

Unified field theory for electromagnetic and gravity fields with the introduction of quantized space–time and zero-point energy

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Abstract

In our previous papers [1,3], using only the concepts of the zero-point energy and quantized space–times, all the fields including gravity were explained. However, the previous papers had the following limitations: First, the concept of the quantized space–time must be experimentally confirmed. Second, we should clarify the meaning of the quantized Einstein’s gravity equation, which is derived in [1]. Moreover, in another paper [2], we succeeded in describing the neutrinos’ self-energy and their oscillations. However, this paper assumes the rest energy of 3-leptons in advance, which is why we needed to uncover the reason why leptons have 3-generations. As mentioned, using the concepts of the zero-point energy and quantized space–times, we derived the quantized Einstein’s gravity equation in our previous paper [1]. The paper provides an analytical solution of this equalized Einstein’s equation, which implies the conservation of angular momentum in terms of quantized space–times. Employing this solution and without the standard big bang model, a unique form of acceleration equation for the acceleration-expansion universe is derived. Moreover, the temperature of the cosmic microwave background (CMB) emission is also obtained. Further, this solution results in an analytical (not numerical) derivation of the gravity wave. Moreover, based on the configuration of quantized space–times in terms of both electric and magnetic fields, we analytically attempted to calculate every equation in terms of electromagnetic and gravity fields, using the solution of the quantized Einstein’s gravity equation. As a result of this theory, first the calculated acceleration and temperature of CMB emission agree with the measurements. Furthermore, the analytical solution of the quantized Einstein’s gravity equation resulted in all the laws of electromagnetic and gravity fields in addition to the analytically derived gravity wave, which agrees well with the recent measurements. Moreover, the calculations of the energies in the basic configuration of the quantized space–times resulted in all 3-leptons’ rest energies. Considering this basic configuration is uniformly distributed everywhere in the universe, we can conclude that τ -particles or static magnetic field energy derived from the basic configuration of the quantized space–times is the identity of dark energy, which also distributes uniformly in the universe.

Keywords: unified field theory, zero-point energy, quantized space–time, quantized Einstein’s gravity equation, conservation of angular momentum in terms of quantized space–times

1. Introduction

1.1 Summary of the entire contents including previous works

This paper introduces concepts of quantized space–times and zero-point energy. By these concepts, we succeed in reinforcing our previously established unified particle theory [1,3] as well as giving the reason for 3-generations of leptons. Furthermore, these concept results in quantization of the Einstein’s gravity equation and its analytical solution imply, in a real sense, the conservation of angular momentums in terms of quantized space–times. This solution solves various current universe problems, such as acceleration–expansion universe, dark energy, and so on. Furthermore, this solution of the quantized Einstein’s gravity equation creates all the laws and equations regarding electromagnetic and gravity fields. Considering our previous works [1,3] described that electromagnetic and gravity fields were created from the zero-point energy in principle, the electromagnetic and gravity fields were related to the weak interaction, strong interaction, neutrinos, quarks, protons, and neutrons, because all these fields are also created from the zero-point energy [1,3]. Thus, we now reinforce the unified field theory on previous paper and present the basic principle that the conservation of angular momentums in terms of quantized space–times, i.e., both the zero-point energy and quantized space–times, creates all physics laws regarding particle physics.

1.2 Background

In our previous papers [1,3], with the zero-point energy and the concepts of quantized space–times, it is succeeded to describe all the interactions of electromagnetic interaction, gravity interaction, weak interaction, and strong interaction with no numerical or fitting methods, with the agreements of the measurements. Moreover, in another paper [2], we described neutrino self-energy and their oscillations analytically, which agrees with the measurements.

However, in these previous papers, we did not describe

1. Rotations of quantized space–times, using the quantized Einstein’s gravity equation, which is derived in the previous paper [1].
2. Comparisons with the measurements for proving existences of the proposed quantized space–times.
3. Our neutrino theory [2] depends on the assumption that 3-leptons’ masses are given.
4. In view of particle physics, what dark energy is.

Regarding 3-leptons masses, we succeed in obtaining these masses from the basic configuration of the quantized space–times, which basically agrees with the measurements. This result is important because our presented concept of the quantized space–times is certified by the measurements. Moreover, because the dark energy generally distributes uniformly, it is allowed to conclude that the energy of this configuration of the quantized space–times implies the dark energy. Furthermore, because this configuration also implies that the static magnetic field energy (GeV order), this fact can explain recent measurements [4,5] that everywhere in the universe there are static magnetic fields even in no macroscopic objects.

One of the significances of this paper is that we succeed in obtaining the analytical (not numerical) solution of the Einstein’s gravity equation. This is because the introduction of quantized space–times results in the quantization of the Einstein’s gravity equation, which enables us to solve this equation analytically. The resultant facts from this analytical solution are

- 1) The temperature for the cosmic microwave background (CMB) emission is predicted, which agrees with the measurements.
- 2) The acceleration-expansion of the universe can be described, which also agrees well with the recent measurements. The expansion of the universe was known in the era of Einstein, but this expansion is taking acceleration, which was reported in [6]. We claim that this fact cannot be explained according to the standard big bang model.
- 3) The gravitational wave is calculated analytically. Thus far, gravitational wave has been calculated only using the numerical methods.
- 4) With the concept of the quantized space–times and with this solution of the quantized Einstein’s equation, the conservation of angular momentums of the quantized space–times creates all the laws in electromagnetism and gravity. That is, with our previous papers [1,3], the unified field theory of particle physics is now reinforced.

Now let us consider the problems in the current standard big bang model.

- 1) It cannot explain acceleration-expansion in the universe with quantity.
- 2) There is a light-element problem. The standard big bang model’s prediction for the amount of Li does not seriously agree with the recent measurement [7].
- 3) As mentioned, CMB emission well be descried without the standard big bang model.
- 4) The most serious problem is that, considering the singularity and acceleration-expansion, the standard big bang model must assume there is infinite energy in the universe. This assumption is strongly against all the general physics equations, because all physics equations generally form under the conservation of energy.
- 5) The big bang model does not describe the dark energy. This paper also clarifies the identity of

dark energy. However, this paper claims that this energy is merely the well-known particle, which obeys general gravitational law. Therefore, this paper claims that the dark energy which exhibits repulsive forces does not exist.

In short, the standard big bang model cannot describe the recent cosmology problems and is not supported by the measurements in a real sense. In particular, the above-mentioned “Li problem” is serious. Therefore a new model has been pursued by other researchers recently.

1.3 Summary of significances in the present paper

As a summary of significances in this paper, confirming the agreements with the measurements, we succeeded in confirming the existence of the basic configuration of the quantized space–times, and the quantized space–times’ concept and zero-point energy concept resulted in an analytical solution of the Einstein’s gravity equation, which implies the conservation of angular momentum of the quantized space–times. This solution creates all the electromagnetic and gravity fields’ laws and equations. In our previous papers [1,3], the weak interaction, strong interaction and particle fields themselves are well described only with the concepts of the zero-point energy and quantized space–times. Therefore we now reach an important principle: all the physical fields and their laws are created only by the conservation of angular momentums in terms of the quantized time-spaces, i.e., the zero-point energy with the introduction of quantized space-times.

In other contribution, this paper could obtain the reason why leptons and neutrinos have 3-generations, which has been puzzled since the establishment of particle physics. Additionally, the main problems in the cosmology have been solved without the standard big bang model.

2. Theory

2.1 Review of the concepts of quantized space–times and of the quantized Einstein’s gravity equation

2.1.1 Concept of a quantized space–time

Starting the result of Dirac equation, which implies that a photon creates an electron and a positron.

$$\hbar\omega_0 = 2m_e c^2, \quad (1)$$

where ω_0 , m_e , and c denote a constant angular frequency, the mass of an electron, and the speed of light, respectively.

This equation can be interpreted as

$$\frac{1}{2}\hbar\omega_0 = m_e c^2. \quad (2)$$

This equation produces the minimum quantized length λ_0 and time t_0 in terms of a space–time:

$$\lambda_0 = \frac{\hbar}{2m_e c}, \quad (3)$$

$$t_0 = \frac{\hbar}{2m_e c^2}. \quad (4)$$

We derive a more general constant quantized space–time’s length and time:

$$\lambda_c = \lambda_0 \sqrt{1 - \frac{v^2}{c^2}}, \quad (5)$$

$$t_c = t_0 \sqrt{1 - \frac{v^2}{c^2}}. \quad (6)$$

We consistently assume that the above length (5) and time (6) are the minimum length and time. Thus, they cannot be divided further. As discussed later, it was found that these concepts are supported by the measurement.

In eq. (2), the left-hand side is identical for the zero-point energy in the Harmonic oscillator Hamiltonian.

$$H = (n + \frac{1}{2})\hbar\omega_0. \quad (7)$$

As every quantum field theory argues, the first term implies AC electromagnetism. However, the second term called zero-point energy, which is neglected in the quantum field theory, is more important because of DC electromagnetism. Note that we have also described that first term photon creates the Maxwell’s time-dependent equations in view of different approaches from the quantum field theory.

Figure 1 shows a schematic of the basic configuration of the quantized space–times. Two quantized space–times in terms of an electric field are rotating with velocity v . Each quantized space–time in terms of an electric field has embedded up- and down-spin electrons, respectively. Note that it is necessary to distinguish these embedded electrons from real-body electrons. In this way, a quantized space–time in terms of a magnetic field is induced. As discussed later, the energies of these two quantized space–times are commonly expressed as the zero-point energy. The forces F is the Lorentz force originating from this static magnetic field, which is identified with the attractive gravity forces F from the gravitational field. This is important because, in this scale, a gravity and a magnetic field are unified, which results in the quantized Einstein’s gravity equation. An important notation that this configuration can be described by the solution of the quantized Einstein’s gravity equation, as will be discussed later.

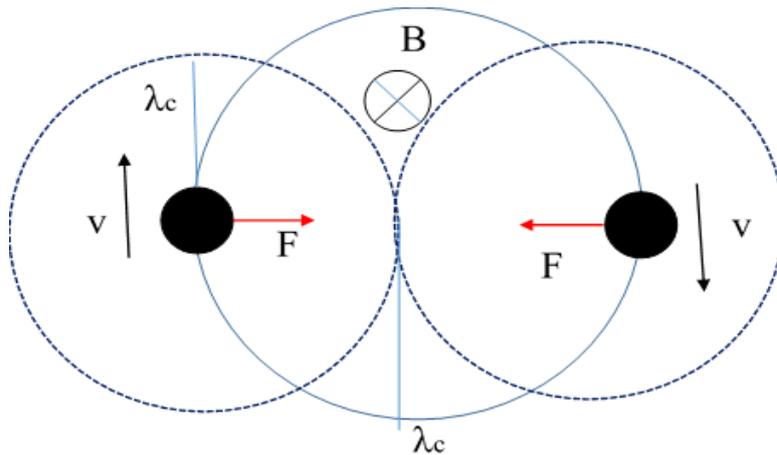


Fig. 1

Schematic diagram of the basic configuration of quantized space-times. Note that this figure was cited from [1,2].

First, two quantized space-times embedding electrons, which are not real bodies, have rotations. An important point is that these two embedded electrons have opposite spins with each other. That is, one has up-spin and the other has down-spin. This fact will take the importance when considering the creation of a μ -particle. The radius of the quantized time-space is λ_c , which is determined using the Dirac equation and Lorentz contraction. For details, refer to our previous paper [1].

By rotations of the embedded electrons, i.e., the rotations of the two quantized space-times, another quantized space-times in terms of a magnetic field is induced. This magnetic field accompanies a concept of flux. That is, as shown, another quantized space-time whose radius is the same as λ_c . Therefore, it is very important to distinguish two quantized space-times as the different ones:

- 1) A quantized space-time accompanying an embedded electron in terms of an electric field
- 2) A quantized space-time induced in terms of a magnetic field

Moreover, a force F is generated as a result of the magnetic field generation, as the Lorentz force. As discussed in our various previous papers, generally when relative momentum in terms of two charged particles is zero, these two particles experience an attractive force, which stems from the Lorentz force. As shown, this attractive force F is identical to an attractive gravity force, which is related to the quantization of the Einstein's gravity equation.

1.1.2 Quantization of the Einstein's gravity equation

According to our previous paper [1], energies' relationship in terms of gravity, static magnetic field and electric field in the scale of the quantized space-times are derived as

$$u_B = -G^2 \frac{1}{r^4} \frac{1}{c^2} u_E \left(\frac{\hbar}{2c^2} \right)^2, \quad u_B = u_G \quad (8)$$

where r , G , u_B , u_G , and u_E denote the distance, the gravitational constant, magnetic field energy, gravity field energy and electric field energy.

From the above equation, we can combine the Einstein's gravity equation with this equation, because both equations include the constant of gravitation G . The existing Einstein's equation is

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (9-1)$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \quad (9-2)$$

where $R_{\mu\nu}$, $T_{\mu\nu}$, $g_{\mu\nu}$ and R denote the Riemann curvature tensor, the energy flux tensor, the metric tensor, and the Ricci tensor.

As a result of substituting the gravitational constant G from eq. (8), we obtain

$$G_{\mu\nu} = \frac{16\pi}{ch} \sqrt{-\frac{u_B}{u_E}} \lambda_c^2 T_{\mu\nu}. \quad (10)$$

Now, it is assumed that the macroscopic tensor $g_{\mu\nu}$ to be approximately the Minkowski tensor g_{ij} because, for a quantized space-time, an analytical differential cannot be defined [1]. That is, it merely implies a division by λ_c [1]. Similarly, the energy density is given using zero-point energy:

$$\varepsilon = \frac{1}{2} \hbar \omega / \lambda_c^3. \quad (11)$$

Thus, $T_{\mu\nu}$ is approximated using the Minkowski tensor:

$$T_{\mu\nu} = \varepsilon g_{ij}. \quad (12)$$

Considering the above, Einstein's gravitational equation is transformed to

$$R_{\mu\nu} = \left(\frac{16\pi}{ch} \sqrt{-\frac{u_B}{u_E}} \frac{|u_G|}{\lambda_c} + \frac{1}{2} R \right) g_{ij}. \quad (13)$$

As mentioned, the energy of a quantized space-time regarding a magnetic field is given as the zero-point energy.

$$|u_G| = \frac{1}{2} \hbar \omega. \quad (14)$$

Assuming the Ricci tensor $1/2R$ to be substantially smaller than the first term, we obtain

$$R_{\mu\nu} = \left(\frac{16\pi}{ch} \sqrt{-\frac{u_B}{u_E}} \frac{1}{\lambda_c} \frac{1}{2} \hbar \omega \right) g_{ij}. \quad (15)$$

Moreover, considering the initial equation (8), the ratio u_B/u_E in eq. (15) is obtained.

$$u_B = -G^2 \frac{1}{r^4} \frac{1}{c^2} u_E \left(\frac{\hbar}{2c^2} \right)^2 = -G^2 \frac{1}{\lambda_c^4} \frac{1}{c^2} u_E \left(\frac{\hbar}{2c^2} \right)^2. \quad (16)$$

Considering this, we obtain conclusively

$$R_{\mu\nu} = \left(G \frac{8\pi}{c^4} \frac{1}{\lambda_c^3} \frac{1}{2} \hbar \omega \right) g_{ij}. \quad (17)$$

2.2 Zero-point energy in quantized space–time for gravity or magnetic field

Let us estimate the zero-point energy in terms of a magnetic or gravity field as well as in terms of an electric field concerning a quantized space–time. Here, we consider the energy level $|\Delta|$ from the special relativity Equation (18)

$$|\Delta| = \frac{m_e c^2}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (18)$$

This energy level is located at the middle position within the band gap of the vacuum, i.e., the energy gap is $2|\Delta|$.

As indicated in the following eq. (19), if $v \neq 0$ in eq. (18), this energy gap $2|\Delta|$ is produced by the product of both the fine-structure constant α and zero-point energy.

$$2|\Delta| = \frac{1}{2} \hbar \omega \times \alpha. \quad (19)$$

This fact will be proved while discussing the CMB in this paper or literature [8] has already proved it. Note that if $v = 0$ in eq. (18), this equation $|\Delta|$ implies the zero-point energy in terms of an electric field related to eq. (2). Conversely, the velocity v in eq. (18) here is assumed to be critical velocity v_c for an electron. That is, when largely accelerated, an electron takes the maximum velocity v_c less than the speed of light c . According to our previous paper [1,2], an electron can accompany an e-neutrino and thus the e-neutrino speed is equal to the critical velocity of an electron. Therefore, in eq. (18), $v = v_c$ is substituted by $0.994c$ [1,2].

As a result, using eq. (19), the calculated zero-point energy for magnetic or gravity field becomes $\frac{1}{2} \hbar \omega = \frac{2}{\alpha} |\Delta| = 9.14 m_e c^2 = 1.23 \times 10^9 \text{eV}$. (20)

It is allowed to consider that this value agrees with the measurement of a τ -particle [9].

In summary, the zero-point energy related to a special relativity energy, eq. (18), where $v = 0$ implies the rest energy of an electron and is related to the quantized space–time in terms of an electric field. While v in eq. (18) is the critical velocity v_c for an electron, the zero-point energy in eq. (19) implies a quantized space–time in terms of magnetic or gravity field.

Related items are listed below:

- 1) As described later, because the τ -particle mass is equal to the energy of a quantized time–space in terms of magnetic or gravity field, we will conclude that dark energy, which distributes uniformly in the universe, is τ -particle mass or magnetic field energy of a quantized space–time.
- 2) As described in the later section, an important point is that the existence of energies of the magnetic field and the gravity field in a quantized space–time will be proved by the presence of a measured τ -particle.

2.3 Three-generations of lepton

2.3.1 Collapse of the basic configuration of quantized space–times

Figure 2 indicates schematic how the magnetic or gravity field in a quantized space–time (i.e., the combination of two embedded electrons in two quantized space–times) is collapsed. Each quantized space–time in terms of a magnetic or gravity field has torque property whose moment corresponds to the magnetic field vector.

By the work of torque property, if two magnetic field vectors of two quantized space–times in terms of a magnetic field take the same direction, a larger magnetic field is generated. On the contrary, however, if two magnetic field vectors of two quantized space–times in terms of magnetic field take reverse directions, the net magnetic field vanishes. In this way, although the combination energy of two embedded electrons in two quantized space–times is quite large [2], a quantized space–time in terms of a magnetic or gravity field collapses. This fact results in creations of τ - and μ -particles as discussed later.

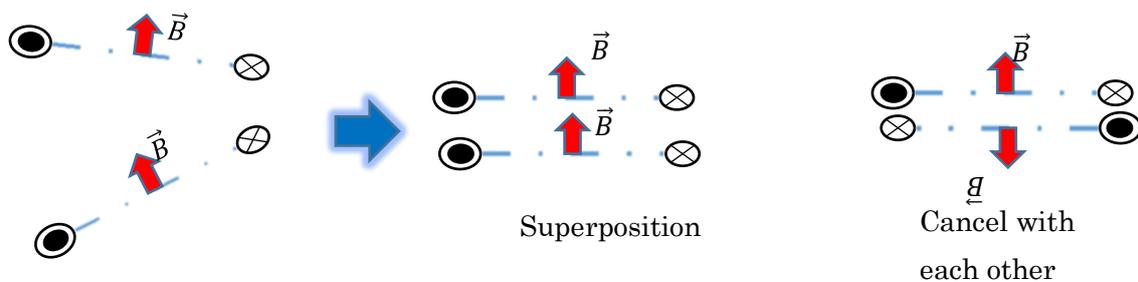


Fig. 2

Schematic that two quantized space–times in terms of a magnetic or gravity field interact with each other and the way how these quantized space–times collapse.

First, a quantized space–time in terms of magnetic field has torque property whose moment corresponds to its magnetic field vector. The superposition case in this figure is that the two magnetic field vectors are maximally strengthened. Generally, in the location of the universe, at which the gravity field becomes extremely strong, this maximum superposition occurs. An important case is the cancelation with each other. In this case, the quantized space–times in terms of a magnetic field collapse, and as discussed in the body in this paper, τ - and μ -particles appear according to the energy conservation. The magnetic field energy is converted to τ -particles, while μ -particle energy comes from the spin interaction of two electrons embedded in quantized space–times in terms of electric field.

2.3.2 Masses of μ - and τ -particles from the basic configuration of the quantized space–times

This paper claims that the masses of 3-generation leptons stem from the abovementioned collapse of the basic configuration of quantized space–times. As a result of the collapse, three energies are generated from a collapsed quantized space–time in terms of a magnetic or gravity field. Based on Fig. 1, we can conclude the following:

1. The combination energy between two embedded electrons in quantized space–times in terms of electric field, i.e., the magnetic field (gravity field) energy in a quantized space–time is converted. This energy corresponds to the rest energy of τ -particle.
2. Each embedded electrons in two quantized space–times, which take rotations and which induce the magnetic field energy in a quantized space–time, have interactions in terms of spins (up- and down-spin). This interaction is converted to the rest energy of μ -particle.
3. As a result of the collapse of the τ - or μ -particle, real electrons automatically appear in the collapsed mode of these particles. This fact comes from the lifetime of τ - or μ -particles.

In the Result section, the actual calculations will be conducted.

2.4 The analytical solution of the quantized Einstein's gravity equation

We now consider the quantized Einstein's gravity (QEG) equation according to our paper [1].

$$R_{ij} = \frac{8\pi G}{c^4} \frac{\hbar\omega}{\lambda_c^3} g_{ij}, \quad (21)$$

giving Minkowski tensor g_{ij}

$$g_{ij} = \begin{pmatrix} & -1 & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & +1 \end{pmatrix} \quad (22)$$

This QEG equation requires Riemann curvature tensor R_{ij} be a specific form, as the following reason.

- 1) Because g_{ij} is a diagonal matrix, Riemann curvature tensor R_{ij} is a diagonal matrix considering eq. (21).
- 2) The QEG equation must automatically express the Lorentz conservation, not to add this Lorentz conservation as a condition.

3) Riemann curvature tensor R_{ij} is a covariance tensor. Thus, this covariance tensor must be composed by direct product of covariance position vector \mathbf{k}_i .

$$\mathbf{k}_i = xe_1 + ye_2 + ze_3 + ict e_4, \quad (23)$$

where i in the fourth term denotes the imaginary unit.

$$R_{ij} \equiv \mathbf{k}_i \times \mathbf{k}_j. \quad (24)$$

(Note that the symbol \times implies direct product of vectors in this paper)

$$e_i \cdot e_j = \delta_{ij}. \quad (25)$$

Therefore

$$R_{ij} = \begin{pmatrix} x^2 & & & 0 \\ & y^2 & & \\ & & z^2 & \\ 0 & & & -(ct)^2 \end{pmatrix} \quad (26)$$

In the QEG, eq. (21) taking the trace Tr to form the Lorentz conservation:

$$x^2 + y^2 + z^2 - (ct)^2 = (-2) \frac{8\pi G}{c^4} \frac{\frac{1}{2}\hbar\omega_i}{\lambda_c^3}. \quad (27)$$

According to this equation, the equation automatically presents the Lorentz conservation.

In the derived equation, time t should be consider a period,

$$r_i \equiv x^2 + y^2 = (-2) \frac{8\pi G}{c^4} \frac{\frac{1}{2}\hbar\omega_i}{\lambda_c^3} + (c \frac{2\pi}{\omega})^2 - z^2, \quad (28)$$

where r_i implies radius of rotation at each location indexed by i , and we assume the cylindrical coordinates. Note that the radius r_i depends on the values of variable z . Moreover, variable angular frequency ω was introduced because time t implies a period.

As described later, the derived equation implies a quantized space–time's rotation and z gives anisotropic property. We will see that this solution of the QEG well describes both electromagnetism and gravity fields in later section.

Considering Equations (21), (22), and (26), each position- and time-variables x , y , z , and t are not independently defined. Thus, this equation obtains the meaning only when the trace Tr of both sides in the equation is considered. This fact is important because this paper claims that moving (rotating) space–time exists. That is, mathematical metrics in terms of Cartesian geometry is an approximated concept in the vacuum. This can be understood by considering an analogy that, while a rigid body has metrics on it like a length of line, an area of square and so on, but at the microscopic scale in this rigid body, many thermally fluctuated lattices (phonons) exist, which implies the mathematical metrics on the rigid body is not formed at the microscopic scale.

Moreover, as time t implies a period, the equation automatically introduces unique time in terms of special relativity.

2.5 Derivation of acceleration $a(r)$

Let us undertake calculations for acceleration-expansion of the universe. First, let us consider the analytical solution of the quantized Einstein's equation again.

$$r_i \equiv x^2 + y^2 = (-2) \frac{8\pi G}{c^4} \frac{\frac{1}{2}\hbar\omega_i}{\lambda_c^3} + (c \frac{2\pi}{\omega})^2 - z^2. \quad (28)$$

Index i is associated with both radius r_i and zero-point energy $\frac{1}{2}\hbar\omega_i$. Note that we assume $z=0$ here.

$$r_i \equiv (-2) \frac{8\pi G}{c^4} \frac{\frac{1}{2}\hbar\omega_i}{\lambda_c^3} + (c \frac{2\pi}{\omega})^2. \quad (29)$$

Because r_i should be considered macroscopic variable of radius in this section, the first term of the right-hand side (i.e., the zero-point energy having the index i) should be neglected in this section. As a result, macroscopic variable radius r appears and thus a unique angular frequency is derived as

$$\omega = \frac{\sqrt{4\pi c}}{r}. \quad (30)$$

Acceleration $a(r)$ is derived as follows:

Note that considering eq. (29), r_i must indicate a radius of the rotation. Thus, it is necessary to introduce the circumference r_L for radius r , and the acceleration is derived as eq. (32).

$$r_L = 2\pi r, \quad (31)$$

$$\frac{d^2}{dt^2} r_L = \frac{2\pi r}{T^2} = 2r\pi \frac{\omega^2}{4\pi^2} = 2 \frac{c^2}{r} \equiv a(r). \quad (32)$$

In the above equation, eq. (30) was used. Moreover, T in eq. (32) implies a period.

Now we calculate a difference in velocity, Δv , considering 1 Mps. Considering the general definition of an acceleration, a differential velocity Δv is expressed as

$$\Delta v = 2 \frac{c^2}{r} \Delta t. \quad (33)$$

The above result and equation (30) include macroscopic variable distance r and this movement implies rotations, which results in acceleration (32). We can take a physical picture that our entire universe is undergoing rotations but its angular frequency and its acceleration is not the conventional and basic form but includes variable distance r , which results from the solution of the quantized Einstein's equation. An important point is that this rotational model in the universe can provide the maximum boundary in the universe, which agrees well with the concept of special relativity. Thus, even with the acceleration in the universe, the velocity must not dominate over the speed of light c . In fact, according

to eq. (32), the acceleration decreases with radius r . Thus, our universe size is finite, which guarantees conservations of energy. Considering this rotational model in the universe, it is allowed that we claim that the true radial acceleration, which has been conventionally discussed, is not occurring in the universe. Note that the standard big bang model cannot explain the abovementioned facts (i.e., acceleration, finite size of the universe, and no contradiction for the special relativity concept.)

Result section will conduct actual calculation using eq. (33).

2.6 Cosmic microwave background (CMB)

First, to the formula of the Prank emission α_T , eq. (34), an angular frequency ω , eq. (30), is substituted, which was derived from the analytical solution of the QEG equation.

$$\alpha_T = \frac{1}{\pi^2} \frac{\hbar\omega/c^2}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}, \quad (34)$$

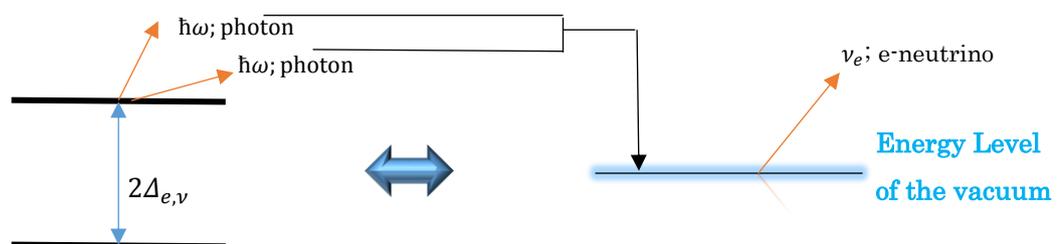
where T and k_B denote the temperature and Boltzmann constant, respectively.

$$\omega = \frac{\sqrt{4\pi c}}{r}, \quad (30)$$

Thus when the exponential function in eq. (34) becomes e^{-1} , the following equation holds.

$$\frac{k_B T_0 r}{\hbar\sqrt{4\pi c}} = 1. \quad (35)$$

In this equation, the temperature T_0 implies one of Prank emission. We claim that a photon of CMB is derived from the energy gap, which fluctuates in the energy level of the vacuum [2] and is related to e-neutrino self-energy. That is, considering that r in eq. (35) is a wavelength, this wavelength can be derived from the fluctuations in the energy level of the vacuum.



Result of the emissions of photons. As a result, the fluctuated energy gap creates.

Result of the absorptions of photons. As a result, the fluctuated energy gap vanishes. Instead, e-neutrino appears

Fig.3

Schematic of creations and absorptions of CMB photons through neutrino's energy gap.

The left-hand side figure indicates a creation of the energy gap which fluctuates in the energy level of the vacuum. By the emissions of photons from the energy level in the vacuum, an energy gap is created. This created energy gap is essentially equal to the self-energy of an e-neutrino. The right-hand side figure shows the disappearance of the energy gap by absorbing the photons. Instead, a real body of e-neutrino is emitted. As mentioned in our previous paper [2], because of many-body interactions of the basic configuration of quantized space-times, an energy gap is again created at the energy level of the vacuum according to the BCS ground state. This is essentially equal to the abovementioned photons' emissions from the energy level of the vacuum. Everywhere in the universe, the left-hand and right-hand sides occur locally with iterations. Thus, the creations and absorptions of photons in terms of CMB arise everywhere and the source of the CMB is not the birth of the universe (i.e., the big bang).

Figure 3 shows the iteration that photons are absorbed or emitted to or from the energy gap. This implies that CMB photons are created and absorbed everywhere in the universe and thus this paper claims that CMB photons are not the source of the birth in terms of big bang on our universe.

In Result section, the actual calculation will be conducted using e-neutrino self-energy.

2.7 Analytical derivation of the gravity wave

In the solution of the QEG equation,

$$r_i^2 = x^2 + y^2 = -\frac{16\pi G}{c^4} \frac{\frac{1}{2}\hbar\omega_i}{\lambda_i^3} + \left(c \frac{2\pi}{\omega}\right)^2 - z^2, (28)$$

$r_i = 0$ is a condition of the generation of gravity wave. Note that the index i is arbitrary. Thus, this implies that previous everywhere rotations ceases. That is, a macroscopic rotation ceased [10,11].

Moreover, depending on the variable z , variable angular frequency ω is varied. Considering the cylindrical coordinates, in this case, the center of the previous rotation is considered. That is, $z=0$ is assumed. This implies that the maximum is considered.

Moreover, for a secondary condition, the zero-point energy is converted to photon by the product of the fine-structure constant α because it is needed to convert the energy level to an energy gap.

When the uncertain relation is introduced, angular frequency- ω equation dependent on Δt is derived.

$$\frac{1}{2}\hbar\omega_i \rightarrow \frac{1}{2}\hbar\omega_i\alpha = n\hbar\omega = |\Delta|, (36)$$

$$|\Delta| \times \Delta t = \hbar. (37)$$

Thus,

$$\left(c \frac{2\pi}{\omega}\right)^2 = \frac{16\pi G}{c^4} \frac{1}{\lambda_c^3} |\Delta|, \quad (38)$$

$$\left(c \frac{2\pi}{\omega}\right)^2 \Delta t = \frac{16\pi G}{c^4} \frac{1}{\lambda_c^3} \hbar. \quad (39)$$

In this equation, the distributed relationship regarding λ_c is employed.

$$\omega^5 = \frac{\pi c^9}{4G} \frac{1}{\hbar} \Delta t. \quad (40)$$

Next, the strain h_{\max} is considered.

$$h_{\max} = \frac{\delta L}{L}. \quad (41)$$

This definition is translated to

$$h_{\max} = \frac{\lambda_c}{L/2}, \quad (42)$$

where

$$\lambda_c = ct_c, \quad (43-1)$$

$$t_c = t_0 \sqrt{1 - \frac{v^2}{c^2}}, \quad (43-2)$$

$$v = v_c = 0.994c. \quad (43-3)$$

As mentioned in Section 2.2, this velocity v_c is the critical speed of an electron, which is equal to that of an e-neutrino. Therefore this implies consideration of the quantized space–times in terms of gravity (magnetic) fields.

Moreover,

$$\frac{L}{2} = c\Delta t. \quad (44)$$

Considering these equations,

$$h_{\max} = \frac{t_c}{\Delta t}. \quad (45)$$

Therefore the Chirp signal is

$$u_p = \frac{t_c}{\Delta t} \cos(\omega t_{00}), \quad (46)$$

where t_{00} is defined as a constant 1[s] because the angular frequency ω is variable dependent on time Δt .

In Result section, this Chirp signal u_p will be depicted.

2.8 The picture of the unified field in terms of an electromagnetic and a gravity field by rotations of quantized space–times

Again the solution of the QEG equation is

$$r_i^2 = x^2 + y^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \frac{1}{\lambda_c^2} \frac{1}{2} \hbar \omega_i, \quad (29)$$

where $z=0$ is assumed.

As mentioned, this equation implies that a quantized time–space's rotation.

2.8.1 The case of DC

2.8.1.1 General notation

Let us see that the basic equation (29) creates every equation regarding electromagnetism and Newtonian gravity. To conduct this, first, the quantized length λ_c should be cancelled in this basic equation, and later we consider general fields.

We assume that electric and magnetic fields only in a quantized space–time have energies as

$$\frac{1}{2} \hbar \omega_i = \frac{1}{2} \varepsilon_0 E_i^2 \lambda_c^3, \quad (47-1)$$

$$\frac{1}{2} \hbar \omega_i = \frac{B_i^2}{2\mu_0} \lambda_c^3. \quad (47-2)$$

Concerning the gravity, in the previous section 2.2, we derived the zero-point energy in terms of gravity field:

$$\frac{1}{2} \hbar \omega_i = \frac{2}{\alpha} |\Delta| = \frac{2}{\alpha} \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (47-3)$$

Concerning the gravity, furthermore, the general wave function is considered:

$$\int |\psi|^2 dv = |\psi_i|^2 \lambda_c^3 = 1. \quad (47-4)$$

Note that, as mentioned, a differential and integral become merely a division and product in the quantized space–time [1].

Each of the above equations is substituted with the basic eq. (29), and thus the general electric field, magnetic field, and gravity field equations are derived as follows:

$$r_i^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \frac{1}{2} \varepsilon_0 E_i^2, \quad (48-1)$$

$$r_i^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \frac{B_i^2}{2\mu_0}, \quad (48-2)$$

$$r_i^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} |\psi_i|^2 \frac{2}{\alpha} \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (48-3)$$

2.8.1.2 Derivation of each Poisson equation

Let us obtain the Poisson equations in terms of electrostatic potential, vector potential, and gravity potential based on the results obtained in the previous section.

First, consider the Poisson equation in terms of electrostatic potential

$$r_i^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \frac{1}{2} \varepsilon_0 E_i^2. \quad (48-1)$$

Then the first term is neglected, and

$$r_i \equiv \lambda_c. \quad (49)$$

Thus,

$$\frac{1}{2} \varepsilon_0 \frac{E_i^2}{\lambda_c^2} = -\frac{c^4}{16\pi G}. \quad (50-1)$$

In this equation, the division by λ_c is translated to the normal differential. That is, the differential is revived.[1]

$$\frac{1}{2} \varepsilon_0 \frac{dE_i^2}{dr^2} = -\frac{c^4}{16\pi G}. \quad (50-2)$$

Thus,

$$\varepsilon_0 E_r \frac{dE_r}{dr} \frac{1}{\lambda_c} = -\frac{c^4}{16\pi G}. \quad (51)$$

Considering the concept of quantized space-times,

$$r = n\lambda_c, \quad (52)$$

$$nE_{r0} \frac{dE_r}{dr} \frac{1}{r} = -\frac{1}{\varepsilon_0} \frac{c^4}{16\pi G}. \quad (53)$$

An electrostatic potential Φ is introduced.

$$E_r = -\frac{d\Phi}{dr}, \quad (54)$$

$$-\frac{d^2\Phi}{dr^2} = -\frac{1}{\varepsilon_0} \frac{c^4}{16\pi G} \frac{1}{nE_{r0}} r. \quad (55)$$

Herein, the following relation is assumed.

$$E_{r0} = -vB_{z0}, \quad (56)$$

where v implies an arbitrary and rotational velocity, not the speed of light c .

$$\frac{d^2\Phi}{dr^2} = -\frac{1}{\varepsilon_0} \frac{c^4}{16\pi G} \frac{1}{v} \frac{1}{B_{z0}} \frac{1}{n} r. \quad (57)$$

Considering the cyclotron angular frequency,

$$\omega_c = \frac{eB_{z0}}{m_e}. \quad (58)$$

Consequently,

$$\frac{d^2\phi}{dr^2} = -\frac{1}{\epsilon_0} \frac{c^4}{16\pi G} \frac{1}{v} \frac{e}{m_e \omega_c} \frac{1}{n} \Gamma, \quad (59)$$

Or using the following equation,

$$v = r\omega_c, \quad (60)$$

$$\frac{d^2\phi}{dr^2} = -\frac{1}{\epsilon_0} \frac{c^4}{16\pi G} \frac{e}{m_e \omega_c^2} \frac{1}{n}. \quad (61)$$

Next, consider the Poisson equation in terms of vector potential.

Similarly, to the case of electric field, using eq. (48-2)

$$B_z \frac{dB_z}{dr^2} = -\mu_0 \frac{c^4}{16\pi G}. \quad (62)$$

$$B_z \frac{dB_z}{dr} \frac{1}{\lambda_0} = n B_{z0} \frac{dB_z}{dr} \frac{1}{r} = -\mu_0 \frac{c^4}{16\pi G}. \quad (63)$$

In this case, cylindrical coordinates are considered. Thus, the component of a vector potential is introduced by

$$B_z = \frac{1}{r} A_\phi. \quad (64)$$

Thus,

$$n B_{z0} \left(-\frac{A_\phi}{r^3} \right) = -\mu_0 \frac{c^4}{16\pi G}. \quad (65)$$

Thus,

$$\frac{d^2 A_\phi}{dr^2} = 6\mu_0 \frac{c^4}{16\pi G} \frac{1}{n B_{z0}} r. \quad (66)$$

By the introduction of the cyclotron angular frequency ω_c ,

$$\frac{d^2 A_\phi}{dr^2} = 6\mu_0 \frac{c^4}{16\pi G} \frac{1}{n} \frac{e}{m_e \omega_c} r. \quad (67)$$

The number 6 appeared because cylindrical coordinates were used. Thus, this coefficient is not essential. Considering that vector potential A_ϕ in the left-hand side is variable and arbitrary, if the number 6 is included in this vector potential A_ϕ , the Poisson equation does not lose generality.

Consequently,

$$\frac{d^2 A_\phi}{dr^2} = \mu_0 \frac{c^4}{16\pi G} \frac{1}{n} \frac{e}{m_e \omega_c} r. \quad (68)$$

Next, let us derive the Poisson equation in terms of gravity.

Again,

$$r_i^2 = \left(c \frac{2\pi}{\omega} \right)^2 - \frac{16\pi G}{c^4} |\psi_i|^2 \frac{2}{\alpha} \frac{m_e c^2}{\sqrt{1 - \frac{v^2}{c^2}}}. \quad (48-3)$$

The first term is neglected, and $r_i \equiv \lambda_c$ is assumed.

$$\lambda_c^2 = -\frac{16\pi G}{c^4} |\psi_i|^2 \frac{2}{\alpha} \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (69)$$

In short,

$$1 = -\frac{16\pi G}{c^4} \frac{|\psi_i|^2}{\lambda_c^2} \frac{2}{\alpha} \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (70-1)$$

Then, normal deferential is revived [1].

$$1 = -\frac{16\pi G}{c^4} \frac{d^2|\psi_i|^2}{dr^2} \frac{2}{\alpha} \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (70-2)$$

Then, the following potential energy Φ_G for gravity is introduced.

$$g\delta(\vec{r})|\psi_i|^2 \equiv \Phi_G, \quad (72)$$

where g is variable and δ is the Dirac function.

Thus,

$$\frac{d^2\Phi_G}{dr^2} = -\frac{c^4}{16\pi G} \frac{\alpha}{2} \frac{\sqrt{1-\frac{v_R^2}{c^2}}}{m_e c^2} g\delta(\vec{r}), \quad (73)$$

where $v \equiv v_R$ denotes a relative velocity between two charged particles, because a vector \mathbf{r} in the Dirac function implies relative distance between the two charged particles.

When relative velocity v_R is zero, generally two charged particles experience strong attractive force with each other, as Lorentz force. For example, this attractive force creates a Cooper pair in high-temperature superconductors [12].

Therefore, when a relative velocity v_R is assumed to be zero, it is approximated as

$$\frac{d^2\Phi_G}{dr^2} = -\frac{1}{16\pi G} \frac{1}{m_e} c^2 g\delta(\vec{r}). \quad (74)$$

In this equation, g should be considered as variable. Note that, as $\frac{\alpha}{2}$ has the meaning only when v_R is not zero but large, the above conclusive equation does not have this factor.

$$\int \delta(\vec{r}) dv = 1. \quad (75)$$

Considering this volume integral, we take volume integral to the above process eq. (74). Note that, only in this case, sphere coordinates are considered, because r implies a relative distance.

$$\int \frac{d^2\Phi_G}{dr^2} 4\pi r^2 dr = -\frac{1}{16\pi G} \frac{1}{m_e} c^2 g, \quad (76)$$

$$\text{The left side} = \int d\Phi_G \frac{d}{dr} 4\pi r^2 = 8\pi r \Phi_G. \quad (77)$$

Thus, finally, we obtain a Newtonian equation.

$$\Phi_G = -\frac{1}{8\pi} \frac{1}{16\pi G} \frac{c^2}{m_e} g \frac{1}{r}. \quad (78)$$

In Result section, we will examine the validity of these derived Poisson equations by actual calculations

2.8.2 Derivation in the case of AC

First, the zero-point energy in the solution of the QEG equation is translated to photon, the energy gap.

$$\frac{1}{2} \hbar \omega_i \rightarrow \frac{1}{2} \hbar \omega_i \alpha = |\Delta| = |E_i - E_j|. \quad (79)$$

Considering the above, the basic solution becomes

$$r_i^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \frac{1}{\lambda_c^3} |E_i - E_j|, \quad (80)$$

where

$$|E_i - E_j| = \int \left| \frac{1}{2} \varepsilon_0 E_i^2 - \frac{B_j^2}{2\mu_0} \right| dv = \left| \frac{1}{2} \varepsilon_0 E_i^2 - \frac{B_j^2}{2\mu_0} \right| \lambda_c^3. \quad (81)$$

Thus,

$$r_i^2 = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \left| \frac{1}{2} \varepsilon_0 E_i^2 - \frac{B_j^2}{2\mu_0} \right|. \quad (82)$$

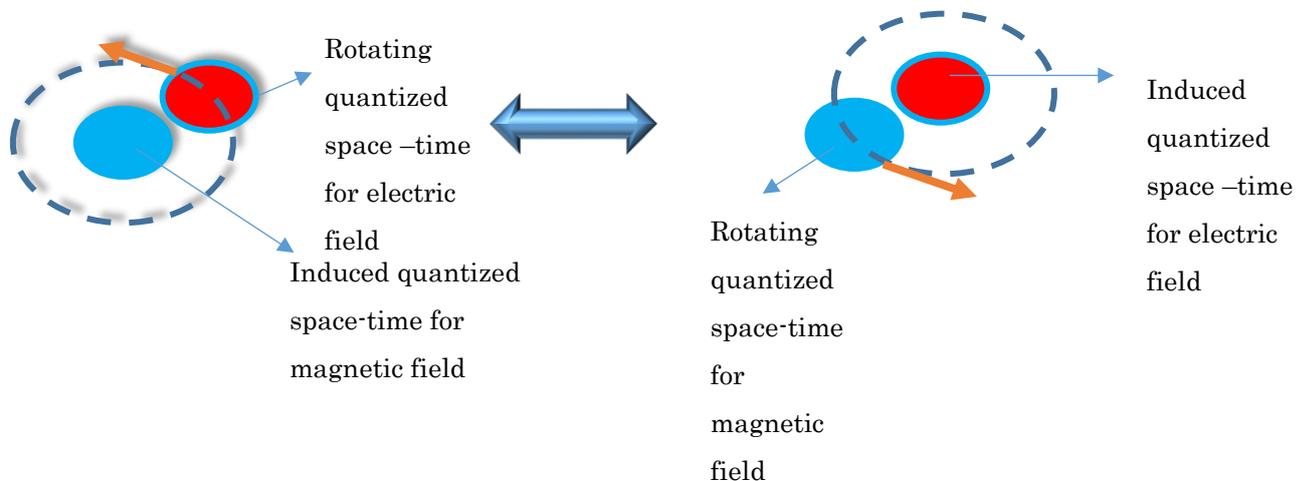


Fig. 4

Schematic of inductions of the quantized space-times both in terms of electric and magnetic fields.

Note that here, we do not consider the previously mentioned and basic configuration of quantized space-times,

i.e., the case of DC. The left figure indicates a phenomenon that a rotation of the quantized space–time for an electric field induces a quantized space–time for magnetic field. On the contrary, the right figure shows a phenomenon that a rotation of the quantized space–time for a magnetic field induces a quantized space–time for electric field.

These two phenomena are supported by the equation in the body;

$$r_i^2 = \left(c \frac{2\pi}{\omega} \right)^2 - \frac{16\pi G}{c^4} \left| \frac{1}{2} \epsilon_0 E_i^2 - \frac{B_j^2}{2\mu_0} \right| \quad \text{or} \quad r_j^2 = \left(c \frac{2\pi}{\omega} \right)^2 - \frac{16\pi G}{c^4} \left| \frac{1}{2} \epsilon_0 E_j^2 - \frac{B_i^2}{2\mu_0} \right| \quad (83)$$

That is, 2 energy levels indexed by both i and j are induced by each other, which is iterated by the angular frequency ω . As derived later, these equations implies the time-dependent Maxwell's equations and thus they indicate the process of the induction of an electromagnetic wave.

As shown in Fig. 4, the above equation (82) implies that a magnetic field energy indexed by j is dependent on the electric field energy indexed by i. Thus, a magnetic field is induced by electric field. If indices i and j are altered, an electric field energy becomes dependent and induced from the magnetic field energy. This physical picture describes the process of an electromagnetic wave, and thus eq. (82) implies time-dependent Maxwell's equations as described later.

Now let us create the Maxwell's time-dependent equations based on this equation.

$$\omega \rightarrow \frac{2\pi}{t_c}, \quad (84)$$

$$r_i \rightarrow \lambda_c, \quad (85)$$

$$\lambda_c^2 = (ct_c)^2 - \frac{16\pi G}{c^4} \left(\frac{1}{2} \epsilon_0 E^2 - \frac{B^2}{2\mu_0} \right). \quad (86)$$

In this equation, E and B are not magnitudes but components. Afterwards, considering generalizations into 3-dimension in view of vector analysis, these components are assumed to be arbitrary components regarding any coordinates.

$$\lambda_c^2 - \frac{16\pi G}{c^4} \left(\frac{B^2}{2\mu_0} \right) = (ct_c)^2 - \frac{16\pi G}{c^4} \left(\frac{1}{2} \epsilon_0 E^2 \right). \quad (87)$$

From this equation, consider the following simultaneous equations.

$$\lambda_c^2 - \frac{16\pi G}{c^4} \left(\frac{B^2}{2\mu_0} \right) = \alpha, \quad (88-1)$$

$$(ct_c)^2 - \frac{16\pi G}{c^4} \left(\frac{1}{2} \epsilon_0 E^2 \right) = \alpha. \quad (88-2)$$

$$\text{eq. (88 - 1)} \rightarrow 1 - \frac{16\pi G}{c^4} \frac{1}{2\mu_0} \frac{B^2}{\lambda_c^2} = \frac{\alpha}{\lambda_c^2}. \quad (89)$$

The deferential is revived and the number 1 is ignored.

$$-\frac{16\pi G}{c^4} \frac{1}{2\mu_0} \frac{dB^2}{dr^2} = \frac{d\alpha}{dr^2}, \quad (90)$$

$$-\frac{16\pi G}{c^4} \frac{1}{2\mu_0} 2B \frac{dB}{dr} \frac{1}{\lambda_c} = \frac{d\alpha}{dr} \frac{1}{\lambda_c}, \quad (91)$$

$$-\frac{16\pi G}{c^4} \frac{1}{\mu_0} \frac{dB}{dr} B = \frac{d\alpha}{dr}. \quad (92)$$

$$\text{eq. (88 - 2)} \rightarrow 1 - \frac{16\pi G}{c^4} \frac{1}{2} \varepsilon_0 \frac{E_i^2}{ct_c^2} = \frac{\alpha}{ct_c^2}, \quad (93)$$

$$-\frac{16\pi G}{c^4} \frac{1}{2} \varepsilon_0 \frac{1}{c} \frac{dE^2}{dt} \frac{1}{t_c} = \frac{d\alpha}{dt} \frac{1}{ct_c}, \quad (94)$$

$$-\frac{16\pi G}{c^4} \varepsilon_0 E \frac{dE}{dt} = \frac{d\alpha}{dt}. \quad (95)$$

At this time, the following Lorentz conservation is assumed:

$$(dr)^2 - c^2(dt)^2 \equiv 0, \quad (96-1)$$

That is,

$$dr = \pm c dt. \quad (96-2)$$

At this time, the sign + is employed.

$$\text{eq. (95)} \rightarrow -\frac{16\pi G}{c^4} \varepsilon_0 E \frac{dE}{dt} = c \frac{d\alpha}{dr}. \quad (95-2)$$

Combining with eq. (92),

$$-\frac{16\pi G}{c^4} \varepsilon_0 E \frac{dE}{dt} = c \left(-\frac{16\pi G}{c^4} \frac{1}{\mu_0} B \frac{dB}{dr} \right), \quad (97)$$

$$\varepsilon_0 E \frac{dE}{dt} = c \frac{1}{\mu_0} B \frac{dB}{dr}. \quad (98)$$

By the way, the ratio E/B is related to the characteristic impedance Z in the vacuum and calculated as

$$\frac{E}{B} = \frac{E}{\mu_0 H} = \frac{1}{\mu_0} Z = \frac{1}{\mu_0} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c. \quad (99)$$

Considering this relation, the above process, eq. (98) becomes

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 \mu_0} \frac{dB}{dr}. \quad (100)$$

In view of vector analysis, this process can be generalized into 3-dimension.

$$\text{rot} \vec{H} = \frac{\partial \vec{D}}{\partial t}. \quad (101)$$

This conclusive equation is identical for the Maxwell's third equation.

Now let us obtain the Maxwell's fourth equation by the same method.

In the solution of the QEG equation, indices i and j are altered:

$$r_j = \left(c \frac{2\pi}{\omega}\right)^2 - \frac{16\pi G}{c^4} \left(\frac{1}{2} \varepsilon_0 E_j^2 - \frac{B_i^2}{2\mu_0}\right). \quad (102)$$

From the similar process, the following simultaneous equations are formed.

$$\lambda_c^2 - \frac{16\pi G}{c^4} \left(\frac{1}{2} \varepsilon_0 E_j^2\right) = \alpha, \quad (103-1)$$

$$(ct_c)^2 - \frac{16\pi G}{c^4} \left(\frac{B_i^2}{2\mu_0}\right) = \alpha. \quad (103-2)$$

From Equation (103-1),

$$1 - \frac{16\pi G}{c^4} \frac{1}{2} \varepsilon_0 \frac{E^2}{\lambda_c^2} = \frac{\alpha}{\lambda_c^2}. \quad (104)$$

Note that E and B are not magnitude, but arbitrary components of vectors regardless of any cooperates.

Similarly, from the division of λ_c , the differential is revived and the number 1 is ignored.

$$-\frac{16\pi G}{c^4} \frac{1}{2} \varepsilon_0 \frac{dE^2}{dr^2} = \frac{d\alpha}{dr^2}, \quad (105)$$

$$-\frac{16\pi G}{c^4} \varepsilon_0 E \frac{dE}{dr} \frac{1}{\lambda_c} = \frac{d\alpha}{dr} \frac{1}{\lambda_c}, \quad (106)$$

$$-\frac{16\pi G}{c^4} \varepsilon_0 E \frac{dE}{dr} = \frac{d\alpha}{dr}. \quad (107)$$

$$eq. (103 - 2) \rightarrow 1 - \frac{16\pi G}{c^4} \frac{1}{2\mu_0} \frac{B^2}{ct_c^2} = \frac{\alpha}{ct_c^2}, \quad (108)$$

Similarly,

$$-\frac{16\pi G}{c^4} \frac{1}{2\mu_0} \frac{dB^2}{cdt^2} = \frac{d\alpha}{cdt^2}, \quad (109)$$

$$-\frac{16\pi G}{c^4} \frac{1}{\mu_0} B \frac{dB}{dt} \frac{1}{c} \frac{1}{t_c} = \frac{d\alpha}{dt} \frac{1}{t_c} \frac{1}{c}, \quad (110)$$

$$-\frac{16\pi G}{c^4} \frac{1}{\mu_0} B \frac{dB}{dt} = \frac{d\alpha}{dt}. \quad (111)$$

From the abovementioned Lorentz conservation,

$$dr = \pm cdt. \quad (96-2)$$

In this case, the sign – is employed.

Thus, the above process becomes

$$-\frac{16\pi G}{c^4} \frac{B}{\mu_0} \frac{dB}{dt} = -c \frac{d\alpha}{dr}. \quad (112)$$

Combining with eq. (107),

$$-\frac{16\pi G}{c^4} \frac{B}{\mu_0} \frac{dB}{dt} = -c \left(-\frac{16\pi G}{c^4} \epsilon_0 E \frac{dE}{dr} \right), \quad (113)$$

$$\frac{B}{\mu_0} \frac{dB}{dt} = -c \epsilon_0 E \frac{dE}{dr}, \quad (114)$$

As mentioned, the ratio E/B is

$$\frac{E}{B} = c, \quad (99)$$

Thus,

$$\frac{dB}{dt} = -c^2 \mu_0 \epsilon_0 \frac{dE}{dr} = -\frac{dE}{dr}. \quad (115)$$

In view of vector analysis, this equation can be generalized to 3-dimension.

$$\frac{\partial \vec{B}}{\partial t} = -\text{rot} \vec{E}. \quad (116)$$

This is how we derived the Maxwell's fourth equation.

In Result section, we will summarize these processes and results.

3. Results

3.1 The masses of 3 leptons

From our previous paper [2], the combination energy (i.e., Lorentz force) in terms of two embedded electrons in a quantized space-time, i.e., the magnetic field (also, gravity field) energy U_B is estimated as

$$|U_B| = |-8.0 \times 10^{-10}| \text{ J}. \quad (117)$$

As mentioned, this energy gives the rest energy of τ -particles. Considering this particle is a fermion,

$$|U_B| = 2M_\tau c^2, \quad (118)$$

where M_τ is the mass of a τ -particle.

As a result,

$$M_\tau = 2.5 \times 10^9 \text{ eV}/c^2. \quad (119)$$

Comparison with the measurement [9] makes me find that the theoretical value has the same order as the measurement but a little bit large, which is in agreement with the measurement. This is because, in the theoretical value, the gravity interaction between two τ -particles includes due to their large masses. That is, strictly speaking, in eq. (118), a very small term regarding gravity interaction between two τ -particles should be added. As mentioned, we also claim that there are dark energies, but they have attractive interactions due to the gravity, not repulsive interactions.

Let us consider the case of μ -particles.

From our previous paper regarding superconductivity [13], the spin interaction V between up- and down-spin electrons is expressed as

$$V = -\frac{e^2\hbar^2}{16\pi^2 m_e^2 (2\lambda_c)^3}, \quad (120-1)$$

where

$$\lambda_c \approx \lambda_0, \quad (120-2)$$

And

e denotes the charge of an electron.

Considering that both μ -particle is a fermion and that from Fig. 1, the relative distance in the abovementioned equation (120-1) is the diameter of a quantized space–time $2\lambda_c$, the rest energy is derived as

$$2M_\mu c^2 = \left| -\frac{e^2\hbar^2}{16\pi^2 m_e^2 (2\lambda_c)^3} \right|, \quad (121)$$

where M_μ denotes the mass of a μ -particle.

$$M_\mu = 1.15 \times 10^8 eV/c^2. \quad (122)$$

Compared with the measurement [9], the agreement is sufficient.

Note that a real electron appears as a result of lifetimes and collapses of τ - or μ -particle.

The significance of the above discussion is that this paper clarified the reason why leptons have 3-generations, from the view of the basic configuration of quantized space–times (Fig.1). In previous paper [2], we calculated self-energy of 3-generation neutrinos. Thus combining with these results, now we obtained the comprehensive understanding of why leptons have 3-generations.

3.2 Acceleration-expansion universe

In the Theory section, we derived the following conclusion.

$$\Delta v = 2 \frac{c^2}{r} \Delta t. \quad (33)$$

In this equation, r and Δt are listed in Table 1.

Table 1 Physical parameters for 1Mps

r	3.0×10^{22} m
Δt	1.0×10^{10} s

Thus

$$\Delta v = 66 \text{ km/s.} \quad (123)$$

The latest measurement [14] is

$$\Delta v = 67 \text{ km/s.} \quad (124)$$

The agreement is sufficient.

3.4 Emission of CMB

Theory section derived the following unique angular frequency.

$$\omega = \frac{\sqrt{4\pi c}}{r}. \quad (30)$$

Thus, when the exponential function in eq. (34) becomes e^{-1} , the following equation was held.

$$\frac{k_B T_0 r}{\hbar \sqrt{4\pi c}} = 1. \quad (35)$$

In this equation, the temperature T_0 implies one of Prank emission.

When r in eq. (35) is considered as a wavelength, the source of this wavelength λ comes from the fluctuation energy gap, which is related to an e-neutrino self-energy [2]. The e-neutrino self-energy is expressed as the following equation [2],

$$2\Delta_{e,\nu} = 0.025 \text{ eV} = 4.0 \times 10^{-21} \text{ J.} \quad (125)$$

$\Delta_{e,\nu}$ implies the energy level for an e-neutrino. Thus, it is necessary to obtain a photon from this energy level. In this case, the product of the fine-structure constant α to this energy level makes it be photon energy gap $\hbar\omega$.

$$2\Delta_{e,\nu} \rightarrow 2(\Delta_{e,\nu} \times \alpha) = 2\hbar\omega = 4.0 \times 10^{-21} \alpha. \quad (126)$$

Thus, ω and λ are calculated as

$$\omega = 1.9 \times 10^{13} \times \frac{1}{137} = 1.3 \times 10^{11} \text{ rad/s.} \quad (127)$$

$$\lambda = \frac{c}{\omega} = 2.3 \times 10^{-3} \text{ m.} \quad (128)$$

The derived wavelength λ is substituted to eq. (35), considering that r is equal to the wavelength λ .

$$T_0 = \frac{\hbar c \sqrt{4\pi}}{k_B \lambda} \approx 3.7 \text{ K.} \quad (129)$$

This value agrees with the temperature of CMB [15]

3.5 Depict of gravitation wave (Chirp signal)

The derived equation in Theory section is again,

$$h_{max} = \frac{t_c}{\Delta t}, \quad (45)$$

$$u_p = \frac{t_c}{\Delta t} \cos(\omega t_{00}), \quad (46)$$

where t_{00} is defined as a constant 1[s] because the angular frequency ω is dependent on time Δt .

Figure 5 shows the result of this analytical calculation of the gravity wave. As shown in this figure, this calculation agrees with the measurements [10]. Considering strain h_{max} implies a quantized space-time t_c , the gravity wave is the universal phenomenon. Moreover, h_{max} was derived by $z=0$ of the solution of the quantized Einstein's equation, and thus the gravity wave has an anisotropic property [16].

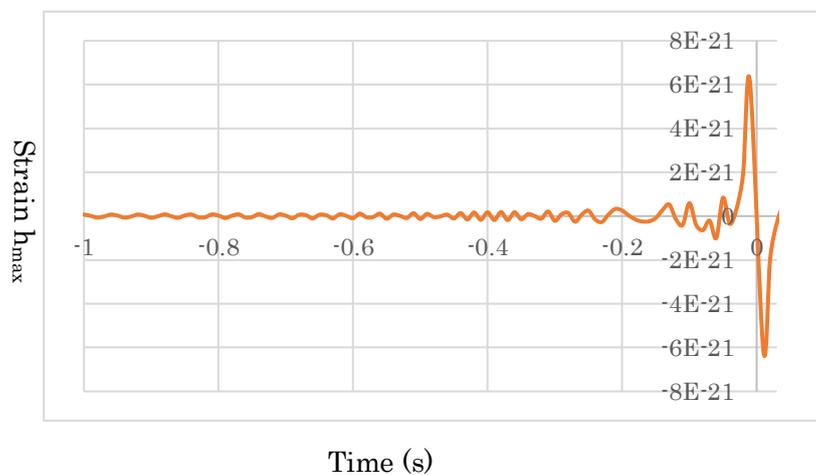


Fig. 5

Result of analytical calculation for the gravidity wave. As shown, the critical point indicates 10^{-21} order and time scale is 0.1 s. Thus the calculation agrees well with the measurements. This result was obtained from the solution of the QEG equation. That is, this result is one case and thus every phenomenon regarding electromagnetic and gravity fields can be uniformly derived only from the solution of the QEG equation.

3.6 Laws of the electromagnetism derived from the solution of the quantized Einstein's equation

3.6.1 Coulomb interaction and Poisson equations

Let us consider the Coulomb interaction and the satisfaction of continuity equation regarding charge density and current density. The Poisson equations derived in the Theory section are

$$\frac{d^2\Phi}{dr^2} = -\frac{1}{\epsilon_0} \frac{c^4}{16\pi G} \frac{1}{v} \frac{e}{m_e \omega_c} \frac{1}{n} \Gamma. \quad (59)$$

Thus, the charge density is

$$\rho = \frac{c^4}{16\pi G} \frac{1}{v} \frac{e}{m_e \omega_c} \frac{1}{n} \Gamma. \quad (59-2)$$

Or eq. (61) is expressed as

$$\frac{d^2\Phi}{dr^2} = -\frac{1}{\epsilon_0} \frac{c^4}{16\pi G} \frac{e}{m_e \omega_c^2} \frac{1}{n}, \quad (61)$$

For vector potential,

$$\frac{d^2 A_\phi}{dr^2} = \mu_0 \frac{c^4}{16\pi G} \frac{1}{n} \frac{e}{m_e \omega_c} r. \quad (68)$$

From the Poisson equation for electrostatic potential,

$$\rho_0 = \frac{c^4}{16\pi G} \frac{e}{m_e \omega_c^2} \equiv \beta_0 e, \quad (130)$$

where $n=1$.

By the way, generally

$$\frac{dQ}{dv} = \beta_0 e, \quad (131)$$

where dv denotes the volume difference.

$$dQ \equiv e. \quad (132)$$

Thus,

$$\beta_0 = \frac{1}{dv} = \delta(\vec{r}). \quad (133)$$

Therefore, as every elementary physics text mentions, the standard Coulomb potential forms:

$$\Phi = \frac{e}{4\pi\epsilon_0 r}. \quad (134)$$

Note that variable r here is a relative distance between two charged particles because of the introduction of the Dirac function having position vector \mathbf{r} .

Next let us consider the satisfaction of the continuity equation. Considering the following elementary equation,

$$v \equiv \frac{dr}{dt}. \quad (135)$$

Moreover from eq. (68) the current density is

$$i = -\frac{c^4}{16\pi G} \frac{1}{n} \frac{e}{m_e \omega_c} r. \quad (136)$$

Thus, following equation is satisfied.

$$\frac{d\rho}{dt} + \frac{di}{dr} = 0. \quad (137)$$

Thus it is allowed to generalize this equation to 3-dimension:

$$\frac{\partial \rho}{\partial t} + \text{div} \vec{i} = 0. \quad (138)$$

Considering the satisfactions both of eq. (134) and (138), the Poisson equation for vector potential is automatically proved. That is, because the charge density and the continuity equations have been proved, the current density, i.e., the Poisson equation for vector potential as eq. (68) has also been proved.

3.6.2 Newtonian equation

In Theory section, we derived

$$\Phi_G = -\frac{1}{8\pi} \frac{1}{16\pi G} \frac{c^2}{m_e} g \frac{1}{r}, \quad (78)$$

Using this equation, let us calculate the gravity energy of a quantized space–time in terms of magnetic or gravity field. In this equation, if Φ_G has the unit [J], then parameter g has the unit [$\text{J} \cdot \text{m}^6$].

Thus for the embedded two electrons in Fig. 1,

$$g \equiv 2m_e c^2 \lambda_c^6 \approx 2m_e c^2 \lambda_0^6, \quad (139)$$

and in eq. (78),

$$r \equiv 2\lambda_c \approx 2\lambda_0. \quad (140)$$

Then, the potential is resulted as

$$\Phi_G = -5.8 \times 10^{-10} \text{ J} \quad (141)$$

This value is approximately equal to U_B in eq. (117).

That is, using eq. (118),

$$M_\tau = 1.8 \text{ GeV}/c^2. \quad (141-2)$$

Thus, this energy is approximately the rest energy of the τ -particle [9], and thus also implies the energy of a quantized space–time in terms of magnetic or gravity field, which agrees with the measurement

and which does not contradict the theory of this paper.

Note that the above Newtonian equation has a different shape from the standard Newtonian gravity equation, which is usually taught in high-schools. However, although the standard Newtonian gravity equation is applied in the scale of the solar system, it is unnatural to consider that it can be applied even in the quantum scale, because every physic equation generally has application scales. For example, the equation “ $ma = F$ ” is well applied in macroscopic scales, but it cannot be applied in the scales less than an atomic one. Thus, the success of Schrodinger’s equation in applying to the H atom comes not from the fact that the value of the standard Newtonian gravity equation is too small but from the fact that it is already considered inapplicable to the scale of an atom.

3.6.3 Derivation of the time- dependent Maxwell’s equations

As mentioned many times, the source is the solution of the QEG equation:

$$r_i \equiv (-2) \frac{8\pi G}{c^4} \frac{\frac{1}{2}\hbar\omega_i}{\lambda_c^3} + (c \frac{2\pi}{\omega})^2. \quad (29)$$

Using this equation, we in the Theory section converted divisions of the quantized space–times λ_c and t_c into the standard differentials [1]. Considering the Lorentz conservation regarding the differentials, we derived the following equations:

$$\text{rot}\vec{H} = \frac{\partial\vec{D}}{\partial t}, \quad (101)$$

$$\frac{\partial\vec{B}}{\partial t} = -\text{rot}\vec{E}. \quad (116)$$

Therefore, we claim that, in a real sense, the above two equations are the same.

4. Discussion

4.1 Summary of key points of this study

Only by introducing quantized space–times derived from the zero-point energy, electromagnetic and gravity fields including dark energy are analytically well explained, involving the quantization of the Einstein’s gravity (QEG) equation. In this point, concepts are only the zero-point energy and the conservation of angular momentum of quantized space–times.

4.2 Analytical solution of the QEG equation

Analytical solutions of the QEG equation result in various significant results:

To begin with, the significance of quantizing Einstein's gravity equation, which can be derived by the introductions of quantized time-spaces and the zero-point energy, enables us to obtain the analytical solution (not numerical solution), and this analytical solution describes every electromagnetic and gravity fields uniformly. According to our previous paper [1,3], the weak and strong interactions essentially equal to electromagnetic fields with consideration of the zero-point energy. Thus, this paper reinforced the results of our previous paper [1,3], which describes the unified field theory in terms of particle physics, according to that only the source of every field is the zero-point energy.

Moreover, the solutions of the QEG equation well describes the existing main problems in terms of the universe.

4.3 Acceleration $a(r)$ resulting in acceleration-expansion universe

From the solution of the QEG equation, the specific form of acceleration a_r was derived. The calculation agrees well with the measurements. The significance is that this paper first succeeded in describing acceleration-expansion universe without any additional concepts. Because acceleration a_r depends on the distance r , this paper claims that a Hubble constant generally varies on distances. For example, although this paper calculated it for 1Mps, its value varies at 10 Mps.

Without introducing the dark energy, which exhibits the repulsive forces, only Einstein's gravity equation well explains acceleration-expansion in the universe. However, it does not imply that there is no dark energy. On the contrary, this paper proved the existence of the dark energy as well-known particles, which obey gravitational attractive forces. An important point is that the model of the universe with acceleration is not radial acceleration, but originated from a rotational model. Thus, this paper claims that our universe has a rotational model with a unique form of acceleration, while the standard big bang mode assumes the singularity and thus infinite energy. If the infinite energy was accepted, all our general physical equations and laws would be meaningless.

4.4 CMB emission

Analytical solution of the QEG equation also describes the emission of CMB. The result implies that we are not employing the standard big bang model. We derived the CMB emission and a unique angular frequency, which is a result of the solution of the QEG equation and which results in a rotational model of the universe. The significance is that the emissions and absorptions of CMB

photons are occurring everywhere in our universe, and they are directly related to the e-neutrino self-energy, which fluctuates in the energy level of the vacuum. Thus, CMB can be described without the standard big bang model, and thus this paper claims that the measured CMB does not have the meaning of initial time of the birth of the universe.

4.5 Unified field in terms of electromagnetic and gravity fields

The analytical solution of the QEG equation also describes the unified field in terms of electromagnetic and gravity fields. This solution implies rotations of quantized space–times both in terms of electric field and magnetic field (gravity field). The results lead to the Poisson equations regarding electrostatic potential, vector potential and gravity potential, respectively. These equations result in the Coulomb equation, Biot–Savart’s law (the derivation of this law is described in every physics text), and Newtonian gravity equation, respectively.

Moreover, using the concepts of photons, the inductions both from electric field to magnetic field and from magnetic field to electric field are derived. Thus, the time-dependent Maxwell’s equations are described.

In short, the existing Einstein’s gravity equation already contained both the properties of electromagnetic and gravity fields. Thus, we claim that to obtain the unified field theory, it is not needed to expand the existing Einstein’s gravity equations like 5-dimension.

The most important point is that all equations from electromagnetic and gravity fields come from the conservation law of angular momentum in terms of quantized space–times. Moreover, as mentioned in our previous paper [1,3], the weak and strong interactions are, in a real sense, equal to electromagnetic fields, and thus all the microscopic fields and basic equations stem from the conservation law of angular momentum in terms of quantized space–times. That is, there is the only source of the zero-point energy to create all the fields.

Furthermore, the result of the analytical solution of the QEG equation leads to analytical derivation of gravity waves automatically. The significance is that, although thus far this gravity waves were obtained only by the numerical analysis of the existing Einstein’s gravity equation, now we derived the same gravity wave from the pure analytical solution of the Einstein’s gravity equation. The reason comes from the fact that we succeeded in the quantization of the Einstein’s gravity equation.

4.6 3-Generation of leptons

Considering the basic configuration including quantized space–times both in terms of electric field and magnetic (gravity) field and the collapse of this configuration, we derived both rest energies of a τ -particle and a μ -particle, which basically agree with the measurements. Considering the real electron

is result of the collapses of a τ -particle or of a μ -particle, we have now succeeded in giving the reason why leptons have 3-generations. The above implies that the concepts of quantized space–times in terms of electric, magnetic or gravity field with the zero-point energy can be proved by the measurements. In our previous paper regarding neutrinos [2], we described 3-generations of neutrino, i.e., the oscillation of neutrinos, under the assumption that the masses of 3 leptons are known in advance. However, we have now clarified all the masses of 3 leptons without an assumption, and the most important mystery of why elementary particles have 3-generations was uncovered.

5. Conclusion

This paper described that, by the introduction of quantized space–times derived from the zero-point energy, the conservation of angular momentum of them, i.e., the analytical solution of the QEG equation, created all the law and equations in terms of electromagnetic and gravity fields. Moreover, the concept of the quantized space–times provides the reason why the leptons and neutrinos have 3-generations

As contributions of cosmology, the solution of the QEG equation results in acceleration-expansion universe, what is the dark energy and analytical derivation of gravity waves. All these results agree well with the measurements.

With the combination of the results from our previous paper [1,3], we now could reinforce the unified field theory in terms of particle physics that only the concepts of the zero-point energy and quantized space–times describe all the fields (i.e., electromagnetic field, gravity field, weak interaction, strong interaction, leptons, neutrinos, quarks, protons and neutrons, and so on).

We select the zero-point energy (i.e., the basic configuration of quantized space–times) as the ultimate source, which describes almost all fields, including the masses of W and Z bosons, but there is a Higgs boson. This paper and our previous papers did not describe this particle. As a follow-up, it is necessary to achieve a consistent description, including this boson.

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Additional information

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Appendix

Theoretical Calculation of the Specific Charge e/m

The specific charge e/m has been measured very accurately, while there is few theoretical report to calculate it. Herein let us calculate the specific charge e/m theoretically without numerical calculations or fitting methods. This calculation will be able when the concept of a quantized space–time is employed.

Generally cyclotron angular frequency ω_c is

$$\omega_c = \frac{e}{m} B_0. \quad (\text{A-1-1})$$

That is,

$$\frac{e}{m} = \frac{\omega_c}{B_0}. \quad (\text{A-1-2})$$

Thus, we attempt to provide ω_c and magnetic field B_0 , respectively.

First, in a static magnetic field, considering the analogy from a dc motor, a magnetic flux Φ is generally is expressed as

$$\Phi = B_0 S \sin(\omega_c t), \quad (\text{A-2})$$

where S and t denote the area and time, respectively.

The above implies a self-rotation-like spin.

From the Faraday law, a voltage V is given as

$$V = -\frac{d\Phi}{dt} = -B_0 S \omega_c \cos(\omega_c t) \quad (\text{A-3})$$

Within a quantized time t_c , the magnitude of this voltage becomes the initial form V_0 .

$$V_0 = B_0 S \omega_c = B_0 \lambda_c^2 \omega_c, \quad (\text{A-4})$$

where

$$S \equiv \lambda_c^2. \quad (\text{A-4-2})$$

On the other hand, the cyclotron angular frequency is simply

$$\omega_c = \frac{\theta_0}{t_c} = \frac{\theta_0}{\lambda_c/c} = \frac{\theta_0 c}{\lambda_c}, \quad (\text{A-5})$$

where θ_0 implies a quantized angle.

Thus eq. (A-4) is further transformed to

$$V_0 = B_0 \lambda_c^2 \frac{\theta_0 c}{\lambda_c} = B_0 \theta_0 c \lambda_c. \quad (\text{A-6})$$

That is,

$$B_0 = \frac{V_0}{\theta_0 c \lambda_c}. \quad (\text{A-6-2})$$

Thus, from eq. (A-1-2),

$$\frac{e}{m} = \frac{\frac{\theta_0 c}{\lambda_c}}{\frac{V_0}{\theta_0 c \lambda_c}} = \theta_0^2 c^2 / V_0. \quad (\text{A-7})$$

The initial voltage V_0 comes from the energy gap of the vacuum, i.e., the zero-point energy;

$$\frac{1}{2} \hbar \omega = m_e c^2 = 0.51 \text{ MeV}. \quad (\text{A-8})$$

Because we are now considering an electron, this energy is interpreted as the initial voltage V_0 :

$$V_0 = 0.51 \text{ MV}. \quad (\text{A-9})$$

Therefore, eq. (A-7) becomes

$$\frac{e}{m} = 1.764 \times 10^{11} \text{ C/kg}, \quad (\text{A-10})$$

where $\theta_0 \equiv 1 \text{ rad}$ is assumed, because generally a quantized space–time has no internal metrics.

That is, an arc and radius in a quantized space–time are not distinguished from each other.

As every physical text reports, the measurement value is

$$\frac{e}{m} = 1.758 \times 10^{11} \text{ C/kg}. \quad (\text{A-11})$$

Compared with each other, the agreement is sufficient.