

Unsteady Analytical Solution of The Influenced of a Thermal Radiation Force Generated from a Heated Rigid Flat Plate on Non-homogeneous Gas Mixture.

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Abstract

In the present paper, the effect of the non-linear thermal radiation on the neutral gas mixture in the unsteady state is investigated for the first time. The unsteady BGK technique of the Boltzmann kinetic equations for a neutral non-homogenous gas is solved. The solution of the unsteady case makes the problem more general significance than the stationary one. For this purpose, the moments' method, together with the traveling wave method, is applied. The temperature and concentration are calculated for each gas component and mixture for the first time.

Furthermore, the study is held for aboard range of temperatures ratio parameter and a wide range of the molar fraction. The distribution functions are calculated for each gas component and the gas mixture. The significant non-equilibrium irreversible thermodynamic characteristics the entire system is acquired analytically. That technic allows us to investigate the consistency of Boltzmann's H-theorem, Le Chatelier principle, and thermodynamics laws. Moreover, the ratios among the different participation of the internal energy alteration are evaluated via the Gibbs formula of total energy. The final results are utilized to the argon-helium non-homogenous gas at different magnitudes of radiation force strength and molar fraction parameters. 3D-graphics are presented to predict the behavior of the calculated variables, and the obtained results are theoretically discussed.

Keywords: Unsteady Exact analytical solutions; Partial differential equations system; Travelling wave method; Moment method; Boltzmann kinetic equation; Neutral non-homogenous gas; Thermal radiation force; Non-equilibrium irreversible thermodynamics; Internal energy.

I. Introduction

Thermal radiation is a deep-rooted part of our environment. Also, at high temperatures, the radiative heat transfer is a remarkable incomplete system analysis. Some application where thermal radiation transfer is of primary significance includes cryogenic fuel storage systems, spacecraft cooling systems, boilers and furnaces, and solar collectors [1, 2]. Although, the statistical-mechanical study of a non-homogenous gas in a non-equilibrium situation is a fascinating subject from theoretical as well as experimental viewpoint. Few papers are dealing with the gas mixture [3-9], compared with the significant number of studies in the case of a single neutral gas [10-16].

It is well known that the particles of a non-homogenous neutral gas radiate and absorb thermal energy. Therefore, in non-homogenous neutral gases, the interaction with thermal radiation is remarkable, unlike solids and liquids, which are capable of undergoing conspicuous volume alteration. Considering this interaction makes the non-homogeneous neutral gas way more realistic. Further, radiative heat transfer in non-homogenous neutral gases has remarkable applications from combustion processes to atmospheric modeling operations. Abourabia and

Abdel Wahid [13] presented a new approach for investigating the effect of the thermal radiation force on the non-homogenous neutral gases. For steady problem, this idea was applied on a half-space filled off by a non-homogenous neutral gas bounded by a fixed heated solid plane. The present paper is extended to investigate the non-stationary problem for a neutral non-homogenous gas affected by a thermal radiation force. Therefore, we should solve two systems of non-linear partial differential equations (PDEs) instead of one system of ordinary differential equations (ODEs), as presented in [13].

The Boltzmann kinetic equation is valid for all ranges of Knudsen number [14], while the Navier-Stokes (N.S.) system is suitable to give us acceptable results for the continuum flow only, (where $Kn = \frac{\lambda}{L}$ is the Knudsen number that measures the rarefaction of any gas molecules and represents the ratio between the mean free path λ to a characteristic length L). For this purpose, we utilize coupled systems of non-stationary BGK kinetic equations, one for each component of the neutral gas. The radiation force effect is inserted into the force term of the Boltzmann kinetic equation. These procedures are done by applying the Liu-Lees model for two-side Maxwell non-equilibrium distribution functions using the moment method. Moreover, the manner of the macroscopic characteristics of the non-homogenous gas is estimated for different radiation force strength according to different fixed, rigid plate temperatures. The temperature and concentration are, in turn, substituted into the related non-equilibrium distribution function. This approach permits us to investigate the manner of the equilibrium, non-equilibrium, and non-stationary distribution functions for different magnitudes of the molar fraction parameters. Also, the remarkable non-equilibrium thermodynamic characteristics of the entire system (neutral non-homogenous gas + heated plane solid plate) are calculated. Especially thermodynamic forces, entropy, entropy generation, entropy flux, and kinetic coefficients are investigated. Furthermore, the consistency of thermodynamics second law, Boltzmann H-theorem, and Onsager's relation are illustrated. The ratios among the different participations of the internal energy alteration are estimated via the Gibbs' formula. The results are applying to the argon-helium neutral non-homogenous gas. Finally, the remarkable conclusions of the paper are indicated.

II. The Physical and Mathematical Formulation of the Problem

Consider a neutral non-homogenous gas consisting of two species, for example, the A-Kind and B-Kind. The gas fills in the upper half of the space in the system ($y \geq 0$), and bounded by an illimitable immobile plane solid plate ($y=0$), in a constant pressure P_s [2, 13]. The solid plate is suddenly heated. That is generating a thermal radiation force. The flow is non-stationary and compressible. In a frame co-moving with the fluid the manner of the neutral non-homogenous gas is studied under the following conditions:

(I) The two velocities of the particles (incident and reflected) are equals at the fixed plane solid plate but in opposite signs. That result happens as a result of Maxwell's formula of momentum defuses reflection. Furthermore, the exchange will be generated by the temperature difference among the particles and the heated plane solid plate. That is taking the form of full energy accommodation [17].

(II) A thermal radiation force is acting from the heated plane solid plate on the neutral non-homogenous gas, written in vector form [17-18] as

$$\vec{f} = \frac{-4\sigma_s}{3n_s c} \nabla T^4(y, t) \text{ here } f_y = \frac{-16\sigma_s T^3}{3n_s c} \frac{\partial T(y, t)}{\partial y} \quad (1)$$

Here σ_s, n_s, c, T , and f_y are the Stefan-Boltzmann constant, the gas concentration at the plane solid plate surface, the velocity of light, the temperature, and the thermal radiation force component along y-axis direction, respectively. For a non-stationary motion, the operation in the entire system subject to f_y (the thermal radiation force) can be formulated in terms of the BGK kinetic equations in the following form [4, 7]:

$$\frac{\partial g_A}{\partial t} + \xi_y \frac{\partial g_A}{\partial y} + \frac{f_y}{m_A} \frac{\partial g_A}{\partial \xi_y} = \omega_{AA} (g_{0A} - g_A) + \omega_{AB} (g_{0B} - g_A), \quad (2)$$

$$\frac{\partial g_B}{\partial t} + \xi_y \frac{\partial g_B}{\partial y} + \frac{f_y}{m_B} \frac{\partial g_B}{\partial \xi_y} = \omega_{BB} (g_{0B} - g_B) + \omega_{BA} (g_{0A} - g_B). \quad (3)$$

Here ξ_y , g_v , and m_v are the molecular velocity of the non-homogenous neutral gas particles component along the y-axis, the two-sided Maxwell non-equilibrium distribution function, and the mass of particles of the v species. The four quantities ω_{AA} , ω_{AB} , ω_{BB} , and ω_{BA} are the collision frequencies that are mentioned in [4, 7] as:

$$\omega_{\gamma\gamma} = n_{s\gamma} \frac{\pi d_\gamma^2}{4} \left[\sqrt{\frac{8KT_s}{\pi m_\gamma}} \right], \quad \omega_{\gamma\nu} = n_{s\gamma} \pi \left(\frac{d_\gamma + d_\nu}{2} \right)^2 \left[\sqrt{\frac{8KT_s}{\pi \mu_{\gamma\nu}}} \right], \quad \mu_{\gamma\nu} = \left(\frac{m_\gamma m_\nu}{m_\gamma + m_\nu} \right),$$

here γ and $\nu \equiv A$ or B where d , $\mu_{\gamma\nu}$ and $n_{s\gamma}$ are the diameter of the effective collisions sphere, the reduced mass, and neutral gas mixture concentration at the solid plate surface, respectively. The local Maxwell non-equilibrium distribution functions g_{0v} are denoted by:

$$g_{0v} = \frac{n_v}{(2\pi RT_v)^{\frac{3}{2}}} \exp \left[\frac{-\xi^2}{(2RT_v)} \right], \quad \text{where } \xi^2 = \xi_x^2 + \xi_y^2 + \xi_z^2.$$

Lee's moment method [19-20] is employed here to gain the solution of the BGK kinetic equation. By adding heat to a non-homogeneous neutral gas, allowing it to expand, it is made rarer than the non-homogeneous gas neighbor sections. It continues to create an upward stream of the heated non-homogenous gas, which is usually followed by a flow in the opposite direction by the more distant parts of the non-homogenous gas. The fresh portions of non-homogenous gas are carried into the heat source neighborhood, taking their weather along with them to other regions [4, 7]. We assume that the temperature of the non-homogenous gas particles rising upward is while the temperature of the non-homogenous gas particles going downward is T_{2v} [13, 21]. The correspondent concentrations are n_{1v} and n_{2v} . We utilize the Liu-Lees model of the two-sided Maxwell non-equilibrium distribution function near the solid plane plate [17] for particles of the v species that can be expressed as:

$$g_v = \begin{cases} g_{1v} = \frac{n_{1v}}{(2\pi RT_{1v})^{\frac{3}{2}}} \exp \left[\frac{-\xi^2}{(2RT_{1v})} \right], \text{ For } \xi_y > 0 & \uparrow \\ g_{2v} = \frac{n_{2v}}{(2\pi RT_{2v})^{\frac{3}{2}}} \exp \left[\frac{-\xi^2}{(2RT_{2v})} \right], \text{ For } \xi_y < 0 & \downarrow \end{cases} \quad (4)$$

By multiplying the BGK kinetic equation by a velocity function $\psi_i(\xi)$ and integrating over the velocity space, we derive Maxwell's moment equations. This formula can be expressed as follows:

Particles of any non-homogeneous neutral gas component v with the second species γ

$$\begin{aligned} & \frac{\partial}{\partial t} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_i g_{2v} d\xi + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_i g_{1v} d\xi \right] + \frac{d}{dy} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_i \xi_y g_{2v} d\xi + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_i \xi_y g_{1v} d\xi \right] \\ & - \frac{F_y}{m_\beta} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{2v} \frac{d\psi_i}{d\xi_y} d\xi + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{1v} \frac{d\psi_i}{d\xi_y} d\xi \right] = \omega_{\beta\beta} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_i g_{0v} d\xi - \right. \\ & \left. \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{2v} + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{1v} \right) \psi_i d\xi \right] + \omega_{\gamma\gamma} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_i g_{0\gamma} d\xi - \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{2\gamma} + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{1\gamma} \right) \psi_i d\xi \right], \end{aligned} \quad (5)$$

where $d\xi = d\xi_x d\xi_y d\xi_z$.

The obtained equations (5) are known as the general equations of transfer [22]. Applying the non-dimensional forms by assuming:

$$y = y^* \left(\tau_{AB} \sqrt{2RT_s} \right), t = t^* \tau_{AB}, \vec{\xi} = \vec{\xi}^* \sqrt{2RT_s}, g_{iv} = \frac{g_{iv}^* (2\pi RT_s)^{\frac{3}{2}}}{n_s}, i = 0, 1, 2. \quad (6)$$

$$N_v = \frac{16\sigma_s T_s^3}{3n_s cm_v R}, T_{1v} = T_{1v}^* T_s, n_{1v} = n_{1v}^* n_s, T_{2v} = T_{2v}^* T_s, n_{2v} = n_{2v}^* n_s, \text{ and } dU_v = dU_v^* K_b T_s.$$

here τ_{AB} , N_v and dU_v are the relaxation time among collisions of the A-B species, a non-dimensional constant, and the internal energy modification of the non-homogenous neutral gas species v , respectively. When the values of g_{0v} , g_{1v} , and g_{2v} are calculated, then the macroscopic quantities (velocity, density, and temperature, etc.) can be evaluated as;

The bulk velocity of the gas:

$$u_v(y, t) = \frac{1}{n_\beta} \int \xi_y g_v(y, t, \xi_y) d\underline{\xi} = \frac{(n_{1v} \sqrt{T_{1v}} - n_{2v} \sqrt{T_{2v}})}{(n_{1v} + n_{2v})}. \quad (7)$$

The number density:

$$n_v(y, t) = \int g_v(y, t, \xi_y) d\underline{\xi} = \frac{(n_{1v} + n_{2v})}{2}. \quad (8)$$

The temperature:

$$T_v(y, t) = \frac{1}{3n_v} \int \xi^2 g_v(y, t, \xi_y) d\underline{\xi} = \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right). \quad (9)$$

The static pressure normal to the heated solid plate:

$$P_{yyv}(y, t) = \int \xi_y^2 g_v(y, t, \xi_y) d\underline{\xi} = \frac{1}{2} (n_{1v} T_{1v} + n_{2v} T_{2v}). \quad (10)$$

The y-component heat flux:

$$Q_{yv}(y, t) = \int \xi_y \xi^2 g_v(y, t, \xi_y) d\underline{\xi} = \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right). \quad (11)$$

The functions $T_{1v}(y, t)$, $T_{2v}(y, t)$, $n_{1v}(y, t)$ and $n_{2v}(y, t)$ in Eq. (4), which are unknowns to be calculated later in this paper for each component of the neutral non-homogenous gas. Therefore, we should examine the problems in a phase space coordinate system in which the velocity \mathbf{u} is at the origin.

Now, let $\psi_1 = \xi^2$ and $\psi_2 = \frac{1}{2} \xi_y \xi^2$. Substituting form Eq. (4) into Eq. (5), taking into account (6), we acquire the

moments of the BGK kinetic equations in non-dimensional form. After dropping the stars, we have eight PDEs for particles of the $v = A$, and B species. The energy conservation equation has the form:

$$\frac{\partial}{\partial t} (n_{1v} T_{1v} + n_{2v} T_{2v}) + \frac{\partial}{\partial y} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) + N_v \left(n_{1v} T_{1v}^{\frac{1}{2}} - n_{2v} T_{2v}^{\frac{1}{2}} \right) \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right)^3 \frac{\partial}{\partial y} \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right) = 0, \quad (12)$$

The y-direction component of the heat flux has the form:

$$\frac{\partial}{\partial t} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) + \frac{5}{4} \frac{\partial}{\partial y} \left[(n_{1v} T_{1v}^2 + n_{2v} T_{2v}^2) \right] + \frac{3N_v}{2} (n_{1v} T_{1v} + n_{2v} T_{2v}) \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right)^3 \frac{\partial}{\partial y} \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right) = \left[\frac{\omega_{vv}}{\omega_{AB}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) + \frac{\omega_{v\gamma}}{\omega_{AB}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) \right]. \quad (13)$$

The equations of state complement the above equations:

$$P_v = n_v T_v = \text{constant}, \quad (14)$$

Thus, we acquire the fourth equation in the form:

$$\left(n_{1v} T_{1v}^{\frac{1}{2}} - n_{2v} T_{2v}^{\frac{1}{2}} \right) = 0 \quad (15)$$

Traveling wave solution method [23] is utilized to solve the problem, by assuming

$$\mathcal{G} = ky - \Omega t, \quad (16)$$

That will transform the dependent variables from functions in (y,t) to functions in \mathcal{G} with the transformation constants k and Ω . Applying Eq. (11) we acquire the derivatives:

$$\frac{\partial}{\partial t} = -\Omega \frac{\partial}{\partial \mathcal{G}}, \quad \frac{\partial}{\partial y} = k \frac{\partial}{\partial \mathcal{G}} \text{ and } \frac{\partial^n}{\partial t^n} = (-1)^n \Omega^n \frac{\partial^n}{\partial \mathcal{G}^n}, \quad \frac{\partial^n}{\partial y^n} = k^n \frac{\partial^n}{\partial \mathcal{G}^n}, \quad (17)$$

where n is a positive integer. Substituting from Eqs. (11-12) into Eqs. (7-8) we acquire:

$$-\Omega \frac{\partial}{\partial \mathcal{G}} (n_{1v} T_{1v} + n_{2v} T_{2v}) + k \frac{\partial}{\partial \mathcal{G}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) + k N_v \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right)^3 \quad (18)$$

$$\frac{\partial}{\partial \mathcal{G}} \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right) \left(n_{1v} T_{1v}^{\frac{1}{2}} - n_{2v} T_{2v}^{\frac{1}{2}} \right) = 0$$

$$-\Omega \frac{\partial}{\partial \mathcal{G}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) + \frac{5}{4} k \frac{\partial}{\partial \mathcal{G}} \left[(n_{1v} T_{1v}^2 + n_{2v} T_{2v}^2) \right] + \frac{3}{2} k N_v \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right)^3 \quad (19)$$

$$\frac{\partial}{\partial \mathcal{G}} \left(\frac{n_{1v} T_{1v} + n_{2v} T_{2v}}{n_{1v} + n_{2v}} \right) (n_{1v} T_{1v} + n_{2v} T_{2v}) = \frac{\omega_{vv}}{\omega_{AB}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) + \frac{\omega_{v\gamma}}{\omega_{AB}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right),$$

Know, we intend to solve Eqs. (14)- (19) with the boundary value problem to evaluate T_{1v} , T_{2v} , n_{1v} and n_{2v} , as follows;

$$n_{2v} \sqrt{T_{2v}} = n_{1v} \sqrt{T_{1v}}. \quad (20)$$

Substituting from Eqs. (14) and (20) into Eq. (18), taking into consideration Eq. (10), we get:

$$k \frac{\partial}{\partial \mathcal{G}} \left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) = k \frac{\partial}{\partial \mathcal{G}} \left(n_{2v} \sqrt{T_{2v}} (T_{1v} - T_{2v}) \right) = 0 \quad (21)$$

Integrating Eq. (21), for \mathcal{G} , we get after factorization

$$\left(n_{1v} T_{1v}^{\frac{3}{2}} - n_{2v} T_{2v}^{\frac{3}{2}} \right) = \left(n_{2v} \sqrt{T_{2v}} (\sqrt{T_{1v}} + \sqrt{T_{2v}}) (\sqrt{T_{1v}} - \sqrt{T_{2v}}) \right) = \theta_{1v} \theta_{2v} = C_{2v}, \quad (22)$$

here we assume

$$\theta_{1v} = n_{2v} \sqrt{T_{2v}} (\sqrt{T_{1v}} + \sqrt{T_{2v}}), \quad \theta_{2v} = (\sqrt{T_{1v}} - \sqrt{T_{2v}}), \quad (23)$$

Moreover, C_{2v} is the integration constants. We can easily show that θ_{1v} , θ_{2v} they are constants.

To make our calculation simpler, and make better use of the Eq. (20), let us introduce the function $H_v(\mathcal{G})$ in the following form:

$$H_v(\mathcal{G}) = n_{2v} \sqrt{T_{2v}} = n_{1v} \sqrt{T_{1v}}. \quad (24)$$

From Eqs. (23 and 24) we can obtain:

$$T_{1v}(\mathcal{G}) = \frac{(\theta_{1v} + \theta_{2v} H_v)^2}{4H_v^2}, \quad T_{2v}(\mathcal{G}) = \frac{(\theta_{1v} - \theta_{2v} H_v)^2}{4H_v^2}, \quad n_{1v}(\mathcal{G}) = \frac{2H_v^2}{(\theta_{1v} + \theta_{2v} H_v)} \text{ and } n_{2v}(\mathcal{G}) = \frac{2H_v^2}{(\theta_{1v} - \theta_{2v} H_v)}. \quad (25)$$

Now, we can integrate equation (19) w.r.t. \mathcal{G} after performing some necessary algebraic manipulations, taking into consideration Eqs. (14) and (22), to get:

$$\frac{5}{4}k(n_{1v}T_{1v}^2 + n_{2v}T_{2v}^2) + \frac{3}{2}kN_v \frac{(n_{1v}T_{1v} + n_{2v}T_{2v})^5}{(n_{1v} + n_{2v})^4} = \left(\left(\frac{\omega_{vv}}{\omega_{AB}} + \frac{\omega_{v\alpha}}{\omega_{AB}} \right) \mathcal{G} + \Omega \right) \theta_{1v} \theta_{2v} + \theta_{3v}, \quad (26)$$

here θ_{3v} is the constant of integration. Substituting from Eqs. (25) into Eq. (26), we have:

$$(H_v^{-8}) \left(\begin{aligned} &3kN_v \theta_{1v}^9 - 12kN_v \theta_{1v}^7 \theta_{2v}^2 H^2 + 18kN_v \theta_{1v}^5 \theta_{2v}^4 H_v^4 + 4k \theta_{1v}^3 H_v^6 (80 - 3N_v \theta_{2v}^6) \\ &+ H_v^8 (-2^{10} \theta_{3v} + \theta_{1v} \theta_{2v} (-2^{10} \Omega + 960k \theta_{2v} + 3kN_v \theta_{2v}^7)) + \left(\frac{\omega_{vv}}{\omega_{AB}} + \frac{\omega_{v\gamma}}{\omega_{AB}} \right) \mathcal{G} \end{aligned} \right) = 0 \quad (27)$$

To solve this equation, we obtain eight roots for $H_v(\mathcal{G})$, using any symbolic software. We take root into account, which preserves the positive signs of both concentration and temperature.

The magnitudes of the integrated constants can be evaluated under the initial and boundary condition $(y, t) = (0, 0) \Rightarrow \mathcal{G} = 0$:

$$\frac{(n_{1v}(\mathcal{G} = 0) + n_{2v}(\mathcal{G} = 0))}{2} = C_v, \text{ where } C_v = \frac{n_{sv}}{n_s} \text{ is the molar fraction.} \quad (28)$$

$$\left(\frac{n_{1v}(\mathcal{G} = 0)T_{1v}(\mathcal{G} = 0) + n_{2v}(\mathcal{G} = 0)T_{2v}(\mathcal{G} = 0)}{n_{1v}(\mathcal{G} = 0) + n_{2v}(\mathcal{G} = 0)} \right) = 1, \quad (29)$$

$$\left(n_{1v}(\mathcal{G} = 0)T_{1v}(\mathcal{G} = 0)^{\frac{1}{2}} - n_{2v}(\mathcal{G} = 0)T_{2v}(\mathcal{G} = 0)^{\frac{1}{2}} \right) = 0. \quad (30)$$

Also, the temperature of the incident particles is T_2 . The heat of the reflected particles from the solid plane plate is represented by temperature T_1 , which can be expressed, as mentioned in [4, 7]:

$$T_{2v}(\mathcal{G} = 0) = \chi_v T_{1v}(\mathcal{G} = 0) : 0 < \chi_v \leq 1, \quad (31)$$

here χ_v is the ratio among the solid plane plate and the temperature of the non-homogenous neutral gas. The parameter χ_v takes an arbitrary positive value less than unity to ensure that the solid plane plate remains hotter than the non-homogenous neutral gas. By solving the system of algebraic Eqs. (28)-(31), we have

$$n_{1v}(\mathcal{G} = 0) = 2C_v \left(\frac{\sqrt{\chi_v} - \chi_v}{1 - \chi_v} \right), n_{2v}(\mathcal{G} = 0) = \frac{2C_v}{1 + \sqrt{\chi_v}}, T_{1v}(\mathcal{G} = 0) = \chi_v^{-0.5} \text{ and } T_{2v}(\mathcal{G} = 0) = \chi_v^{0.5}. \quad (32)$$

The obtained formulae represent the initial and boundary conditions of the problem.

Substituting the calculated quantities T_1, T_2, n_1 and n_2 into the two-side Maxwell non-equilibrium distribution function in the form

$$g = \begin{cases} g_{1v}(y, t) = \frac{n_{1v}(y, t)}{T_{1v}^{\frac{3}{2}}(y, t)} e^{\frac{-\xi^2}{T_{1v}(y, t)}}, \text{ For } \xi_y > 0 \\ g_{2v}(y, t) = \frac{n_{2v}(y, t)}{T_{2v}^{\frac{3}{2}}(y, t)} e^{\frac{-\xi^2}{T_{2v}(y, t)}}, \text{ For } \xi_y < 0 \end{cases} \quad (33)$$

we can acquire the sought non-equilibrium distribution functions, which enable us to investigate the manner of the non-homogenous neutral gas particles. That was not possible if we utilized the N.S. equations system instead of the BGK kinetic equations.

III. The Non-Equilibrium Thermodynamic Characteristics of the System:

The entropy S per unit mass of the non-homogenous neutral gas has the form [24-26]:
 $S(y,t) = \rho^{-1} \sum_{v=A}^B \rho_v S_v$, where S_v is the entropy of the non-homogenous neutral gas species of the v species

denoted by $S_v(y,t) = - \int \varphi_v \text{Log} \varphi_v d\underline{\xi} = \frac{\pi^{\frac{3}{2}}}{8} \left(n_{1v} \left[3 - 4 \ln \left(\frac{n_{1v}}{T_{1v}^{\frac{3}{2}}} \right) \right] + n_{2v} \left[3 - 4 \ln \left(\frac{n_{2v}}{T_{2v}^{\frac{3}{2}}} \right) \right] \right)$. The y-component of

the entropy flux vector is [24-26]: $J_y(y,t) = \sum_{v=A}^B J_{yv}(y,t)$ where J_{yv} is the y-component of the entropy flux of the non-homogenous neutral gas species of the v species denoted by

$$J_{yv}(y,t) = - \int \xi_y \varphi_v \text{Log} \varphi_v d\underline{\xi} = \frac{\pi}{2} \left(n_{1v} \sqrt{T_{1v}} \left[1 - \text{Log} \left(\frac{n_{1v}}{T_{1v}^{\frac{3}{2}}} \right) \right] - n_{2v} \sqrt{T_{2v}} \left[1 - \text{Log} \left(\frac{n_{2v}}{T_{2v}^{\frac{3}{2}}} \right) \right] \right),$$

while the Boltzmann's entropy generation in the non-stationary state σ_s is formulated as mentioned in:

$$\sigma_s(y,t) = \frac{dS}{dt} = \frac{\partial S(y,t)}{\partial t} + \vec{\nabla} \cdot \vec{J}(y,t).$$

According to the theory of thermodynamics [4, 7, and 13], the thermodynamic forces can be evaluated:

The first is X_n that related to the modification in the concentration $X_n(y,t) = \frac{\Delta y}{n(y,t)} \frac{\partial n(y,t)}{\partial y}$.

The second thermodynamic force is X_T that related to the alteration in the temperature,

$$X_T(y,t) = \frac{\Delta y}{T(y,t)} \frac{\partial T(y,t)}{\partial y}.$$

The third thermodynamic force is X_R , which related to the modification in the radiation force energy,

$$X_R(y,t) = \frac{\Delta y}{U_R(y,t)} \frac{\partial U_R(y,t)}{\partial y}, \text{ where } U_{Rv}(y,t) = \left(\frac{16\sigma T_s^4}{3cn_s K_B T_s} \right) T_v^4(y,t) \text{ is the non-dimensional radiation force}$$

energy that affected the non-homogenous neutral gas particles and Δy is the thickness of the layer near to the solid plane plate that has units as the mean free path.

After calculating the thermodynamic forces and the entropy generation, the kinetic coefficients L_{ij} can be obtained from the following relation [24-26]:

$$\sigma_s(y,t) = \sum_i \sum_j L_{ij} X_i X_j = (X_1 \ X_2 \ X_3) \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \geq 0.$$

The Gibbs relation for the modification of the internal energy $dU(y,t)$ utilized to the entire system (neutral non-homogenous gas + heated plane solid plate) is $dU(y,t) = dU_s(y,t) + dU_v(y,t) + dU_R(y,t)$. Moreover, the internal energy change according to the modification of the extensive variables (entropy dU_s , volume dU_v and besides the temperature gradient generated by the thermal radiation force dU_R), are respectively donated for a non-homogenous neutral gas as follows:

$$dU_s(y,t) = \rho^{-1} \sum_{v=A}^B \rho_v dU_{sv}(y,t), \text{ here } dU_{sv}(y,t) = T_v dS_v(y,t),$$

$$dU_v(y,t) = \rho^{-1} \sum_{v=A}^B \rho_v dU_{vv}(y,t), \text{ here } dU_{vv}(y,t) = -P_v dV_v(y,t),$$

$$dU_R(y, t) = \rho^{-1} \sum_{v=A}^B \rho_v dU_{R_v}(y, t), \text{ here } dU_{R_v}(y, t) = \zeta_v \frac{\partial T_v^4(y, t)}{\partial y} \Delta y, \zeta_v = \frac{16\sigma T_s^4}{3cn_s K_B T_s}.$$

The pressure and modification in the volume are $P_v = n_v T_v$ and $dV_v = -\frac{dn_v}{n_v^2}$ respectively, and

$$dn(y, t) = \frac{\partial n}{\partial t} \delta t + \frac{\partial n}{\partial y} \delta y, dS(y, t) = \frac{\partial S}{\partial t} \delta t + \frac{\partial S}{\partial y} \delta y \text{ and } \delta y = 2.5, \delta t = 1.$$

IV. Results and Discussion

The manner of the neutral non-homogenous gas was examined under the influence of a thermal radiation force in the non-stationary state of a plane heat transfer problem in the entire system (neutral non-homogenous gas + heated plane solid plate). Also, the thermal radiation is presented in the force term of the Boltzmann kinetic equation for a non-homogenous neutral gas subject to the inequality $\lambda \gg \delta \gg d$, where $\delta = n^{-1/3}$ is the average spacing among the molecules, both λ and d are the mean free path and the molecular diameter, respectively. In all calculations and figures, we take the magnitudes of the following parameters from [28-29] for the argon-helium neutral non-homogenous gas, where the particles of the A and B components are the argon and helium non-homogenous neutral gases, respectively:

$$T_s = 1200K; \sigma_s = 5.670367 \times 10^{-8} \text{ kg s}^{-3} \text{ K}^{-4}; R = 8.3145 \text{ J.K}^{-1} \text{ mol}^{-1}; K_B = 1.3807 \times 10^{-23} \text{ J.K}^{-1}; c = 2.9979 \times 10^8 \text{ m.sec}^{-1};$$

$$n_s = 3 \times 10^{18} \text{ m}^{-3}; m_A = 39.948 m_u; m_B = 4.0026 m_u; m_u = 1.993 \times 10^{-26} \text{ Kg is atomic mass unit}; d_A = 3.84 \times 10^{-10} \text{ m}; \lambda_A = 1.017 \text{ m};$$

$$V_{TA} = 7.067 \times 10^2 \text{ m.sec}^{-1}; \tau_A = 1.439 \times 10^{-3} \text{ sec}; d_B = 6.2 \times 10^{-11} \text{ m}; \lambda_B = 39.356 \text{ m}; V_{TB} = 2.232 \times 10^3 \text{ m.sec}^{-1}; \tau_B = 1.748 \times 10^{-2} \text{ sec};$$

for a fixed $C_A = 0.5$, where K_B , V_{Tv} and τ_v are Boltzmann constant, thermal velocity, and the relaxation time of the non-homogenous neutral gas species of the v species, respectively. Using the idea behind the shooting method [2], we evaluate the transformation constants to obtain $k = 1.5$ and $\Omega = 3$. Dimensionless N has the values $N_A(1200K) = 1.5355$, $N_B(1200K) = 2.628$. All figures clarify that all variables satisfy the equilibrium condition:

1- Wherein the equilibrium as $(y, t) = (0, 0)$:

$$n(y=0, t=0) = 1, T(0, 0) = 1, S(0, 0) = \text{Its maximum value}, \sigma(0, 0) = 0, dU_s(0, 0) = 0, X_T(0, 0) = 0, X_n(0, 0) = 0, X_r(0, 0) = 0, \text{ as shown in Figs. (1, 2-c, 4-12)}$$

2- Wherein the equilibrium as $\chi = 1$:

$$n(\chi=1) = 1, T(\chi=1) = 1, S(\chi=1) = \text{Its maximum value}, \sigma(\chi=1) = 0, dU_s(\chi=1) = 0, dU_v(\chi=1) = 0, dU_r(\chi=1) = 0, X_r(\chi=1) = 0, X_n(\chi=1) = 0, X_r(\chi=1) = 0, \text{ as shown in Figs. (36-46)}$$

Now, we will discuss the manner of the non-homogenous neutral gas particles far from the equilibrium state:

Figures (1-a, b, c) clarify the meaning that the gas concentration is an extensive parameter, while figures (2-a, b, c) elucidate the implication that the gas concentration is an intensive parameter. Similar manner introduced by Figs. (19, 20-A, B, C) and Figs. (36, 37-A, B, C), while the number density n decreases, with time, the temperature T growing, these happen for all magnitudes of t . That is, according to the fact of the uniform pressure, check Figs. (1, 2-c). Similarly, while the number density n decreases, with time, the temperature T growing, these happen for all magnitudes of C_A . That is according to the fact of the uniform pressure, check figures (19, 20- c). The number density n decreases, with time, the temperature T growing, these happen for all magnitudes of χ . That is according to the fact of the uniform pressure, as shown in Figs. (36, 37-c).

Figures (3a, 3b, 3c) shed light upon an increment in f_{1v} and a decrement in f_{2v} compensating for each other, compared with f_0 for each gas and gas mixture, too. That gives a great qualitative agreement in the behavior of the distribution functions with [21]. That is explained by a departure from the equilibrium state, where the gas particles, having temperature T_1 and density n_1 , after being heated, are replaced by a counterchange by the non-homogenous neutral gas particles having temperature T_2 and density n_2 . This behavior agrees with Le-Chatelier's principle.

The entropy S always growing with time, and the entropy production σ has non-negative magnitudes for all values of t , χ and the molar fraction C_A . That gives a complete satisfaction of the second law of thermodynamics and the Boltzmann H-theorem. This manner agrees with Le Chatelier's principle, as shown in Figs. (4-5, 21-22, 38-39).

The manner of the different participation of the internal energy modification can be clarified in Figs. (7-9, 24-26, 41-43). The Gibbs formula estimates the numerical ratios among the diverse involvement of the internal energy alteration based upon total derivatives of the extensive parameters. Taking into consideration their different tendencies, the maximum numerical magnitudes of the three participations at different radiation force strengths (correspondent to different plane rigid plate temperatures), are ordered in quantity as follows:

(i) For fixed values ($\chi = 0.5$, and $C_A = 0.5$) and variable magnitudes of y and t in the considered range ($0 < t < 1$ and $0 < y < 2.5$), we have $dU_s(1200K) : dU_v(1200K) : dU_R(1200K) \approx 1.0 : 10^{-2} : 0.4$,

as shown in Figs. (7-9). The participation of dU_R reaches its maximum numerical value at the magnitudes of $y = 2.5$ that represents the far distance from the heated solid plate. That is due to the X_R manner related to each component in the non-homogenous gas, as shown in Fig. (14). The X_T (according to the gradient of temperature) will have the opposite direction to the X_n (according to the gradient of the density), as shown in Figs. (10-11). That gives a qualitative agreement in the manner with the thermodynamic forces determined in [28], comparing with the same neutral non-homogenous gas (Ar-He) at the same molar fraction $C_A = 0.5$.

(ii) For a fixed value ($\chi = 0.5$) and variable magnitudes of C_A in the considered range ($0.2 < C_A < 0.95$), we have $dU_s(1200K) : dU_v(1200K) : dU_R(1200K) \approx 1.0 : 0.1 : 2.0$,

as shown in Figs. (24-26). The participation of dU_R reaches its maximum numerical value at the quantities of $C_A = 0.2$ (i.e. $C_B = 0.8$) that represents the concentrations of the heavier non-homogenous neutral gas (argon) indirectly, and the lighter non-homogenous neutral gas (helium) respectively. That means that, the more, the lighter mass of the non-homogenous neutral gas species, the more the efficiency of the thermal radiation energy participation in the entire system total energy modification, as shown in Fig. (26). That is also according to the manner of the correspondent value of the X_R itself related to each component in the neutral non-homogenous gas, as shown in Fig. (29). The X_T (according to the gradient of temperature) will have the opposite direction to the X_n (according to the slope of the density), as shown in Figs. (27-28). That gives a qualitative agreement in the manner with the thermodynamic forces determined in [28], comparing with the same neutral non-homogenous gas (Ar-He) at the same molar fraction $C_A = 0.5$.

(iii) For a steady value $C_A = 0.5$ and variable values of χ in the considered range ($0.1 < \chi < 1$), we have

$$dU_s(1200K) : dU_v(1200K) : dU_R(1200K) \approx 1 : 10^{-1} : 1.2, \text{ as shown in Figs. (41-43).}$$

Figure (45) indicated that, dU_R reaches its maximum numerical value at $\chi = 0.1$ that means that the more the temperature difference (among the fixed plate surface temperature and the neutral non-homogenous gas temperature), the more will be the active participation of the thermal radiation energy in the entire system total energy modification. That is attributed to the manner of the correspondent magnitudes of the X_R itself, as shown in Fig. (44).

According to our calculations, the constraints imposed on the kinetic coefficients L_{ij} are fulfilled. $L_{11} \geq 0$, $L_{22} \geq 0$ and $L_{33} \geq 0$, for all magnitudes in the considered ranges of both χ and C_A . Onsager's reciprocal relations are fulfilled, as we have ($L_{12} \equiv L_{21}$, $L_{13} \equiv L_{31}$ and $L_{32} \equiv L_{23}$), for all magnitudes in considered ranges of t , χ , and C_A , as shown in Figs. (13-18, 30-35, 47-52). By using our model, we find that Onsager's kinetic inequality is fluctuating in the order of $\pm 10^{-14}$, $\pm 10^{-13}$, and $\pm 10^{-11}$, as shown in Figs. (6, 23, and 40), respectively, which is a very acceptable error in our calculations.

V. Conclusions

The calculated results were demonstrated to conclude the following:

- a) The predominant vital factors that are directly proportional to the efficiency of the radiation energy in the total energy modifications of the entire system are:
 - i) The ratio among the temperature of the plane solid plate surface and the neutral non-homogenous gas particles.
 - ii) The ratio among the mass of each component of the neutral non-homogenous gas.
 - iii) The molar fraction of each component of the non-homogenous gas.
- b) At a temperature ($T \geq 1200\text{K}$), the radiation energy participation in the modified total internal energy becomes the predominant one.
- c) The lighter non-homogenous neutral gas component, helium, is influenced by a thermal radiation force more than the more massive gas component, argon.
- d) The first laws of thermodynamics (notably, the second law), the Le-Chatelier's rule, the Boltzmann H-theorem, and the Onsager's reciprocal relations, are all fulfilled for the investigated non-homogenous gas entire system.
- e) The negative sign at specific kinetic coefficients, related to cross effects, implies in such cases that according to the imposed thermodynamic force (gradient), there is a heat flux in the opposite direction to the main flux. For instance, the negative sign in front of $L_{Tn} = L_{12}$ and $L_{nR} = L_{23}$ implies that there is a flow caused by the temperature gradient, from a lower to a higher temperature, known as thermal diffusion (or effect of Soret), which gives a qualitative agreement with the investigation results [23, 28].

Data Availability

- No data, models, or code were generated or used during the study.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

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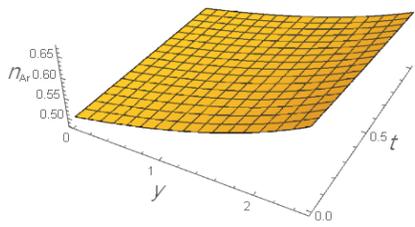


Figure (1-A): Concentrations n_A vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

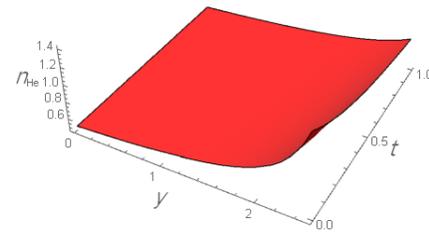


Figure (1-B): Concentrations n_B vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

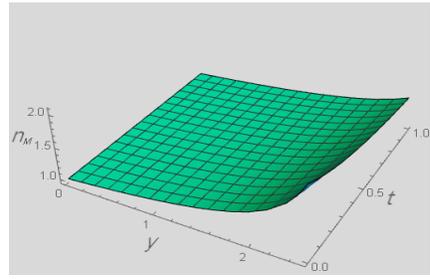


Figure (1-C): Concentrations n vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

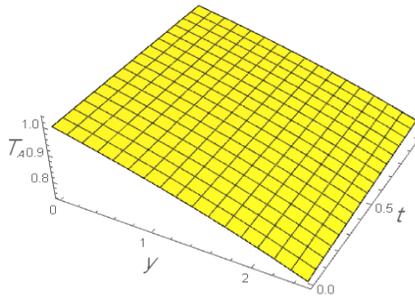


Figure (2-A): Temperature T_A vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

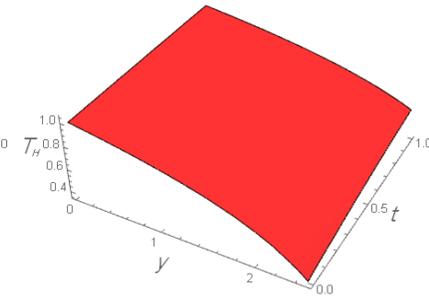


Figure (2-B): Temperature T_B vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

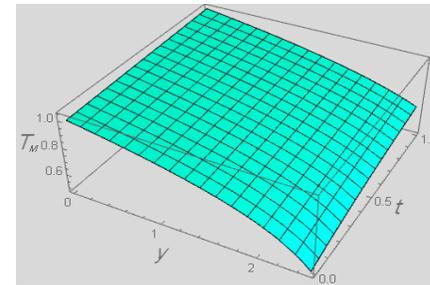
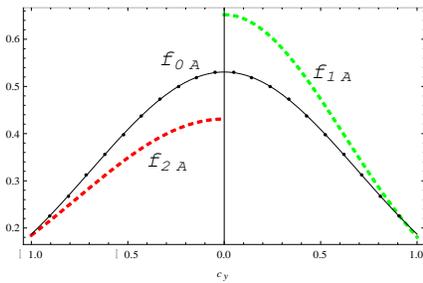
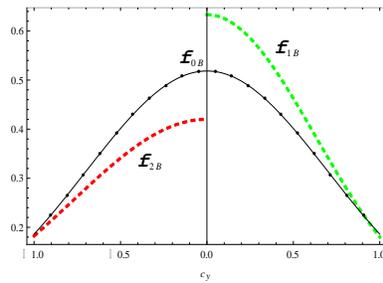


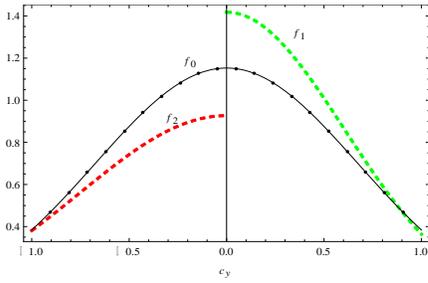
Figure (2-C): Temperature T vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.



Figure(3-A): The comparison between the combined perturbed velocity distribution functions f_A [f_1, f_2] and equilibrium velocity distribution function f_{0A} at ($\chi = 0.5$) for a fixed y value ($y=0.4$).



Figure(3-B): The comparison between the combined perturbed velocity distribution functions f_B [f_1, f_2] and equilibrium velocity distribution function f_{0B} at ($\chi = 0.5$) for a fixed y value ($y=0.4$).



Figure(3-C): The comparison between the combined perturbed velocity distribution functions f [f_1, f_2] and equilibrium velocity distribution function f_0 at ($\chi = 0.5$) for a fixed y value ($y=0.4$).

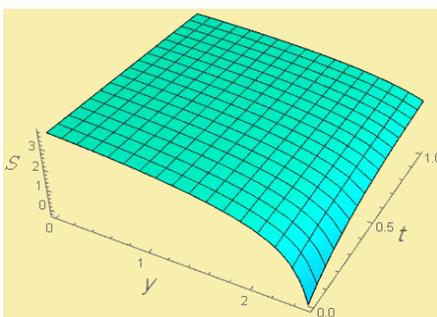


Figure (4): Entropy S vs. y and t for fixed

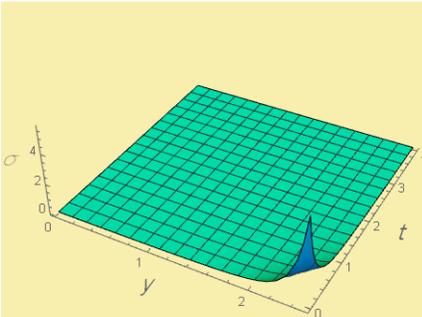


Figure (5): Entropy production σ , vs. y and

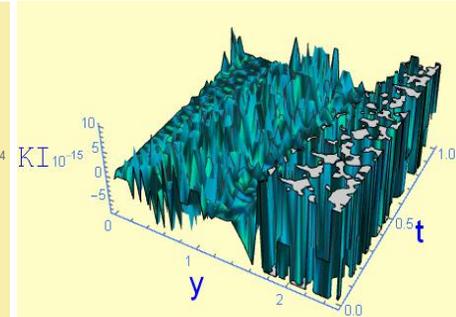


Figure (6): Kinetic Inequality vs. y and t for

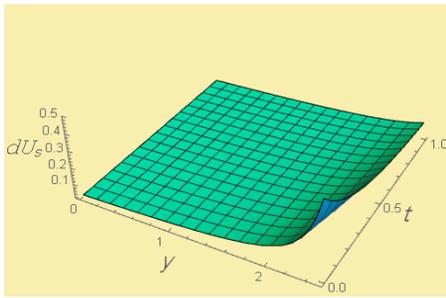


Figure (7): dU_s vs. y and t for fixed $\chi=0.5$, and

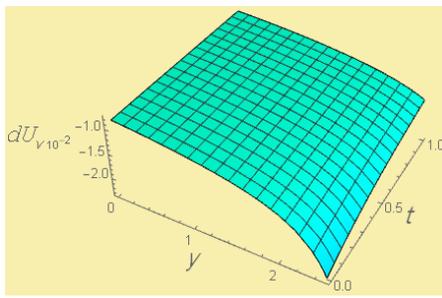


Figure (8): dU_v vs. y and t for fixed $\chi=0.5$, and

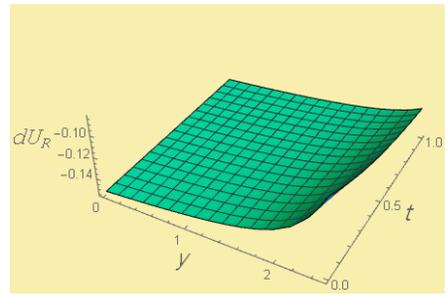


Figure (9): dU_R vs. y and t for fixed $\chi=0.5$, and

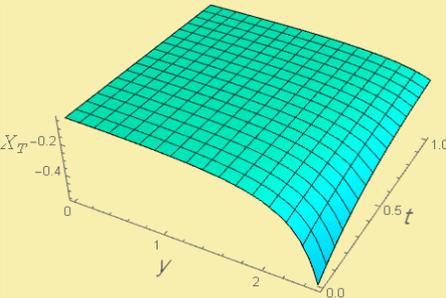


Figure (10): Thermodynamic force X_T vs. y

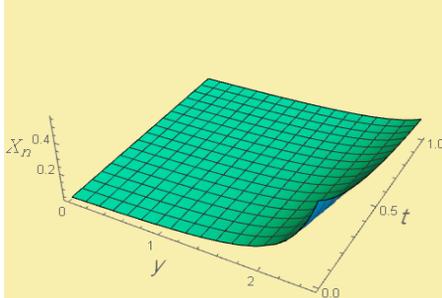


Figure (11): Thermodynamic force X_n vs. y

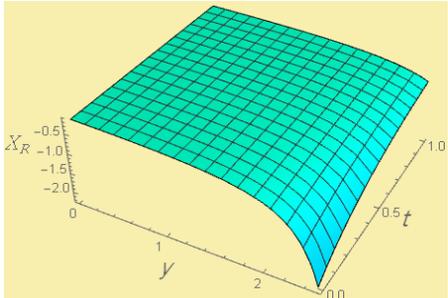


Figure (12): Thermodynamic force X_R vs. y

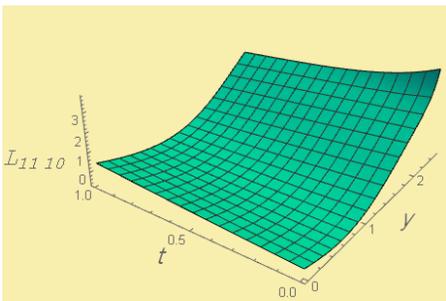


Figure (13): Kinetic coefficient $L_{11} \geq 0$ vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

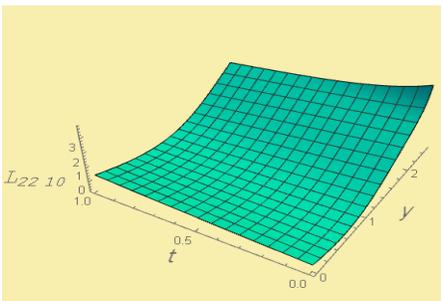


Figure (14): Kinetic coefficient $L_{22} \geq 0$ vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

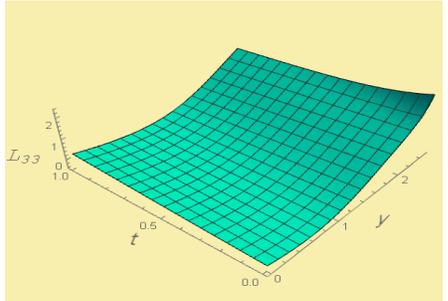


Figure (15): Kinetic coefficient $L_{33} \geq 0$ vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

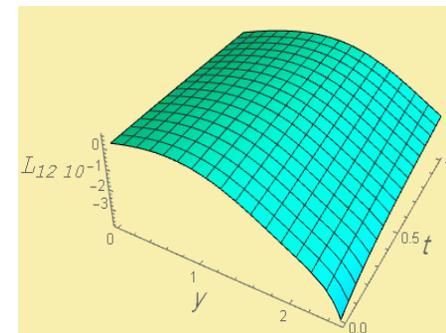


Figure (16): Kinetic coefficient $L_{12}=L_{21}$ vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

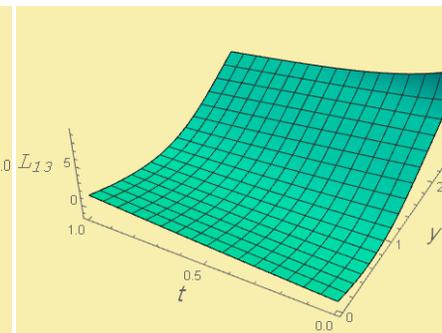


Figure (17): Kinetic coefficient $L_{23}=L_{32}$ vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

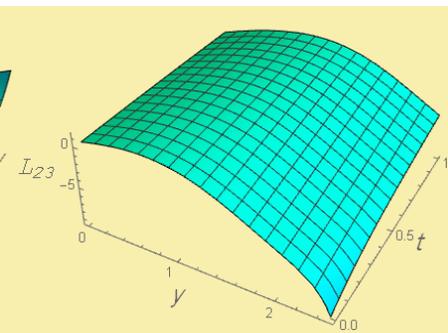


Figure (18): Kinetic coefficient $L_{31}=L_{13}$ vs. y and t for fixed $\chi=0.5$, and $C_A=0.5$.

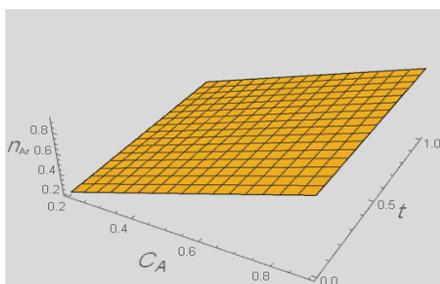


Figure (19-A): Concentrations n_A vs. t and C_A for fixed $\chi=0.5$, and $y=0.4$.

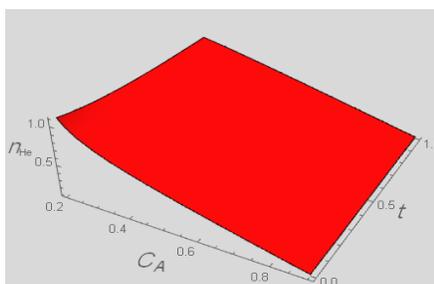


Figure (19-B): Concentrations n_B vs. t and C_A for fixed $\chi=0.5$, and $y=0.4$.

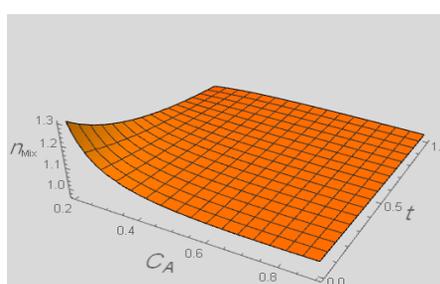


Figure (19-C): Concentrations n vs. t and C_A for fixed $\chi=0.5$, and $y=0.4$.

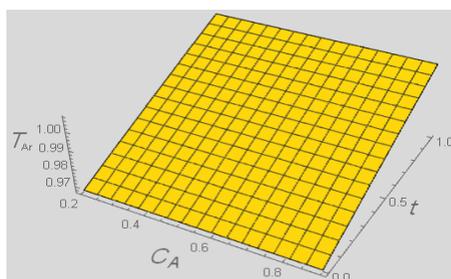


Figure (20-A): Temperature T_A vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

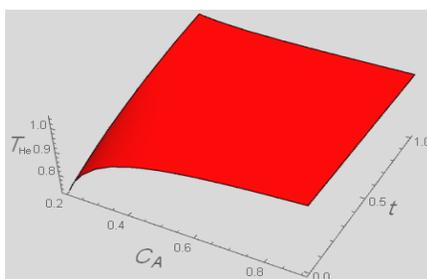


Figure (20-B): Temperature T_B vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

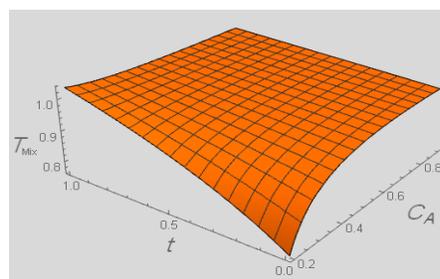


Figure (20-C): Temperature T vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

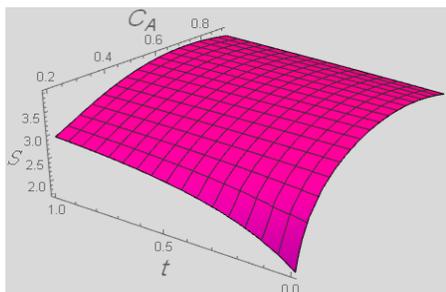


Figure (21): Entropy S vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

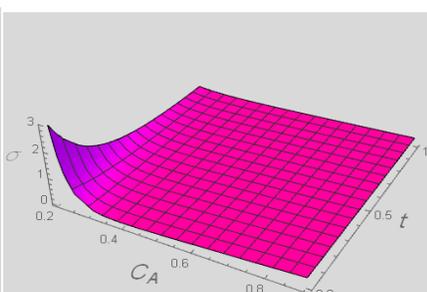


Figure (22): Entropy production σ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

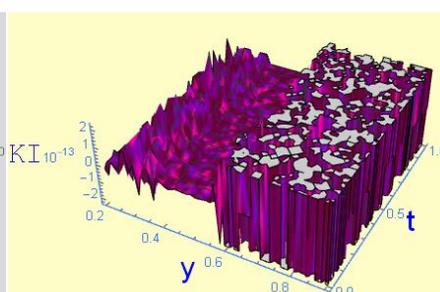


Figure (23): Kinetic Inequality vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

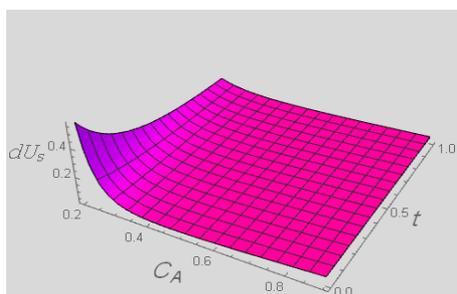


Figure (24): dU_S vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

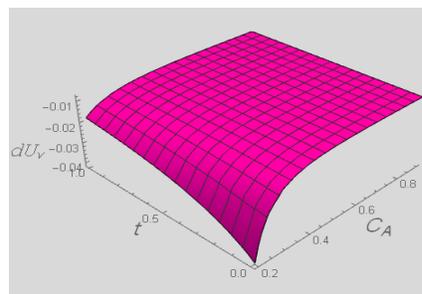


Figure (25): dU_V vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

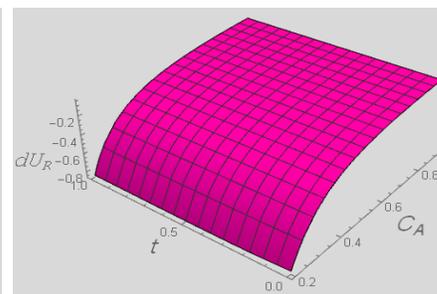


Figure (26): dU_R vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

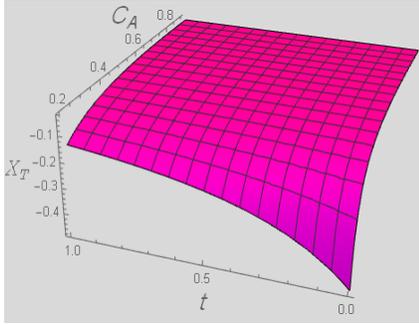


Figure (27): Thermodynamic force X_T vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

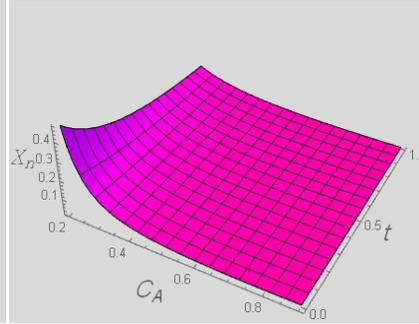


Figure (28): Thermodynamic force X_n vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

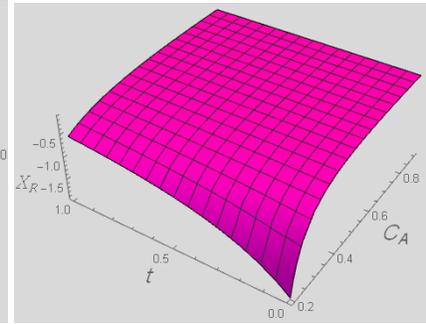


Figure (29): Thermodynamic force X_R vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

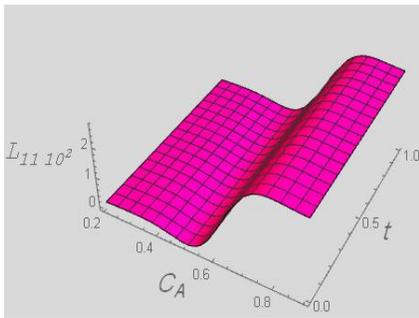


Figure (30): Kinetic coefficient $L_{11} \geq 0$ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

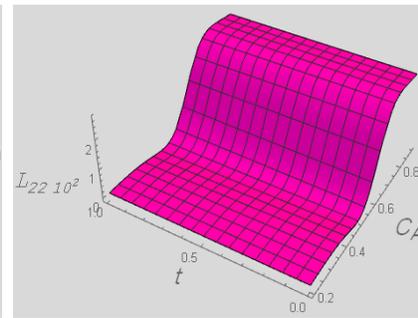


Figure (31): Kinetic coefficient $L_{22} \geq 0$ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

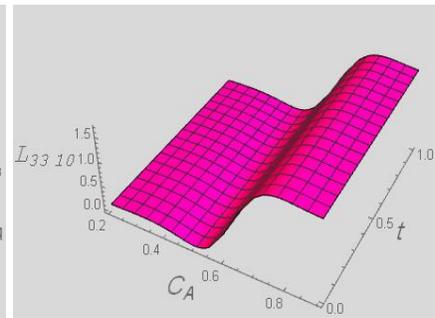


Figure (32): Kinetic coefficient $L_{33} \geq 0$ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

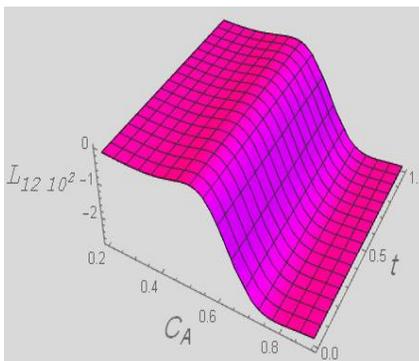


Figure (33): Kinetic coefficient $L_{12} = L_{21}$ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

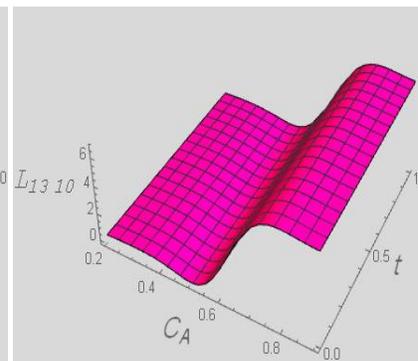


Figure (34): Kinetic coefficient $L_{23} = L_{32}$ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

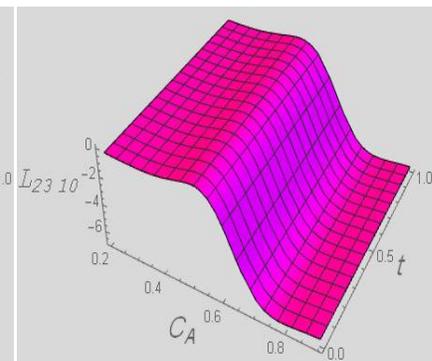


Figure (35): Kinetic coefficient $L_{31} = L_{13}$ vs. t and C_A for a fixed $y=0.4$, and $\chi=0.5$.

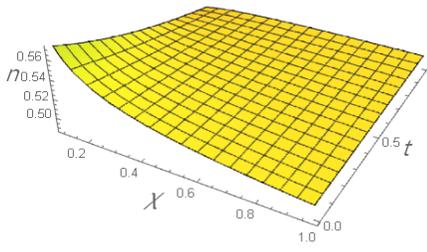


Figure (36-A): Concentrations n_A vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

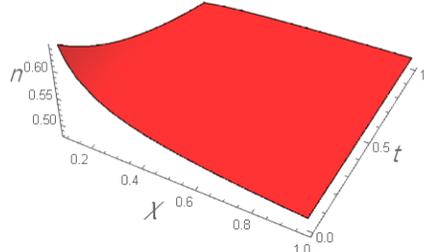


Figure (36-B): Concentrations n_B vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

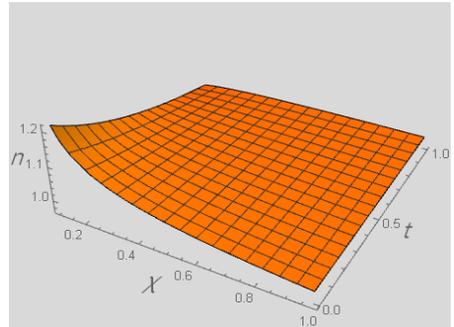


Figure (36-C): Concentrations n vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

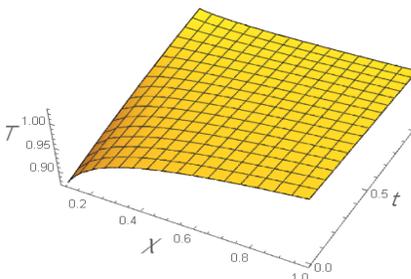


Figure (37-A): Temperature T_A vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

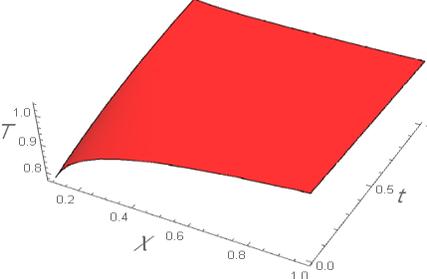


Figure (37-B): Temperature T_B vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

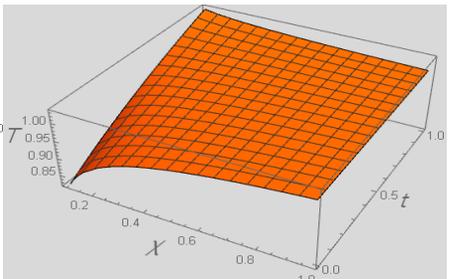


Figure (37-C): Temperature T vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

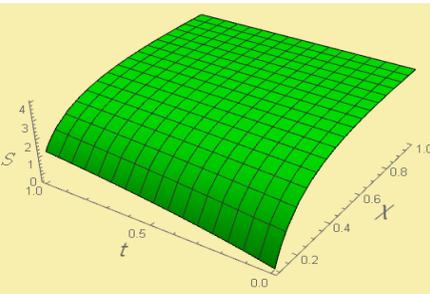


Figure (38): Entropy S vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

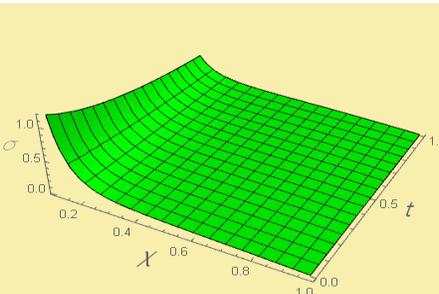


Figure (39): Entropy production σ_s vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

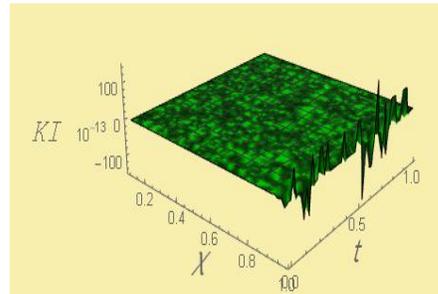


Figure (40): Kinetic Inequality vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

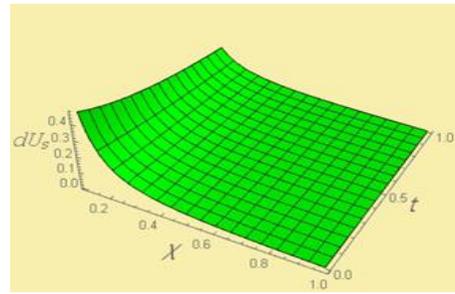


Figure (41): dU_S vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

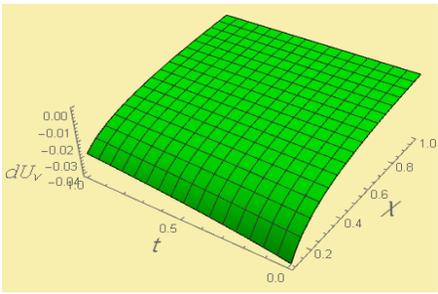


Figure (42): dU_V vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

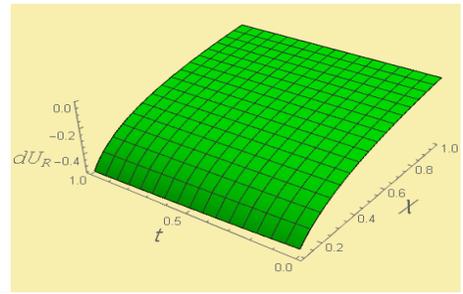


Figure (43): dU_R vs. t and χ for a fixed $y=0.4$, and $C_A=0.5$.

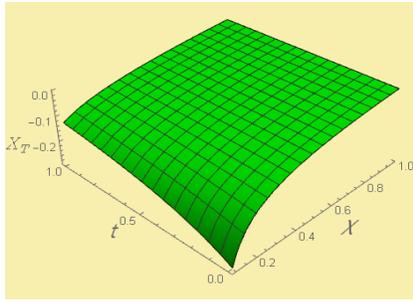


Figure (44): Thermodynamic force X_T vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

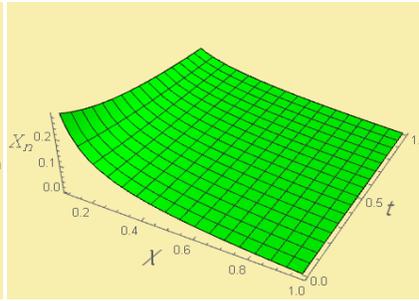


Figure (45): Thermodynamic force X_n vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

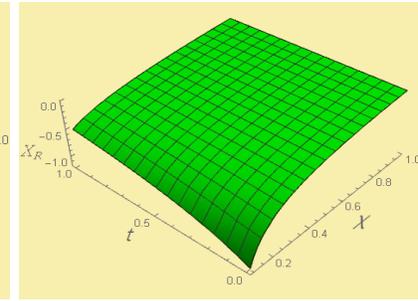


Figure (46): Thermodynamic force X_R vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

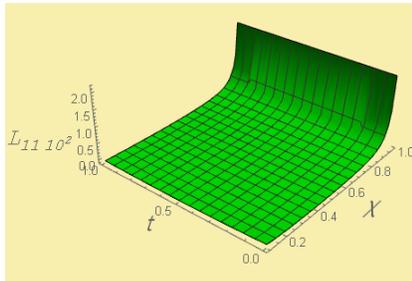


Figure (47): Kinetic coefficient $L_{11} \geq 0$ vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

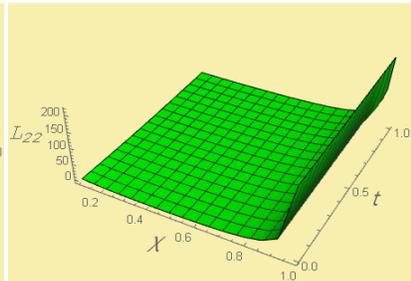


Figure (48): Kinetic coefficient $L_{22} \geq 0$ vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

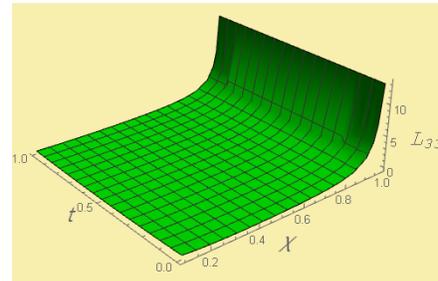


Figure (49): Kinetic coefficient $L_{33} \geq 0$ vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

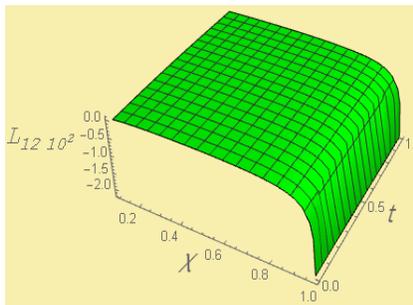


Figure (50): Kinetic coefficient $L_{12} = L_{21}$ vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

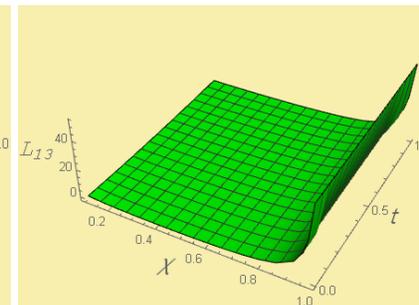


Figure (51): Kinetic coefficient $L_{23} = L_{32}$ vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.

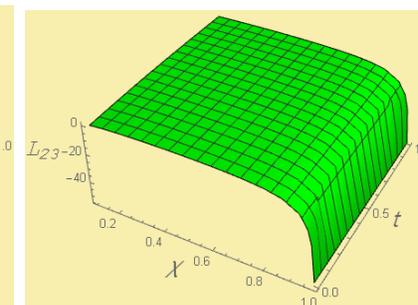


Figure (52): Kinetic coefficient $L_{33} \geq 0$ vs. t and χ for a fixed $y=0.4$, and $C_A = 0.5$.