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Article

Reconstructed $f(R)$ gravity and its cosmological consequences in chameleon scalar field with a scale factor describing the pre-bounce ekpyrotic contraction

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Abstract: Inspired by the work of S. D. Odintsov and V. K. Oikonomou, *Phys. Rev. D* **92**, 024016 (2015) [1], the present study reports a reconstruction scheme for $f(R)$ gravity with the scale factor $a(t) \propto (t_* - t)^{\frac{2}{c^2}}$ describing the pre-bounce ekpyrotic contraction, where t_* is the big crunch time. The reconstructed $f(R)$ is used to derive expressions for density and pressure contributions and the equation of state parameter resulting from this reconstruction is found to behave like "quintom". It has also been observed that the reconstructed $f(R)$ has satisfied a sufficient condition for a realistic model. In the subsequent phase the reconstructed $f(R)$ is applied to the model of chameleon scalar field and the scalar field ϕ and the potential $V(\phi)$ are tested for quasi-exponential ex pansion. It has been observed that although the reconstructed $f(R)$ satisfies one of the sufficient conditions for realistic model, the quasi-exponential expansion is not available due to this reconstruction. Finally, the consequences pre-bounce ekpyrotic inflation in $f(R)$ gravity are compared to the background solution for $f(R)$ matter bounce.

Keywords: pre-bounce ekpyrotic contraction; $f(R)$ gravity; reconstruction.

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1. Introduction

Observational evidence in support of the late time acceleration of the universe is documented in plethora of literatures [2,3,4,5]. An exotic matter, characterized by negative pressure, is considered to be the responsible for this accelerated expansion and is dubbed as “dark energy" (DE) [6,7,8]. Reviews on various candidates of DE have been made in a considerable number of literatures. Some significant ones include [6,8,9,10]. An approach, alterative to dark energy, also known as “modified gravity" have some relative advantages. Although DE and modified gravity theories have some similarities in their basic approach, the modified gravity has some features that have made it attractive in the study of the late time acceleration of the universe. One very promising modified theory of gravity is $f(R)$ gravity [11,12,13,14,15,16,17,18].

A scheme of reconstruction of modified gravity, that is capable of realizing an unification of the acceleration of late time and early inflation, was demonstrated in [19]. In this reconstruction scheme, [19] reported models of modified gravity that capable of presenting a successful transition from the phase of matter dominance to the late time accelerated phase of the universe. Another noteworthy reference in this context is [20] that reported $f(R)$ gravities that are viable and are reproduced through

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e-folding numbers. The unification of late- and early-time acceleration was also reported in this study. Another $f(R)$ reconstruction by the imposition of restrictions of dynamical nature was presented in [21]. Realization of ΛCDM through $f(R)$ gravity was elaborated in [22]. Reference [23] demonstrated how reconstruction schemes can be extended to modified gravity models so as to realize the transition from matter dominance to dark energy dominance. It is due to the non-linear character the $f(R)$ gravity is relevant to early inflation for large R [24]. The unification approach, as stated above, has also been reported in [25,26,27,28].

To avoid the measurable corrections to the occurrence of the local gravity, the models have to apply the chameleon mechanism [29,30] to overcome the solar system tests. References [31,32,33, 34] suggested the models which satisfies both the cosmological and the local gravity constraints. Studies including [35,36,37,38] demonstrated exact solutions that are capable of explaining the current acceleration. For more details, see Ref. [39,40]. Exact behaviour of the exotic matter that is thought to be responsible for the acceleration of the current unverse is yet to be fully understood. As a consequences, various candidates have been proposed for DE till date and modified theories are also having a considerable variability in approach and accordingly the cosmological parameters have been studied. Among the several candidates proposed so far for DE, holographic dark energy (HDE) is considered to be of immense potential. The HDE was proposed on the basis of holographic principle [41,42,43,44]. The present work, as one of its primary objectives, aims to reconstruct modified gravity based on holographic dark energy. Holographic reconstruction of modified gravity has already been reported in the cosmological literatures. Various authors [39,40,45,46,47] have reconstructed different candidates of modified gravity from holographic dark energy (HDE) with different IR cut-offs.

One natural query of the present day cosmology is whether the universe evolution actually follows standard inflationary paradigm. In order the standard inflation to follow, an initial singularity has to exist. Otherwise, the cosmological bounce is to be considered, in which case there does not exist any initial singularity [48]. Big bounce scenarios provide us with an alternative An alternative to the Big Bang cosmologies. A bounce cosmology in $f(R)$ cosmological settings was reported in [48], where it was observed that the bouncing point is characterized by type-IV singularity. In another interesting study, Odintsov and Oikonomou [49] provided an $F(R)$ gravity description of a ΛCDM bouncing model without depending upon any matter fluid or cosmological constant. In another study the authors [50] demonstrated by conformally transforming the Jordan frame singular bounce that the Einstein frame metric leads to a Big Rip singularity. The present work endevours to demonstrate a reconstruction scheme for $f(R)$ gravity with a scale factor describing the pre-bounce ekpyrotic contraction and to study the cosmological consequences for chameleon scalar field model. Rest of the paper is organised as follows: in section II, a brief overview of $f(R)$ gravity has been presented. A reconstruction scheme for $f(R)$ gravity has been presented in section III and section IV has demonstrated the cosmology of chameleon scalar field under reconstructed $f(R)$ gravity. In section V we have discussed the background evolution of matter bounce in $f(R)$ gravity for a different choice of bouncing scale factor. The results and concluding remarks have been presented in section VI.

1. Brief Overview of $f(R)$ Gravity

The action of $f(R)$ gravity is given by [16,18]

$$S = d^4x\sqrt{-g}\left[\frac{f(R)}{2\kappa^2} + \mathcal{L}_{matter}\right] \tag{1}$$

where $\kappa^2 = 8\pi G$, g is the determinant and $\mathcal{L}_{matter}$ is the matter Lagrangian. In the remaining part of the paper, we shall take $\kappa^2 = 1$. The $f(R)$ is a non-linear function of R that contains corrections to the EH action. The gravitational field equations are given by

$$H^2 = \frac{1}{3f'(R)}\left(\rho_m + \rho_R\right) \tag{2}$$

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$$\dot{H} = -\frac{1}{2f'(R)}\left(\rho_m + p_m + \rho_R + p_R\right) \tag{3}$$

where ρ_R and p_R are the density and pressure generated due to $f(R)$ gravity and they have the forms

$$\rho_R = \frac{1}{2}\left[-f(R) + Rf'(R)\right] - 3H\dot{R}f''(R) \tag{4}$$

and

$$p_R = \frac{1}{2}\left[f(R) - Rf'(R)\right] + \left[2H\dot{R} + \ddot{R}\right]f''(R) + \dot{R}^2 f'''(R) \tag{5}$$

respectively. The density due to dark matter in $f(R)$ gravity is

$$\rho_m = 3H^2 f'(R) - \rho_R \tag{6}$$

and we are considering pressureless dark matter, $p_m = 0$. In the subsequent section, we will consider the $f(R)$ gravity with the scale factor describing pre-bounce ekpyrotic contraction.

2. A reconstruction scheme for $f(R)$ gravity

In this section we describe reconstruction scheme for $f(R)$ gravity in a bounce model developed by Cai et al. [51] and further demonstrated by Cohen et al. [52] and Odintsov et al. [53]. In the model demonstrated by [52] a scalar field ϕ with non-canonical kinetic terms and a potential $V(\phi)$ was used to develop the cosmological model in a non-supersymmetric framework. In the present work instead of taking the potential as ekpyrotic potential i.e. $V(\phi) \approx -V_0 e^{-c(\phi)\phi}$ we have considered the chameleon scalar field in the framework of $f(R)$ gravity reconstructed for a scale scale factor describing a pre-bounce ekpyrotic contraction, as was mentioned in [52]. It may also be noted that the reconstruction procedure is similar to that of [53]. However instead of introducing the e-folding number N, contrary to what was presented in [53] we have demonstrated a reconstruction scheme for $f(R)$ through the cosmic time t and subsequently in terms of Ricci scale factor R. The presence of dark matter has also been considered. In Eq. (7), t_* is the big crunch time. However instead of constraining c by the lower bound of $\sqrt{6}$ we have constrained it by non-negativity. The scale factor describing a pre-bounce ekpyrotic contraction is

$$a(t) \propto (t_* - t)^{\frac{2}{c^2}} \tag{7}$$

giving rise to

$$\dot{a} = -\frac{2(t_* - t)^{\frac{2}{c^2}-1}}{c^2} \tag{8}$$

For the above choice scale factor the Hubble parameter and its derivatives come out to be as follows:

$$H = \frac{\dot{a}}{a} = -\frac{2}{c^2(t_* - t)},\quad \dot{H} = -\frac{2}{c^2(t_* - t)^2},\quad \ddot{H} = \frac{4}{c^2(t_* - t)^3}. \tag{9}$$

Using the form of Hubble parameter derived above the Ricci scalar as well as its derivatives are derived below:

$$R = 6\left(2H^2 + \dot{H}\right) = -\frac{12(c^2-4)}{c^4(t_* - t)^2},\quad \dot{R} = \frac{24(4-c^2)}{c^4(t_* - t)^3},\quad \ddot{R} = \frac{72(4-c^2)}{c^4(t_* - t)^4}. \tag{10}$$

It has already been stated that the purpose of the present work is to reconstruct $f(R)$ gravity and to demonstrate the cosmology of chameleon scalar field under this reconstruction. In view of the above the $f(R)$ gravity would now be reconstructed using Eqs. (9–10). Hence the modified field Eqs. give rise to the Friedmann equation in FRW geometry as follows:

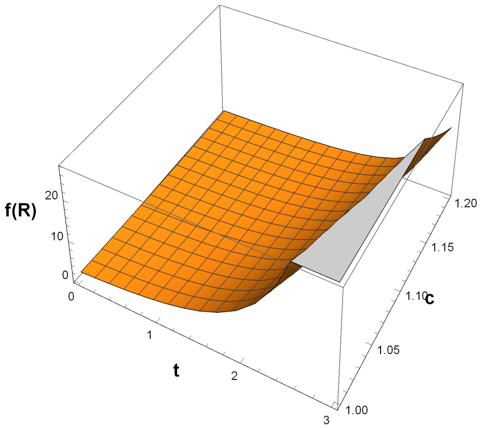

Figure 1. Evolution of reconstructed $f(R)$ (Eq. (12)).

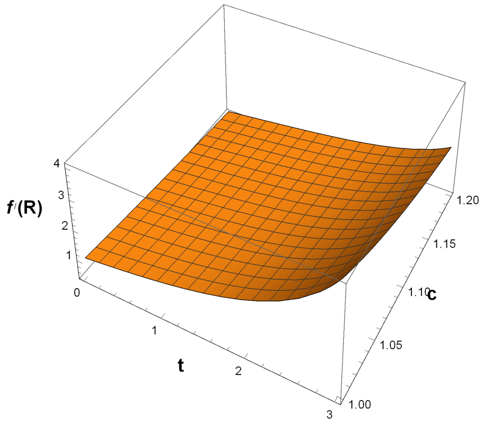

Figure 2. Evolution of reconstructed $f'(R)$.

$$-18\left(4H^2\dot{H}+H\ddot{H}\right)f''(R)+3\left(H^2+\dot{H}\right)f'(R)-\frac{f(R)}{2}+\rho_m=0, \tag{11}$$

where $\rho_m = \rho_{m_0} a(t)^{-3}$ indicates the dark matter density and R is the Ricci scalar as stated in Eq. (10). In Eq. (10) we have shown how R can be expressed in terms of t based on the choice of the scale factor. Hence, the modified Friedmann equation (11) written above gives rise to a differential equation with t as the independent variable which on solving gives us the solution for $f(R)$ in terms of cosmic time t as follows:

$$F(t)=1728^{\frac{1}{c^2}}A_1\left[-\frac{-4+c^2}{c^4(t_*-t)^2}\right]^{\frac{3}{c^2}}+12^{A_2}\left[-\frac{-4+c^2}{c^4(t_*-t)^2}\right]^{A_2}C_1+12^{A_3}\left[-\frac{-4+c^2}{c^4(t_*-t)^2}\right]^{A_3}C_2 \tag{12}$$

where, $F(t) = f(R(t))$. The above Eq. (12) is converted to a function of R and finally the reconstructed $f(R)$ takes the following the form:

$$f(R)=A_1R^{\frac{3}{c^2}}+R^{A_2}C_1+R^{A_3}C_2, \tag{13}$$

where

$$\begin{aligned}
A_1 &= \frac{1}{24-13c^2+c^4}\left[2^{1-\frac{6}{c^2}}3^{-\frac{3}{c^2}}\rho_{m_0}c^2(-4+c^2)\left(\frac{4-c^2}{c^4}\right)^{-\frac{3}{c^2}}\right],\\
A_2 &= -\frac{1}{4c^2}\left[2+c\left\{-3c+\frac{(4+c)\sqrt{c^4+20c^2+4}}{c^2-4}\right\}\right]+\frac{2\sqrt{c^4+20c^2+4}}{c^2(c^2-4)},\\
A_3 &= -\frac{1}{4c^2}\left[2+c\left\{-3c+\frac{(4+c)\sqrt{c^4+20c^2+4}}{c^2-4}\right\}\right]+\frac{\sqrt{c^4+20c^2+4}}{2(c^2-4)}.
\end{aligned} \tag{14}$$

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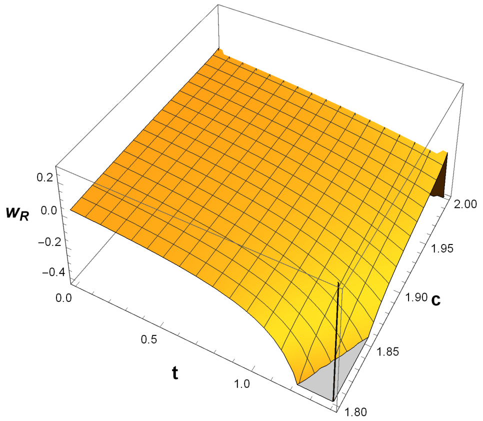

Figure 3. The EoS parameter $w_R = \frac{p_R}{\rho_R}$ (Eqs. (21)–(22)) due to reconstructed $f(R)$ gravity.

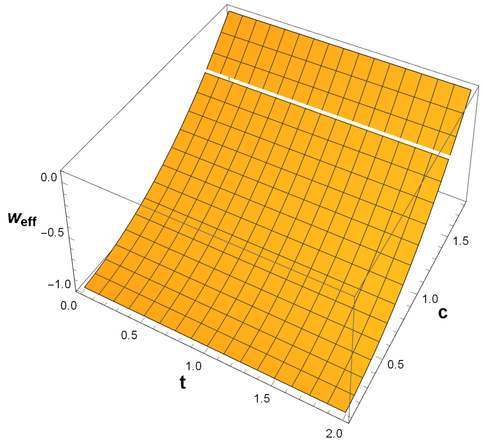

Figure 4. The behaviour of effective EoS parameter ($w_{eff} = \frac{p_R}{\rho_R+\rho_m}$) (Eqs. (21)–(23)).

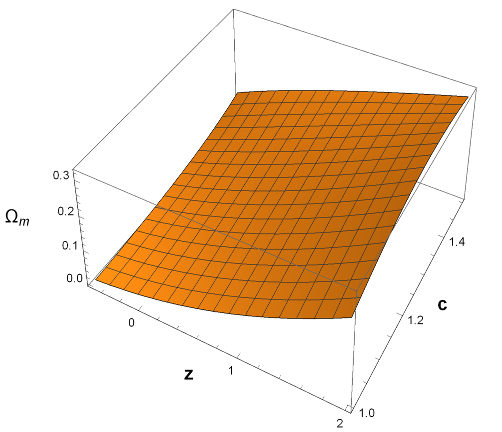

Figure 5. The evolution of the fractional matter density $\Omega_m = \frac{\rho_m}{3H^2}$ in the reconstructed $f(R)$ gravity.

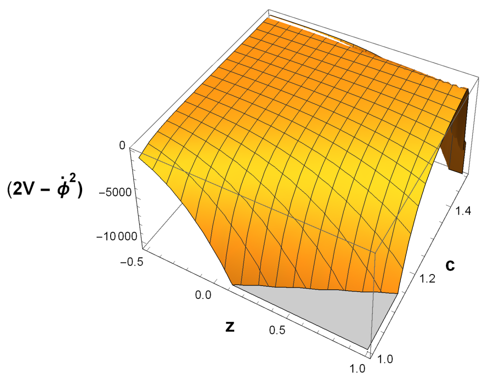

Figure 6. Evolution of $2V - \dot{\phi}^2$ in the reconstructed $f(R)$ gravity framework.

The derivatives of $f(R)$ up to various orders are computed below

$$f'(R) = \frac{1}{R}\left[\frac{3A_1R^{\frac{3}{c^2}}}{c^2} + A_2R^{A_2}C_1 + A_3R^{A_3}C_2\right], \tag{15}$$

$$f''(R) = \frac{1}{R^2}\left[\frac{3A_1(3-c^2)R^{\frac{3}{c^2}}}{c^4} + (-1+A_2)A_2R^{A_2}C_1 + (-1+A_3)A_3R^{A_3}C_2\right], \tag{16}$$

$$f'''(R) = \frac{1}{R^3}[\frac{3A_1(9-9c^2+2c^4)R^{\frac{3}{c^2}}}{c^6} + (-2+A_2)(-1+A_2)A_2R^{A_2}C_1 + (-2+A_3)(-1+A_3)A_3R^{A_3}C_2]. \tag{17}$$

It is clear from the above expressions that reconstructed $f(R)$ is a real solution to Eq. (11) and derivatives up to the orders shown above exist. In Fig. 1 we are observing that $f(R)$ is showing an increasing pattern with cosmic time for $0 < c^2 < 4$. This indicates that $\frac{d}{dt}f(R) > 0$ for $0 < c^2 < 4$. Also, Eq. (10) shows that $\dot{R} > 0$ for $0 < c^2 < 4$. Thus, $f'(R) = \frac{f(\dot{R})}{\dot{R}} > 0$ for $0 < c^2 < 4$. Hence, it is observed that $f(R)$ gravity so obtained is free from ghost instability. Also, from Fig.2 we observe that for $0 < c^2 < 4$, we have an increasing pattern of $f'(R)$ with cosmic time t. Hence, time derivative of $f'(R)$ would be positive. Therefore, we will be having $\frac{f'(\dot{R})}{\dot{R}} > 0$. Thus, the model of $f(R)$ obtained through the reconstruction scheme presented above is free from tachyonic instability. This proves the physical viability of the reconstructed $f(R)$ model.

3. Chameleon scalar field under modified $f(R)$ gravity

In this section the reconstructed $f(R)$ gravity would be considered to reconstruct the chameleon scalar field and chameleon potential to investigate whether they can lead to quasi-exponential expansion in the framework of the reconstructed $f(R)$ gravity. In this connection, it may be noted that an equivalence between $f(R)$ and scalar–tensor theories has been discussed in [56] and [57]. In these references it has been discussed how one may consider $\phi = R$ to reproduce the original action of chameleon mechanism. In a flat homogeneous universe the action for the relevant scalar field and potential is given by [55]

$$S = \sqrt{-g}d^4x\left[f(\phi)\mathcal{L} + \tfrac{1}{2}\phi_{,\mu}\phi^{,\mu} + \tfrac{R}{16\pi G} - V(\phi)\right] \tag{18}$$

where ϕ is the chameleon scalar field and $V(\phi)$ is the chameleon potential. R and G represents Ricci scalar and Newtonian constant of gravity respectively. $f(\phi)$ is an analytic function of ϕ and $f(\phi)\mathcal{L}$ is the modified matter Lagrangian. Variation of the action with respect to metric tensor components in a FRW cosmology leads to the following modified field equation (assuming $8\pi G = 1$).

$$H^2 = \frac{1}{3}\left[(\rho_m + \rho_R)f(R) + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \tag{19}$$

$$2\dot{H} + 3H^2 = \frac{1}{2}\left[-p_R f(R) - \frac{1}{2}\dot{\phi}^2 + V(\phi)\right]. \tag{20}$$

It may be noted that as we are considering the chameleon scalar field in $f(R)$ gravity framework we choose the density to be $\rho_m + \rho_R$, the pressure to be p_R and the analytic function to be replaced by a function of the Ricci scalar R.

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Solving Eqs. (4) and we get the reconstructed density contribution due to $f(R)$ gravity as follows:

$$\begin{aligned}\rho_R = & -\frac{1}{2c^2(-4+c^2)}\Bigg[1728^{\frac{1}{c^2}} A_1\left(30-13c^2+c^4\right)\left\{\frac{4-c^2}{c^4(t_*-t)^2}\right\}^{\frac{3}{c^2}} + \\ & c^2\Bigg\{12^{A_2}(-1+A_2)\left\{4+(-1+2A_2)c^2\right\}\left\{-\frac{-4+c^2}{c^4(t_*-t)^2}\right\}^{A_2} C_1 + \\ & 12^{A_3}(-1+A_3)\left\{4+(-1+2A_3)c^2\right\}\left\{-\frac{-4+c^2}{c^4(t_*-t)^2}\right\}^{A_3} C_2\Bigg\}\Bigg].\end{aligned} \tag{21}$$

Similarly, from Eq. (5) we derive the expression for pressure contribution for this reconstructed $f(R)$ gravity as follows:

$$\begin{aligned}p_R = & \frac{1}{6c^2(-4+c^2)}\Bigg[-2^{1+\frac{6}{c^2}}3^{1+\frac{3}{c^2}} A_1\left(-3+c^2\right)\left\{\frac{4-c^2}{c^4(t_*-t)^2}\right\}^{\frac{3}{c^2}} + \\ & c^2\Bigg\{-12^{A_2}(-1+A_2)\left\{-12+(3-4A_2)c^2+A_2(-1+2A_2)c^4\right\}\left\{-\frac{-4+c^2}{c^4(t_*-t)^2}\right\}^{A_2} C_1 - \\ & 12^{A_3}(-1+A_3)\left\{-12+(3-4A_3)c^2+A_3(-1+2A_3)c^4\right\}\left\{-\frac{-4+c^2}{c^4(t_*-t)^2}\right\}^{A_3} C_2\Bigg\}\Bigg]\end{aligned} \tag{22}$$

Now using the reconstructed $f(R)$ and ρ_R in Eq. (6) the form of dark matter density in $f(R)$ gravity framework comes out to be the following:

$$\begin{aligned}\rho_m = & \frac{1}{2(-4+c^2)}\left[\frac{1728^{\frac{1}{c^2}} A_1\left(24-13c^2+c^4\right)\left\{\frac{4-c^2}{c^4(t_*-t)^2}\right\}^{\frac{3}{c^2}}}{c^2} + \right. \\ & 12^{A_2}\left\{-4+c^2+A_2\left\{2+(-3+2A_2)c^2\right\}\right\}\left\{-\frac{-4+c^2}{c^4(t_*-t)^2}\right\}^{A_2} C_1 + \\ & \left. 12^{A_3}\left\{-4+c^2+A_3\left\{2+(-3+2A_3)c^2\right\}\right\}\left\{-\frac{-4+c^2}{c^4(t_*-t)^2}\right\}^{A_3} C_2\right].\end{aligned} \tag{23}$$

The equation of state parameters, as defined below, can be modified using the reconstructed pressure and densities elaborated above.

$$w_R = \frac{p_R}{\rho_R} \tag{24}$$

$$w_{eff} = \frac{p_R}{\rho_m + \rho_R} \tag{25}$$

At this juncture, we consider the modified field equations for reconstruction of the chameleon scalar field. Using modified field equations (19) and (20) we can have

$$2V - \dot{\phi}^2 = 8\dot{H} + 12H^2 + 2p_R f(R) \tag{26}$$

where, H, p_R and $f(R)$ have already been derived. The cosmological parameters derived above would now be explored through plots and the outcomes would be discussed in the subsequent section.

4. Background solution for $f(R)$ matter bounce

In the previous sections we have discussed the various outcomes of a scale factor describing the pre-bounce ekpyrotic contraction. In this section, we demonstrate a comparison of the results with the bounce scale factor proposed in [61]

$$a(t) = a_0\left(1 + \frac{3\sigma t^2}{2}\right)^{\frac{1}{3}} \tag{27}$$

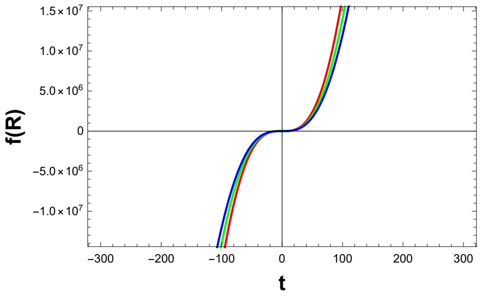

Figure 7. Evolution of the reconstructed $f(R)$ gravity over cosmic time t for scale factor $a(t) = a_0 \left(1 + \frac{3\sigma t^2}{2}\right)^{\frac{1}{3}}$.

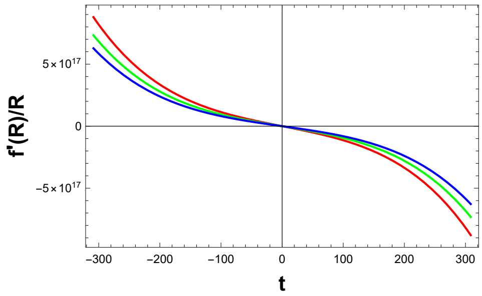

Figure 8. Evolution of $\frac{f'(R)}{R}$.

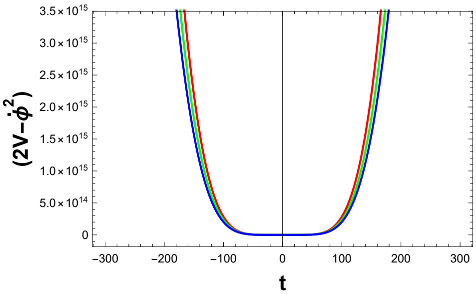

Figure 9. Evolution of the reconstructed $2V - \dot{\phi}^2$ over cosmic time t in the reconstructed $f(R)$ gravity (Fig. 7) framework.

where, a_0 is a scale factor at the bouncing point and σ is a positive parameter. With the help of this scale factor, we have derived Hubble parameter (H), Ricci scalar (R) and their derivatives in terms of cosmic time t as follows:

$$H = \frac{2t\sigma}{2 + 3t^2\sigma}, \quad \dot{H} = \frac{2\sigma(2 - 3t^2\sigma)}{(2 + 3t^2\sigma)^2}, \quad \ddot{H} = \frac{36t\sigma^2(t^2\sigma - 2)}{(2 + 3t^2\sigma)^3} \tag{28}$$

$$R = \frac{12\sigma(2 + t^2\sigma)}{(2 + 3t^2\sigma)^2}, \quad \dot{R} = -\frac{24t\sigma^2(10 + 3t^2\sigma)}{(2 + 3t^2\sigma)^3}, \quad \ddot{R} = \frac{24\sigma^2\left(-20 + 3t^2\sigma(44 + 9t^2\sigma)\right)}{(2 + 3t^2\sigma)^4} \tag{29}$$

This choice of $a(t)$ presents the bouncing behavior corresponding to matter-dominated contraction and expansion. Furthermore, t ranges from $-\infty$ to $+\infty$. Bounce occurs at $t = 0$. Using the expression for R, we can straightway express t as

$$t(R) = \pm \left(\sqrt{\frac{2}{3}} \sqrt{\frac{-R\sigma + \left(\sigma^2 \pm \sqrt{\sigma^3(4R + \sigma)}\right)}{R\sigma^2}} \right) \tag{30}$$

The above inversion is valid if $-\sqrt{\frac{2}{3\sigma}} < t < \sqrt{\frac{2}{3\sigma}}$. At the bouncing point we take $a_0 = 1$ and $\rho_{m0} = 1.41 \times 10^{-5}$. Roughly based on CMB spectrum, $\sigma = 7 \times 10^{-6}$.

The H, R and $a(t)$ discussed above. Clearly, with this choice,

$f'(R) = -\frac{(2+3t^2\sigma)^3 f'(t)}{24t\sigma^2(10+3t^2\sigma)}$

and

$f''(R) = \frac{(2+3t^2\sigma)^5\left(\left(-20+132t^2\sigma+27t^4\sigma^2\right)f'(t)+t\left(20+36t^2\sigma+9t^4\sigma^2\right)f''(t)\right)}{576t^3\sigma^4(10+3t^2\sigma)^3}$.

Because of the complicated form the coefficients the differential equation (Eq. 11) with $f'(R)$, $f''(R)$ in the above forms could not be solved analytically. The reconstructed $f(R)$ is obtained numerically and the solution in terms of cosmic time t in graphically presented in Fig. 7. Here in the numerical solutions, the positive parameter σ played a significant role. So, we have taken three values of $\sigma = 0.75,\ 1.25,\ 1.75$ from the interval $(0.7, 2)$ for the entire calculations. In the following figures, the red, green and blue lines indicate the values $\sigma = 0.75, 1.25,$ and 1.75 respectively. It is observed in Fig. 7 that $f(R) \to 0$ as $t \to 0$ before as well as after bounce. Before bounce the reconstructed $f(R)$ is tending to 0 from negative side. However, after attaining 0 at the bouncing point it starts increasing towards positive direction. Hence, $f(R) \to 0$ irrespective of pre or post bounce scenario. Hence, it is apparent that a realistic solution is available with this choice of bounce scale factor. Secondly, Fig. 8 shows that the $\frac{f'(R)}{R} > 0$ before bounce and < 0 after bounce. Hence in the pre-bounce phase the model is not affected by tachyon instability and this is consistent with the pre-bounce ekpyrotic contraction presented in the previous section. However, after the bounce, $\frac{f'(R)}{R} < 0$ after the bounce. Hence, the post-bounce scenario is characterized by tachyon instability. As we study $2V - \dot{\phi}^2$ we observe that it is positive in the pre as well as post-bounce scenario. However, for $t^- < t < t^+$, i.e. in the vicinity of the bouncing point, $2V - \dot{\phi}^2$ is nearly flat and after the bounce, it starts increasing sharply and hence, we may consider it to be consistent with the inflationary expansion.

5. Results and Conclusions

In the present work we have carried out a reconstruction scheme for $f(R)$ gravity with the scale factor in the form $a(t) = (t_* - t)^{\frac{2}{c^2}}$. Initially a Hubble parameter H has been computed and its time derivatives of different orders have also been derived for this scale factor to get the Ricci scalar R. Subsequently, the field equation for $f(R)$ gravity Eq. (11) has been solved in presence of dark matter. The reconstructed $f(R)$ has been obtained this way as a function of t in Eq. (12), which has been reexpressed as a function of R in Eq. (13). This reconstructed $f(R)$ has been plotted again cosmic time t for a range of values of $0 < c^2 < 4$ in Fig. 1. It has been observed in Fig. 1 that $f(R)$ has an increasing pattern with t. Also, Eq. (13) shows that $\lim_{R\to 0} f(R) = 0$. This satisfies a sufficient condition for a realistic model [58,59]. Furthermore, the non-existence of ghost and tachyonic instability have been established. Hence, it can be said that the reconstructed $f(R)$ model is a realistic model. It may be noted that $C_1 = 0.5, C_2 = 0.3, t_* = 4.1, \rho_{m_0} = 0.32$ and $0 < c^2 < 4$ have been used while creating the plots. Choice of $C_1,\ C_2$ have been made through trial and error, t_* and c^2 have been set keeping the real solution for $f(R)$ in mind. It may be noted that the formulation of the reconstruction approach is inspired by [23].

It has further been observed that the increasing pattern of $f(R)$ is influenced by the value of c. For larger values of c, $f(R)$ maintains approximately flat pattern for a considerable period of cosmic time and exhibits sudden increase at a later stage of the universe. Different derivatives of this reconstructed $f(R)$ has also been computed and presented in Eqs. (15), (16) and (17). In the next of this study we have demonstrated behaviour of chameleon scalar field under this modified form of $f(R)$. The modified field equations in presence of chameleon scalar field and potential have been presented in Eqs. (19) and (20). While reconstructing chameleon scalar field and potential under this reconstructed $f(R)$ model we have first reconstructed the density and pressure due to $f(R)$ using Eq. (13). Based on the reconstructed p_R and ρ_R we have demonstrated the behaviour of EoS parameter w_R due to $f(R)$ gravity in Fig. 3 and effective EoS parameter w_{eff} in Fig. 4. It is observed in Fig. 3 that w_R stays in the negative level for the evolution of the universe. This holds true for the entire range of c. Hence it may be concluded that w_R due to reconstructed $f(R)$ gravity behaves like phantom. The phantom behaviour is stronger for lower values of c than the higher values. However if we consider w_{eff} with the same choice of parameters it is found that for $0 < c0.5$, $w_{eff} \approx -1$ i.e. behaving approximately like cosmological constant. In Fig. 5 it is observed that the fractional energy density $\Omega_m = \frac{\rho_m}{3H^2}$ based on the reconstructed ρ_m and the scale factor describing the pre-bounce ekpyrotic contraction is staying at positive level. This satisfies the weak energy condition (WEC). Furthermore, the model is producing fractional matter density that is decaying with the evolution of the universe. This indicates transition from matter domination in the early stage to dark energy domination in the late stage.

In Fig. 6, the evolution of $2V - \dot{\phi}^2$ has been studied against z for a range of values of c. This figure shows that $2V - \dot{\phi}^2 \leq 0$ which violates the condition for inflationary expansion. Hence quasi-exponential expansion is not available with the chameleon scalar field considered in the framework of modified $f(R)$ gravity in presence of the scale factor $a(t) = (t_* - t)^{\frac{2}{c^2}}$ describing the pre-bounce ekpyrotic contraction. This observation is in contradiction to the study of Chattopadhyay et. al. [54], where quasi-exponential expansion was found to be possible for a linear $f(T)$ gravity based on holographic Ricci dark energy.

While concluding, we should mention that the reconstruction of $f(R)$ gravity has been demonstrated in the model to unify the the bouncing behavior in the early universe and the late-time accelerated expansion of the universe at the dark energy dominated stage can occur within a unified model. In view of the same, a scale factor describing the pre-bounce ekpyrotic contraction has been chosen and the reconstructed $f(R)$ has been demonstrated that is capable of transiting the universe from matter dominated to dark energy dominated phase. Also, the reconstructed $f(R)$ has been found to be free from ghost and tachyonic instabilities and hence it has been interpreted that the reconstructed $f(R)$ is realistic. We could explicitly derive the $f(R)$ gravity model capable of demonstrating the pre-bounce ekpyrotic contraction and the late time acceleration in a single model framework. This kind of reconstruction approach has earlier been demonstrated in [60] in the framework of $f(G)$ gravity. As future study, we propose to demonstrate whether this reconstruction scheme works for any arbitrary choice of scale factor and to compare the results with conventional reconstruction scheme using e-foldings.

While concluding we have demonstrated the background matter bound for $f(R)$ with another bouncing scale factor introduced in [61] and found that although the pre bounce scenario the model is having tachyonic stability and the tachyonic stability is lost after the bouncing point. At the bouncing point we have found that $f(R) \to 0$ for $t \to 0^-$ as well as $t \to 0^+$. At this juncture we must mention some important works in the direction of matter bounce solutions in modified gravity framework. With the scale factor in the form $a(t) = (a_0 t^2 + 1)^n$ [62] demonstrated that Lagrange multiplier $f(R)$ is more adequate than standard $f(R)$ in realizing the cosmological bounce. The matter bounce scenario, the singular bounce, the superbounce and a symmetric bounce scenario in modular $f(R)$ was discussed by [63]. The present work primarily focused on pre-bounce ekpyrotic contraction scenario. Nevertheless, the bouncing scenarios demonstrated in [63] are proposed to be incorporated into the study of exit from the pre-bounce ekpyrotic contraction in modified gravity framework.

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References

[1] S. D. Odintsov, V.K. Oikonomou, Phys. Rev. D **92**, 124024 (2015).
[2] A. G. Riess et al., Astron. J. **116**, 1009 (1998).
[3] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999).
[4] P. de Bernardis et al., Nature **404**, 955 (2000)
[5] S. Perlmutter et al., Astrophys. J. **598**, 102 (2003).
[6] T. Padmanabhan, Phys. Rep. **380**, 235 (2003).

[7] P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[8] E. J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
[9] V. Sahni, Lect. Notes Phys. **653**, 141–180 (2004).
[10] K. Bamba, S. Capozziello, S. Nojiri, S. D. Odintsov, Astrophys. and Space Sci. **342**, 155 (2012).
[11] S. Nojiri, S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007).
[12] T. P. Sotiriou, V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).
[13] S. Tsujikawa, Lect. Notes Phys. **800**, 99 (2010).
[14] S. Nojiri, S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **11**, 1460006 (2014).
[15] S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2012).
[16] S. Capozziello, S. Carloni, A. Troisi, Recent Res. Dev. Astron. Astrophys **1**, 625 (2003).
[17] S. Nojiri, S. D. Odintsov, Phys. Lett. B **576**, 5 (2003).
[18] S. M. Carroll, V. Duvvuri, M. Trodden, M. S. Turner, Phys. Rev. D **70**, 043528 (2004).
[19] S. Nojiri, S. D. Odintsov, Phys. Rev. D **74**, 086005 (2006).
[20] S. Nojiri, S. D. Odintsov, D. Saez-Gomez, Phys. Lett. B **681**, 74 (2009).
[21] S. Carloni, R. Goswami, P. K. S. Dunsby, Class. Quant. Grav. **29**, 135012 (2012).
[22] P. K. S. Dunsby et al., Phys. Rev. D **82**, 023519 (2010).
[23] S. Nojiri, S. D. Odintsov, J. Phys. Conf. Ser. **66**, 012005 (2007).
[24] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).
[25] S. Nojiri, S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003).
[26] S. Nojiri, S. D. Odintsov, Phys. Lett. B **657**, 238 (2007).
[27] S. Nojiri, S. D. Odintsov, Phys. Rev. D **77**, 026007 (2008).
[28] S. Nojiri, S. D. Odintsov, Phys. Rev. D **77**, 046009 (2008).
[29] S. Capozziello, S. Tsujikawa, Phys. Rev. D **77**, 107501 (2008).
[30] P. Brax, C. Van de Bruck, A. Davis, D. J. Shaw, Phys. Rev. D **78**, 104021 (2008).
[31] A. A. Starobinsky, JETP Lett. **86**, 157 (2007).
[32] S. A. Appleby, R. A. Battye, Phys. Lett. B **654**, 7 (2007).
[33] S. Nojiri, S. D. Odintsov, Phys. Lett. B **652**, 343 (2007).
[34] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, L. Sebastiani, S. Zerbini, Phys. Rev. D **77**, 046009 (2008).
[35] K. Bamba, S. Nojiri, S. D. Odintsov, JCAP **0810**, 045 (2008).
[36] J. D. Barrow, T. Clifton, Class. Quant. Grav. **23**, L1 (2005).
[37] T. Clifton, J. D. Barrow, Class. Quant. Grav. **23**, 2951 (2006).
[38] S. Capozziello, A. De Felice, JCAP **0808**, 016 (2008).
[39] K. Karami, M. S. Khaledian, J. High Energy Phys. **86**, (2011).
[40] L. N. Granda, Int. J. Mod. Phys. D **28**, 1950052 (2019).
[41] P. Horava, D. Minic, Phys. Rev. Lett. **85**, 1610 (2000).
[42] P. Horava, D. Minic, Phys. Rev. Lett. **509**, 138 (2001).
[43] S. Thomas, Phys. Rev. Lett. **89**, 081301 (2002).
[44] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
[45] I. G. Salako, A. Jawad, S. Chattopadhyay, Astrophys. Space Sci. **358**, 13 (2015).
[46] S. Chattopadhyay, A. Jawad, S. Rani, Adv. High Energy Phys., Article ID 798902 (2015).
[47] A. Jawad, A. Pasqua, S. Chattopadhyay, Eur. Phys. J. Plus **128**, 156 (2013).
[48] S. D. Odintsov and V. K. Oikonomou, Int. J. Mod. Phys. D **26**, 1750085 (2017).
[49] S. D. Odintsov and V. K. Oikonomou, Phys. Rev. D **91**, 064036 (2015).
[50] S. D. Odintsov and V. K. Oikonomou, Phys. Rev. D **90**, 124083 (2014).
[51] Y. F. Cai, D. A. Easson, R. Brandenburgers, JCAP. **1208**, 020 (2012).
[52] M. Cohen, J. L. Lehners, B. A. Ovrut, Phys. Rev. D. **90**, 025005 (2014).
[53] S. D. Odintsov, V. K. Oikonomou, E. N. Saridakis, Annals of Phys. **363**, 141 (2015).
[54] S. Chattopadhyay, A. Pasqua, I. Radinschi, A. Beesham, Int. J. Geom. Methods Mod. Phys. **15**, 1850167 (2018).
[55] S. Chattopadhyay, U. Debnath, Int. J. Mod. Phys. D **20**, 1135 (2011).
[56] T. Chiba, Phys. Lett. B **575**, 1 (2003).
[57] C. Burrage, J. Sakstein, J. Living Rev. Relativ. **21**, 1 (2018).
[58] S. Nojiri, S. D. Odintsov, Phys. Lett. B **657**, 238 (2007).

[59] S. Nojiri, S. D. Odintsov, Phys. Rev. D **77**, 026007 (2008).
[60] K. Bamba, A. N. Makarenko, A. N. Myagky, S. D. Odintsov, Phys. Lett. B **732**, 349 (2014).
[61] Y-F. Cai, S-H. Chen, J. B. Dent, S. Dutta, and E. N. Saridakis, Class. Quantum Grav. **28**, 215011 (2011).
[62] S. Nojiri, S. D. Odintsov, V. K. Oikonomou and T. Paul, Phys. Rev. D **100**, 084056 (2019).
[63] S. Nojiri, S.D. Odintsov, V. K. Oikonomou, Phys. Rev. D **93**, 084050 (2016).