure 1. The hypersphere of a compact core of a galaxy (the red-orange 4D cloud-world) along with its travel and spin through the conformal spacetime (the blue-purple 4D bulk representing the bulk of distinctive curvature evolving over the conformal time.

4. Gravitational, Electromagnetic and Quantum Interaction Field Equations

In this section, the action in Equation (3) is expanded to investigate the interaction of a quantum field with the electromagnetic fields under the influence of vacuum energy of field strength reliant on the curvature of the cloud-world and the bulk. The modulus can be expressed in terms of the stress-energy tensors of quantum fields and their curvatures in terms of the four-momentum to the curvature of the background as follows

$$E_Q = \frac{\frac{1}{n}L_{\mu\alpha}L_\nu^\alpha - \frac{1}{4n}L_{\alpha\beta}L^{\alpha\beta}q_{\mu\nu}}{p_\mu p_\nu/\pi_\mu\pi^\nu} = \frac{-\hat{\mathcal{F}}_{\alpha\beta}\hat{\mathcal{F}}^{\alpha\beta}}{4\mu_0} \tag{16}$$

where $L_{\mu\alpha}L_\nu^\alpha$ denotes the Lagrangian densities of two entangled quantum fields of metric tensor $q_{\mu\nu}$ and four-momentum $p_\mu p^\nu$ while $\pi_\mu\pi^\nu$ denotes the four-momentum of the vacuum energy density of field strength tensor $\hat{\mathcal{F}}_{\alpha\beta}$, and n is a constant. By including the electromagnetic field $F_{\alpha\beta}$ that interacts with quantum fields, the action is formulated as

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}}{4\mu_0}\right]\sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}}\right]\sqrt{-g}$$
$$\int_Q \left[\frac{p_\mu p_\nu q^{\mu\nu}}{\pi_\mu\pi_\nu g^{\mu\nu}} + \frac{L_{\alpha\beta}q^{\alpha\lambda}L_{\lambda\gamma}q^{\beta\gamma}/n + F_{\alpha\beta}q^{\alpha\lambda}F_{\lambda\gamma}q^{\beta\gamma}/4\mu_0}{\mathcal{L}_{\mu\nu}g^{\mu\nu}}\right]\sqrt{-q}\, d^4\alpha\, d^4\rho\, d^4\sigma \tag{17}$$

The action should hold for any variation as

$$\delta S = \int_B \begin{bmatrix} -\frac{\mathcal{F}_{\lambda\rho}\mathcal{F}_\gamma^\rho\delta\tilde{g}^{\lambda\gamma}}{2\mu_0} \\ -\frac{\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\delta\mathcal{F}_\gamma^\rho}{2\mu_0} + \\ \frac{\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}\tilde{g}_{\mu\nu}\delta\tilde{g}^{\mu\nu}}{8\mu_0} \end{bmatrix}\sqrt{-\tilde{g}} \int_C \begin{bmatrix} \frac{R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}} \\ -\frac{\mathcal{R}_{\mu\nu}\,\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R \\ -\frac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{R}}R \end{bmatrix}\sqrt{-g}$$
$$\int_Q \begin{bmatrix} \frac{p_\mu p_\nu\delta q^{\mu\nu} + q^{\mu\nu}\delta(p_\mu p_\nu)}{\pi_\mu\pi^\nu} - p_\mu p^\nu\frac{(\pi_\mu\pi_\nu)\delta g^{\mu\nu} + g^{\mu\nu}\delta(\pi_\mu\pi_\nu)}{(\pi_\mu\pi^\nu)^2} - \frac{p_\mu p^\nu}{\pi_\mu\pi^\nu}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{2} \\ +2\frac{L_{\alpha\gamma}L_\beta^\gamma\delta q^{\alpha\lambda} + q^{\alpha\lambda}L_{\alpha\beta}\delta L_\beta^\gamma}{n\mathcal{L}} - 2L_{\alpha\beta}L^{\alpha\beta}\frac{\mathcal{L}_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{n\mathcal{L}^2} - L_{\alpha\beta}L^{\alpha\beta}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{2n\mathcal{L}} \\ +\frac{F_{\alpha\gamma}F_\beta^\gamma\delta q^{\alpha\lambda} + q^{\alpha\lambda}F_{\alpha\beta}\delta F_\beta^\gamma}{2\mu_0\mathcal{L}} - F_{\alpha\beta}F^{\alpha\beta}\frac{\mathcal{L}_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{2\mu_0\mathcal{L}^2} - F_{\alpha\beta}F^{\alpha\beta}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{8\mu_0\mathcal{L}} \end{bmatrix}\sqrt{-q}\, d^4\alpha\, d^4\rho\, d^4\sigma \tag{18}$$

By applying the principle of stationary action to the gravitational and electromagnetic parts of the action yield

$$\delta S = \hat{\mathcal{T}}_{\mu\nu}\int_Q \begin{bmatrix} \frac{p_\mu p_\nu\delta q^{\mu\nu} + q^{\mu\nu}\delta(p_\mu p_\nu)}{\pi_\mu\pi^\nu} - p_\mu p^\nu\frac{(\pi_\mu\pi_\nu)\delta g^{\mu\nu} + g^{\mu\nu}\delta(\pi_\mu\pi_\nu)}{(\pi_\mu\pi^\nu)^2} - p_\mu p^\nu\frac{q_{\mu\nu}\delta q^{\mu\nu}}{2\pi_\mu\pi^\nu} \\ +2\frac{L_{\alpha\gamma}L_\beta^\gamma\delta q^{\alpha\lambda} + q^{\alpha\lambda}L_{\alpha\beta}\delta L_\beta^\gamma}{n\mathcal{L}} - 2L_{\alpha\beta}L^{\alpha\beta}\frac{\mathcal{L}_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{n\mathcal{L}^2} - L_{\alpha\beta}L^{\alpha\beta}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{2n\mathcal{L}} \\ +\frac{F_{\alpha\gamma}F_\beta^\gamma\delta q^{\alpha\lambda} + q^{\alpha\lambda}F_{\alpha\beta}\delta F_\beta^\gamma}{2\mu_0\mathcal{L}} - F_{\alpha\beta}F^{\alpha\beta}\frac{\mathcal{L}_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{2\mu_0\mathcal{L}^2} - F_{\alpha\beta}F^{\alpha\beta}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{8\mu_0\mathcal{L}} \end{bmatrix}\sqrt{-q}\, d^4\alpha \tag{19}$$

where $\hat{\mathcal{T}}_{\mu\nu}$ represents the overall stress-energy tensor of the cloud-world and the bulk.

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By transforming the boundary terms and applying the principle of stationary action to the quantum part of the action give

$$\frac{p_\mu p_\nu}{\pi_\mu \pi^\nu} - \frac{p_\mu p^\nu}{\pi_\mu \pi^\nu}\frac{q_{\mu\nu}+2g_{\mu\nu}}{2} - \frac{1}{2}\frac{J^u A_\mu J^\nu A_\nu}{\pi_\mu \pi^\nu} + \frac{1}{2}J^u C_\mu J^\nu C_\nu \frac{p_\mu p^\nu}{(\pi_\mu \pi^\nu)^2} = \frac{\hat{T}_{\mu\nu}\hat{T}^{\mu\nu}}{n\hat{\mathcal{T}}_{\mu\nu}} + \frac{1}{2}J^u B_\mu J^\nu B_\nu \quad (20)$$

where $\hat{T}_{\mu\nu}\hat{T}^{\mu\nu}$ are the stress-energy tensors of the two entangled quantum fields including their boundary terms. A_μ is the electromagnetic field's four-potential generated by the quantum field itself, B_μ is the external electromagnetic field's four-potential applied on the quantum field, C_μ is the electromagnetic field's four-potential applied on the cloud-world boundary and J^u is the four-current. By utilizing the modulus in Equation (16), the field equations can be simplified to

$$p_\mu p_\nu - p_\mu p^\nu \frac{q_{\mu\nu}+2g_{\mu\nu}}{2} - \frac{1}{2}J^u A_\mu J^\nu A_\nu + \frac{1}{2}J^u C_\mu J^\nu C_\nu \frac{p_\mu p^\nu}{\pi_\mu \pi^\nu} = \frac{\varphi}{n}\hat{T}_{\mu\nu}\hat{T}^{\mu\nu} + \frac{1}{2}J^u B_\mu J^\nu B_\nu \quad (21)$$

where φ is a parameter that depends on the gravitational field strength. The field equations can be represented using the components of the four-momentum as follows

$$\left(\frac{E}{c} - \vec{\sigma}.\vec{p}\right)^2 - \frac{1}{2}\left(\frac{E}{c} - \vec{\sigma}.\vec{p}\right)\left(\frac{E}{c} + \vec{\sigma}.\vec{p}\right)(q_{\mu\nu}+2g_{\mu\nu}) - \frac{1}{2}J^u A_\mu J^\nu A_\nu$$
$$+\frac{1}{2}J^u C_\mu J^\nu C_\nu \frac{\left(\frac{E}{c} - \vec{\sigma}.\vec{p}\right)\left(\frac{E}{c} + \vec{\sigma}.\vec{p}\right)}{\left(\frac{E^2}{c^2} - \vec{p}^{\,2}\right)} = \frac{\varphi}{n}\hat{T}_{\mu\nu}\hat{T}^{\mu\nu} + \frac{1}{2}J^u B_\mu J^\nu B_\nu \quad (22)$$

where $E^2/c^2 - \vec{p}^{\,2}$ is Klein–Gordon field. By separating the two entangled quantum fields:

$$\left(\frac{E}{c} - \vec{\sigma}.\vec{p}\right) - \frac{1}{2}\left(\frac{E}{c} + \vec{\sigma}.\vec{p}\right)(q_{\mu\nu}+2g_{\mu\nu}) - \frac{1}{2}J^u A_\mu + \frac{1}{2}J^u C_\mu \frac{\left(\frac{E}{c} + \vec{\sigma}.\vec{p}\right)}{\left(\frac{E^2}{c^2} - \vec{p}^{\,2}\right)} = \frac{\varphi}{n}\hat{T}_{\mu\nu} + \frac{1}{2}J^u B_\mu \quad (23)$$

5. Reproducing Quantum Electrodynamics from the Interaction Field Equations

This section utilized the interaction field equations in Equation (23) for a flat quantum field metric $q_{\mu\nu}$ while disregarding both the background curvature and the interaction term on the cloud-world boundary in order to reproduce the quantum electrodynamics as follows

$$\hbar\left(\frac{i}{c}D\partial_t + A\partial_x + B\partial_y + C\partial_z\right)\psi - \frac{1}{2}\hbar\left(\frac{i}{c}D\partial_t - A\partial_x - B\partial_y - C\partial_z\right) q_{\mu\nu}\psi - \frac{1}{2}J^u A_\mu = \frac{\varphi}{n}\left(\frac{mc^2}{V}\right)\psi + \frac{1}{2}J^u B_\mu \psi \quad (24)$$

where ψ denotes the quantum field wavefunction. By choosing the quantum field metric signature as $(1,-1,-1,-1)$ give

$$\frac{1}{2}\hbar\left(\frac{i}{c}D\partial_t + A\partial_x + B\partial_y + C\partial_z\right)\psi - \frac{1}{2}J^u A_\mu \psi = \frac{\kappa\varphi}{n}m\psi + \frac{1}{2}J^u B_\mu \psi \quad (25)$$

where κ represents a parameter depends on the quantum field type. For electron of charge e and mass m, Equation (25) gives

$$i\hbar\gamma^\mu\partial_\mu\psi - \frac{2\kappa\varphi}{n}m\psi = e\gamma^\mu(A_\mu + B_\mu)\psi \quad (26)$$

where γ^μ are Dirac matrices.

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6. Conclusions and Future Works

This study has presented interaction field equations in terms of the brane-world modified gravity counting for conformal curvature and the boundary terms. The study has considered the implied positive curvature of the early Universe y as the curvature of the background or the 4D bulk and distinguished it from the localized curvature that is induced into the bulk by the presence of comparably smaller celestial objects that are regarded as 'relativistic 4D cloud-worlds'; similarly, the quantum fields and clouds are regarded as '4D relativistic branes' embedded in vacuum energy of a field strength reliant on the background curvature due to gravity. Due to possible background curvature evolution, the bulk curvature should be taken into consideration along with the energy densities of the cloud-worlds when finding their induced curvatures. The new boundary term is only significant at high-energy limits such as within black holes and it can remove the singularities from the theory. Additionally, since the preferred length scale of any given mass changes proportionally to the change in the conformal bulk metric, the corresponding change in Gt accommodates the effective gravitational forces of the given mass, which could satisfy a conformal invariance theory. This work will be utilized to study the evolution of the Universe and the formation of galaxies and their rotational curves. Finally, this theoretical work will be tested using observational data in future works along finding the values of the new parameters.

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