

# ON THE FUNDAMENTAL CONSTANTS OF NATURE

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## ABSTRACT

Planck's constant and the gravitational constant are comprised of more fundamental quantities of length, mass, and time. Reformulating traditional equations in terms of these fundamental units offers a more granular view of the physical transformations encoded in the equations of physics. The composite structure of  $\hbar$  and  $G$  conceals a simple model in which maximum unit potentials are reduced by dimensionless proportionality operators. Natural symmetries correlate the three unit dimensions, yielding predictable quantities of physical dynamics. Insights are organized into a New Foundations Model of physics that reformulates traditional constants and equations in elementary form. The New Foundations Model offers a common language for describing quantum mechanical, gravitational, and electromagnetic phenomena.

**Keywords** Planck units · natural units · fundamental constants · physical constants · Planck's constant · gravitational constant · particle mechanics

## 1 Introduction

At the close of the 19th century, Max Planck unveiled the constant of proportionality that bears his name today [1–3]. Planck's discovery ushered in a new era of quantum physics and  $\hbar$  became ubiquitous in equations describing the physical universe on small scales. At the same time, Planck showed that combining  $\hbar$ ,  $G$ , and  $c$  in the right proportions creates natural quantities of length, mass, and time.

More than a century later, the system of derived Planck units—featuring enigmatic ratios of the same three constants—has expanded deeper into the fields of quantum mechanics, thermodynamics, electromagnetism, and gravity [4–9]. Despite sharing the same building blocks, these formulas offer little insight into the physical meaning of  $\hbar$  and  $G$ , or the Planck units they are used to define.

A deeper examination of the physical constants and the entire system of Planck units reveals a beautifully simple structure that warrants re-evaluation of what is *fundamental*. Formulas defining  $\hbar$  and  $G$  in fundamental quantities of length, mass, and time show that physical transformations are determined by the finer structure of natural Planck units rather than the composite values of the traditional constants. The fundamental Planck units not only explain the entire system of Planck units more simply and elegantly; they redefine physical constants and equations in ways that expand our understanding of the natural universe.

Planck's constant and the gravitational constant can be stated as

$$\hbar = l_P m_P c \quad (1)$$

$$G = \frac{l_P}{m_P} c^2. \quad (2)$$

Similarities between the two constants are immediately apparent in these simple forms, most notably in the relationship between length and mass. The product form of the relationship found in Planck's constant is ideal for determining the

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extensive properties of elementary particles, whereas the quotient form of the gravitational constant is better suited for calculating the intensive properties of gravitational bodies.

The composition of Planck's constant and the gravitational constant are demonstrated in table 1 using CODATA values.

Table 1: Planck's constant and the gravitational constant are comprised of fundamental units of length, mass, and time.

Constant	Unit	Value
$\hbar =$	$l_P$	$1.616255 \times 10^{-35} \text{ m}$
	$\times m_P$	$2.176434 \times 10^{-8} \text{ kg}$
	$\times c$	$299,792,458 \text{ m/s}$
	$=$	$1.054572 \times 10^{-34} \text{ kgm}^2/\text{s}^2$
$G =$	$l_P$	$1.616255 \times 10^{-35} \text{ m}$
	$\div m_P$	$2.176434 \times 10^{-8} \text{ kg}$
	$\times c$	$299,792,458 \text{ m/s}$
	$\times c$	$299,792,458 \text{ m/s}$
	$=$	$6.67430 \times 10^{-11} \text{ m}^3/\text{kg s}^2$

Replacing  $\hbar$  and  $G$  with fundamental Planck units offers a basis for reinterpreting equations in which these two constants appear. For all of the equations featuring Planck's constant and the gravitational constant, it is the relationships between the three fundamental units that quantify the natural world.

## 2 Are Planck's constant and the gravitational constant fundamental?

Equations 1 and 2 offer compelling evidence that Planck's constant and the gravitational constant are *not* fundamental, but comprised of more elementary quantities of length, mass, and time. For example, we can restate the traditional formula for Schwarzschild radius

$$r_s = \frac{2GM}{c^2}$$

in Planck units by replacing the Gravitational constant with Planck length, mass, and time according to 1 and 2, where  $c = l_P/t_P$

$$\begin{aligned} r_s &= 2 \left( \frac{l_P}{m_P} c^2 \right) \left( \frac{M}{c^2} \right) \\ &= 2 \left( \frac{M}{m_P} \right) \left( \frac{l_P^2}{l_P^2} \right) l_P. \end{aligned}$$

In the elementary form of the equation, it becomes evident that two superfluous quantities of  $c$  embedded in the gravitational constant are removed by the formula's inputs. It begs the question of whether  $c^2$  plays a role in determining the Schwarzschild radius at all or is merely included in the constant for use by other gravitational equations.

Removing  $c^2$  from the formula gives the elementary form of the Schwarzschild radius formula

$$r_s = 2 \left( \frac{M}{m_P} \right) l_P. \quad (3)$$

It is possible to write equations for Planck length, mass, and time in terms of  $\hbar$  and  $G$ , but this is like calling  $\sqrt{4} + 1$  a fundamental unit and the number 3 an artifact. Throughout this paper I'll demonstrate that where  $\hbar$  and  $G$  are used, it is the smaller structure of the Planck units that characterize physical systems and not the composite values. The following formulas offer a demonstration of how the fundamental units embedded in  $\hbar$  and  $G$  combine with formula inputs to produce observed quantities of physical phenomena.

The mixture of Planck units and dimensions comprising the gravitational constant are illustrated in figure 1.



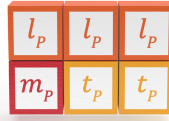


Figure 1: The Gravitational Constant is a mixture of six fundamental Planck units

## 2.1 Gravitational acceleration

The traditional formula for gravitational acceleration

$$g = -\frac{GM}{r^2}$$

can be re-written in elementary form by expanding the gravitational constant

$$g = -\left(\frac{l_p^3}{m_p t_p^2}\right) \frac{M}{r^2}.$$

Re-grouping terms in the equation reveals important ratios

$$g = -\left(\frac{l_p}{r}\right)\left(\frac{M}{m_p}\right)\left(\frac{l_p}{r}\right)\left(\frac{l_p}{t_p^2}\right).$$

Figure 2 illustrates the combined function in which inputs shown in gray pair up with fundamental Planck units embedded in the gravitational constant. The function can be characterized in two parts. The first component consists



Figure 2: Gravitational acceleration is determined by a mixture of proportionality operators acting on the Planck acceleration

of dimensionless operators generated from the ratios of formula inputs to Planck units. The second part can be characterized as the maximum *potential* of the unit dimensions we're calculating—in this case acceleration. The dimensionless operators act on the Planck acceleration, reducing its value proportional to the operators. Re-stating the formula as a combination of operators and potential gives

$$g = -\left(\frac{l_p}{r}\right)\left(\frac{M}{m_p}\right)\left(\frac{l_p}{r}\right)a_p \quad (4)$$

where  $a_p$  is the maximum acceleration potential.

## 2.2 Gravitational potential energy

The formula for gravitational potential energy

$$U_g = -G\frac{Mm}{r}$$

can be restated in elementary form by replacing the gravitational constant with fundamental Planck units

$$\begin{aligned} U_g &= -\left(\frac{l_p^3}{m_p t_p^2}\right) \frac{Mm}{r} \\ &= -\left(\frac{l_p}{r}\right)\left(\frac{M}{m_p}\right)mc^2. \end{aligned}$$

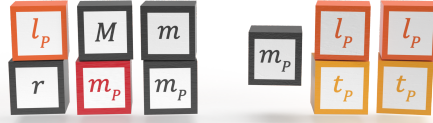


Figure 3: Gravitational potential energy is determined by a mixture of proportionality operators acting on the Planck energy

The equation shows a pair of dimensionless operators acting on a quantity of energy. The formula can be expressed in terms of the Planck energy potential by identifying a third operator assumed in the mass of the second body. Replacing  $m$  with the operator  $m/m_p$  acting on the Planck mass  $m_p$  gives the formula illustrated in figure 3. The formula can be stated in terms of three dimensionless operators acting on the Planck energy potential

$$U_g = - \left( \frac{l_p}{r} \right) \left( \frac{M}{m_p} \right) \left( \frac{m}{m_p} \right) E_p. \quad (5)$$

### 2.3 Gravitational force

The traditional formula for gravitational force

$$F = G \frac{Mm}{r^2}$$

can be expressed in elementary form as

$$\begin{aligned} F &= \left( \frac{l_p^3}{m_p t_p^2} \right) \frac{Mm}{r^2} \\ &= \left( \frac{l_p}{r} \right) \left( \frac{M}{m_p} \right) \left( \frac{l_p}{r} \right) m a_p. \end{aligned}$$

The equation gives a set of dimensionless operators acting on a quantity of force. Identifying the mass operator of the second body gives the formula illustrated in figure 4. The formula can be stated in terms of four dimensionless

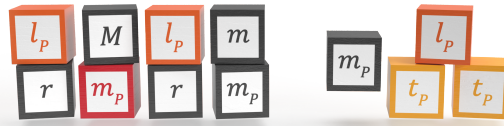


Figure 4: Gravitational force is determined by a mixture of proportionality operators acting on the Planck force

operators acting on the Planck force potential

$$F = \left( \frac{l_p}{r} \right) \left( \frac{M}{m_p} \right) \left( \frac{l_p}{r} \right) \left( \frac{m}{m_p} \right) F_p. \quad (6)$$

The elementary forms of these equations reveal why the quantity  $c^2$  is *not* needed in the Schwarzschild radius formula. In each of the equations, the operators  $l_p/r$  and  $M/m_p$  quantify the gravitation field produced by the first body while operators and potentials formed out of  $c^2$  quantify the properties of a second body at some distance from the first. The quantity  $c^2$  plays no role in determining the Schwarzschild radius of the first body.

In light of the elementary form of the equations, it is unlikely the gravitational constant has meaning apart from the mixture of Planck units it generates in formulas. Restated formulas suggest that the ratio of Planck length to mass says something fundamental about gravity, but not the aggregate dimensions  $L^3 M^{-1} T^{-2}$ . While formulas like 3 remove Planck units from  $\hbar$  and  $G$  using additional inputs, the same formulas can be stated efficiently in Planck units without removing any quantities or dimensions. Formulas can be expressed in Planck units without excess baggage.

Table 2 summarizes the operators, potentials, and natural formulas describing gravitational potentials.

Table 2: A summary of operators, potentials, and formulas that determine gravitational field potentials.

Physical Property	Potential	Operators	Natural Formula	Composite formula
Length	$l_P$	$\frac{M}{m_P}$	$r_s = 2 \left( \frac{M}{m_P} \right) l_P$	$r_s = \frac{2GM}{c^2}$
Energy	$E_P$	$\frac{l_P}{r}, \frac{M}{m_P}, \frac{m}{m_P}$	$U_g = - \left( \frac{l_P}{r} \right) \left( \frac{M}{m_P} \right) \left( \frac{m}{m_P} \right) E_P$	$U_g = - \frac{GMm}{r}$
Acceleration	$a_P$	$\frac{l_P}{r}, \frac{M}{m_P}, \frac{l_P}{r}$	$g = - \left( \frac{l_P}{r} \right) \left( \frac{M}{m_P} \right) \left( \frac{l_P}{r} \right) a_P$	$g = - \frac{GM}{r^2}$
Force	$F_P$	$\frac{l_P}{r}, \frac{M}{m_P}, \frac{l_P}{r}, \frac{m}{m_P}$	$F = \left( \frac{l_P}{r} \right) \left( \frac{M}{m_P} \right) \left( \frac{l_P}{r} \right) \left( \frac{m}{m_P} \right) F_P$	$F = \frac{GMm}{r^2}$
Escape Velocity	$c$	$\frac{l_P}{r}, \frac{M}{m_P}$	$v_e = \sqrt{2 \left( \frac{l_P}{r} \right) \left( \frac{M}{m_P} \right) c}$	$v_e = \sqrt{\frac{2GM}{r}}$

Like the gravitational constant, Planck's constant is a mixture of fundamental Planck units that combine with formula inputs to calculate physical phenomena. The simple form of Planck's constant

$$\hbar = l_P m_P c$$

is shown in elementary form in figure 5. Planck's constant is convenient for calculating the mechanical properties of

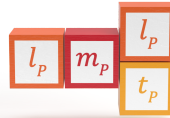


Figure 5: Planck's constant is a mixture of four fundamental Planck units.

elementary particles and systems as demonstrated in the following equations.

## 2.4 Photon momentum

The traditional formula for photon momentum

$$p_\gamma = \frac{\hbar}{\lambda}$$

can be re-stated as

$$\begin{aligned} p_\gamma &= \frac{l_P p_P}{\lambda} \\ &= \left( \frac{l_P}{\lambda} \right) p_P \end{aligned} \quad (7)$$

where the ratio of Planck length to photon wavelength is a dimensionless operator acting on the Planck momentum potential to give the photon's momentum. The formula's input pairs up with the Planck length as illustrated in figure 6.



Figure 6: Photon momentum is determined by a proportionality operator acting on the Planck momentum

## 2.5 Photon energy

The traditional formula for photon energy

$$E_\gamma = hf$$

can be expressed in Planck units as

$$E_\gamma = l_P m_P c \left( \frac{c}{\lambda} \right).$$

Re-grouping and simplifying the formula gives

$$\begin{aligned} E_\gamma &= \left( \frac{l_P}{\lambda} \right) m_P c^2 \\ &= \left( \frac{l_P}{\lambda} \right) E_P. \end{aligned} \quad (8)$$

The restated equation includes a dimensionless operator in the ratio of Planck length to photon wavelength, and the maximum energy potential given by the Planck energy. The elementary form of the equation is illustrated in figure 7.



Figure 7: Photon energy is determined by a proportionality operator acting on the Planck energy.

## 2.6 Compton wavelength

The formula for reduced Compton wavelength

$$\lambda_C = \frac{\hbar}{m_0 c}$$

can be re-stated in Planck units as

$$\lambda_C = \frac{l_P m_P c}{m_0 c}$$

where  $m_0$  is the rest mass. Grouping related terms gives the form

$$\lambda_C = \left( \frac{m_P}{m_0} \right) \left( \frac{c}{c} \right) l_P.$$

The formula creates an operator from the ratio of rest mass to Planck mass and a second operator from the ratio  $c/c$ . These act on the Planck length giving the particle's wavelength at the speed of light. The simplified form of the equation

$$\lambda_C = \left( \frac{m_P}{m_0} \right) l_P \quad (9)$$

is produced from the Planck units in  $\hbar$  and formula inputs as shown in figure 8.

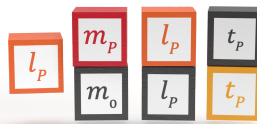


Figure 8: The Compton wavelength is determined by a proportionality operator acting on the Planck length. An instance of  $c$  is removed from Planck's constant to determine the wavelength.

## 2.7 de Broglie wavelength

The de Broglie wavelength formula requires a pair of operators to produce the wavelength of a particle with rest mass. The traditional formula for reduced de Broglie wavelength

$$\lambda = \frac{\hbar}{m_0 v}$$

can be written in Planck units as

$$\lambda = \left(\frac{m_P}{m_0}\right)\left(\frac{c}{v}\right) l_P. \quad (10)$$

The de Broglie formula has the same mass operator as the Compton wavelength formula, but adds a second operator in the ratio  $c/v$ . The two operators act on the Planck length producing the wavelength of a massive particle in motion, as shown in figure 9.

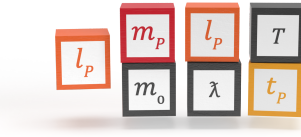


Figure 9: The de Broglie wavelength is determined by a pair of proportionality operators acting on the Planck length

Table 3 summarizes the four natural formulas produced by restating Planck's constant in fundamental units of length, mass, and time.

Table 3: A summary of operators, potentials, and formulas that determine the mechanical properties of elementary particles and systems.

Physical Property	Potential	Operators	Natural Formula	Composite formula
Photon Momentum	$p_P$	$\frac{l_P}{\lambda}$	$p = \left(\frac{l_P}{\lambda}\right) p_P$	$p = \frac{\hbar}{\lambda}$
Photon Energy	$E_P$	$\frac{l_P}{\lambda}$	$E = \left(\frac{l_P}{\lambda}\right) E_P$	$E = \frac{\hbar c}{\lambda}$
Compton Wavelength	$l_P$	$\frac{m_0}{m_P}$	$\lambda_C = \left(\frac{m_0}{m_P}\right) l_P$	$\lambda_C = \frac{\hbar}{mc}$
de Broglie Wavelength	$l_P$	$\frac{m_0}{m_P}, \frac{v}{c}$	$\lambda = \left(\frac{m_0}{m_P}\right)\left(\frac{v}{c}\right) l_P$	$\lambda = \frac{\hbar}{mv}$

The elementary formulas created from Planck's constant and the Gravitational constant suggest there is nothing inherently *quantum mechanical* about  $\hbar$  or *gravitational* about  $G$ . Any gravitational formula can be restated using  $\hbar$  and any quantum mechanical formula can be written in terms of  $G$ , provided we use the right formula inputs. The relationship between the two constants can be demonstrated in Planck units as follows

$$G = \frac{l_P}{m_P} c^2 = \left(\frac{c}{m_P^2}\right) l_P m_P c = \left(\frac{c}{m_P^2}\right) \hbar.$$

The required ratios of Planck units dictate how traditional composite constants may be used in formulas. Oftentimes it is difficult to find physical meaning in the way traditional constants are used, while the Planck units give more intuitive answers. Comparing formulas for black hole temperature gives an example to this effect.

## 2.8 Black hole temperature

The traditional equation for black hole temperature

$$T = \frac{\hbar c^3}{8\pi G k_B M}$$

can be written in terms of energy

$$k_B T = \frac{\hbar c^3}{G} \frac{1}{8\pi M}$$

and stated in Planck units as

$$E = \frac{(\cancel{h} m_P c) \cancel{c^2}}{\left(\frac{\cancel{h} \cancel{c^2}}{m_P}\right)} \frac{1}{8\pi M}.$$

The simplified equation

$$E = m_P^2 c^2 \frac{1}{8\pi M}$$

can be written as a mass operator and a constant acting on the Planck energy

$$E = \frac{1}{8\pi} \left(\frac{m_P}{M}\right) E_P.$$

The elementary form of the equation concisely describes a quantity of black hole energy proportional to the ratio of Planck mass to the body's mass.

### 3 New Foundations Model of physics

The equations in section 2 reveal a general formula for calculating the physical properties of elementary particles and systems. This formula establishes a basis in the *potentials* of certain physical properties quantified by the Planck units. The function transforms inputs of measured values into one or more proportionality operators based on their ratios to the Planck unit values. The magic of the function is that a simple reduction from the maximum potential determines the output.

The following sections introduce a New Foundations Model of physics in which historical constants and equations are reformulated in terms of fundamental units of length, mass, and time. The model reveals a new pattern behind the simple, non-relativistic form of physical equations that is hidden by the composite structure of the traditional constants.

An example of a reformulated equation is shown in figure 10. According to The New Foundations Model, the kinetic energy of elementary particles with rest mass is given by four dimensionless operators acting on the maximum Planck energy potential. Each of the four operators represents a physical property of the particle or system explained by the model.

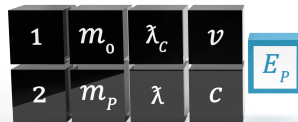


Figure 10: The New Foundations Model characterizes physical phenomena using maximum unit potentials and dimensionless proportionality operators.

#### 3.1 Benefits of the New Foundations Model

A principal goal of the New Foundations Model is to reduce the level of abstraction found in mathematical descriptions of the quantum world. Physical quantities calculated using the New Foundations Model are consistent with traditional formulas given equations 1 and 2; but while the results are the same, the New Foundations Model offers new perspectives on the meaning of abstract constants and equations. These physical descriptions are not part of the standard lexicon today but follow naturally from the relationships between the fundamental units.

The New Foundations Model offers the following advantages:

1. The model presents more granular forms of traditional physics equations. It explicitly identifies each term in an equation including terms hidden by the composite structure of the traditional constants.
2. The model gives meaning to abstract quantities and transformations found in physical equations.

3. The model effectively characterizes the mechanical, gravitational, and electromagnetic properties of elementary particles and systems using a common framework of operators and potentials.

The reformulated model is called the New Foundations Model because  $\hbar$  and  $G$  have served as foundational concepts of physics for well over a century, while the Planck units have largely been ignored. The New Foundations Model asserts that formulas based on natural units of length, mass, and time yield a better description of the natural universe and should replace the traditional constants.

The New Foundations Model does not try to solve the measurement problem. Rather, it presumes that the properties of elementary particles—characterized mathematically by wave functions and matrices—are genuine physical properties and not abstract quantities in configuration space. New Foundations Model formulas describe the fluid, deterministic evolution of particle field oscillations without incorporating the stochastic localization brought about by observation and measurement. The physical descriptions, therefore, are inferred from statistical ensembles and not from individual measurements.

### 3.2 Three Fundamental Units

The New Foundations Model is built on three fundamental quantities of length, mass, and time defined by the Planck units. Physical phenomena emerge in the form of elementary particles in which length and mass characterize a particle's spatial configuration, while time quantifies rates of change in its configuration. The three unit dimensions are defined by the International System of Units (SI) as meters, kilograms, and seconds, shown in figure 11. The three unit dimensions

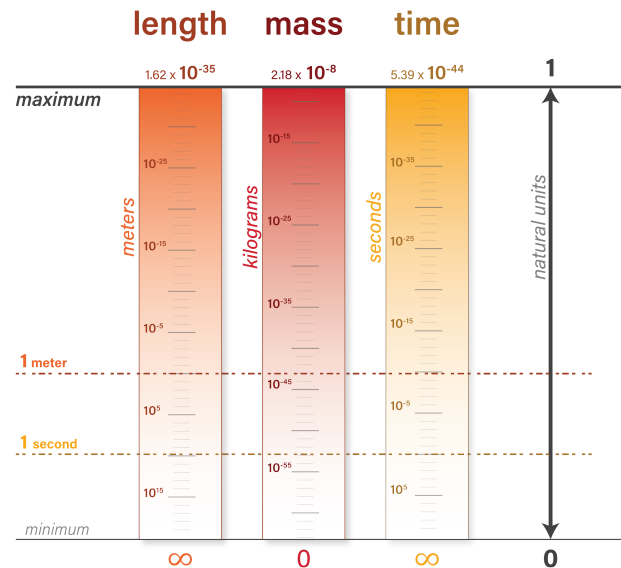


Figure 11: The Planck units are quantities of fundamental unit potentials. Length, mass, and time have definitive maximum potentials at the Planck scale and arbitrarily small potentials asymptotically approaching zero.

have two important characteristics described by the model:

1. The three unit dimensions coincide at the Planck scale where the Planck length, mass, and time represent definitive limits in all three dimensions.
2. All three unit dimensions are quantized in intervals of length, so that length ratios define corresponding quantities of mass and time.

### 3.3 Potentials

The New Foundations Model defines a maximum *potential* for each Planck unit and combination of units used to characterize the natural world. Each unit has a finite limit at its maximum potential and an arbitrarily small potential approaching zero asymptotically. Length and time potential are *inverse* to their unit values, so that maximum length and time potentials correspond with the shortest unit values. Length and time potential become arbitrarily small over larger scales of space and time.

Mass potential is maximized at the Planck scale, where the Planck mass defines the largest possible quantity of mass for an individual, elementary particle. Mass potential grows arbitrarily small over larger distances and timescales.

The finite, maximum potential of each fundamental unit provides a definitive basis for calculating physical properties whereas the asymptotic limit offers no basis. Operators are therefore always calculated as ratios of the maximum potential.

A physical property or dynamic is defined by its unique combination of unit dimensions, giving meaning to the concepts of momentum, energy, acceleration, force, etc. Each of these physical phenomena has a maximum potential quantified by the Planck unit values in the given dimensions. The model considers the phenomena we observe as redistributions of conserved potentials—diluted by the spatial and temporal configurations of elementary particles.

Table 4 presents a list of fundamental unit potentials, dimensions, and values based on CODATA values of Planck units,  $\hbar$ , and  $c$ .

Table 4: A summary of fundamental unit potentials that characterize physical systems.

Physical property	Potential	Value
Length	$l_P$	$1.616255 \times 10^{-35} \text{ m}$
Mass	$m_P$	$2.176434 \times 10^{-8} \text{ kg}$
Time	$t_P$	$5.391247 \times 10^{-44} \text{ s}$
Momentum	$\frac{l_P m_P}{t_P}$	$6.524786 \text{ kgm/s}$
Energy	$\frac{l_P^2 m_P}{t_P^2}$	$1.956081 \times 10^9 \text{ kgm}^2/\text{s}^2$
Velocity	$\frac{l_P}{t_P}$	$299,792,458 \text{ m/s}$
Acceleration	$\frac{l_P}{t_P^2}$	$5.560725 \times 10^{51} \text{ m/s}^2$
Force	$\frac{l_P m_P}{t_P^2}$	$1.210255 \times 10^{44} \text{ kgm/s}^2$
Action	$\frac{l_P^2 m_P}{t_P}$	$1.054572 \times 10^{-34} \text{ kgm}^2/\text{s}$
Mass Density	$\frac{l_P}{m_P}$	$7.426160 \times 10^{-28} \text{ m/kg}$
Mass Length	$l_P m_P$	$3.517673 \times 10^{-43} \text{ kgm}$

The New Foundations Model is a natural unit system in which the three fundamental units are quantified on a scale of 0 to 1, where 1 represents the maximum potential. This natural unit scale does two things:

1. By setting the maximum potential equal to one, any unit *distance* from the Planck scale can be quantified as the ratio of one divided by the distance.
2. Quantities of the three different unit types are normalized such that a ratio of one unit type coincides with the same ratio of another unit type.

The practice of assigning constants like  $c$ ,  $\hbar$  and  $G$  a value of 1 follows naturally from the principles described by the New Foundations Model. These systems work because nature provides a normalized scale across different unit dimensions—the key features of which include a common point of maximum potential, common intervals in each dimension, and symmetries that correlate changes in each unit dimension.

### 3.4 Operators

The New Foundations Model defines a set of proportionality operators representing the physical attributes of elementary particles and systems. Operators have the following properties:



- An operator is the ratio of length, mass, or time to its maximum potential in the same unit dimension—Planck length, mass, and time. For example, the ratio of a photon's wavelength to the Planck length is an operator giving the photon's momentum and energy. Both terms in an operator's ratio have the same unit dimension, making operators dimensionless.
- Operators are defined on a scale of 0 to 1, where one represents the maximum potential and zero represents the limit of minimum potential. The maximum potential serves as the basis of proportionality.
- Operators are designated by the symbol  $\beta$  with a subscript for the operator type.

The New Foundations Model defines the following mechanical operators.

### 3.4.1 Wavelength operator

$$\frac{l_P}{\lambda}$$

The wavelength operator quantifies the ratio of Planck length to an elementary particle's reduced wavelength. The wavelength operator is defined as

$$\beta_\lambda = \frac{l_P}{\lambda}.$$

The wavelength operator was demonstrated in equations 7 and 8. In traditional physics equations, the operator is generated by the ratio  $\hbar/\lambda$ . The Planck length embedded in Planck's constant pairs with the wavelength to form the operator, leaving the Planck momentum as the formula's maximum potential. The formula  $\hbar c/\lambda$  produces the wavelength operator and Planck energy.

The equivalence of operators across unit types gives two additional forms of the wavelength operator in dimensions of mass and time

$$\beta_\lambda = \frac{m}{m_P} = \frac{t_P}{T}.$$

### 3.4.2 Rest mass operator

$$\frac{m_0}{m_P}$$

The rest mass operator quantifies the rest mass of an elementary particle as the ratio of mass to the maximum Planck mass potential. It is defined as

$$\beta_m = \frac{m_0}{m_P}$$

Rest Mass can also be expressed in terms of wavelength as demonstrated in 9. Re-arranging 9 gives an equality between Compton wavelength and rest mass

$$\frac{l_P}{\lambda_C} = \frac{m_0}{m_P}. \quad (11)$$

where the Compton wavelength is a special case of the wavelength operator quantifying rest mass.

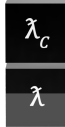
The rest mass operator can be demonstrated by the rest masses and Compton wavelengths of the charged leptons. Table 5 applies the rest mass operator to the Planck length and Planck mass producing known values of Compton wavelength and rest mass. In addition, the table shows that wavelength and rest mass intervals between lepton generations are equivalent.

The equivalence of wavelength and rest mass ratios to their maximum potentials is used in formulas 9 and 10 to determine wavelength from mass, and mass from wavelength. It is also used in equation 3 to determine length from mass in a gravitational system of particles.

Table 5: The Wavelength and rest mass operators produce the Compton wavelength and rest masses of the charged leptons. Intervals between generations of charged leptons show the equivalence of length and mass ratios.

Lepton	$\beta_m$	$l_p/\beta_m$ (m)	$\beta_m m_p$ (kg)	Interval between	Wavelength ratio	Rest mass ratio
e	$4.1855 \times 10^{-23}$	$3.8616 \times 10^{-13}$	$9.1094 \times 10^{-31}$	-	-	-
$\mu$	$8.6542 \times 10^{-21}$	$1.8676 \times 10^{-15}$	$1.8835 \times 10^{-28}$	$\mu$ and e	206.77	206.77
$\tau$	$1.4554 \times 10^{-19}$	$1.1105 \times 10^{-16}$	$3.1675 \times 10^{-27}$	$\tau$ and $\mu$	16.817	16.817

### 3.4.3 Momentum operator



The momentum operator is used to quantify momentum for particles with rest mass. It is defined as the ratio of Compton wavelength to de Broglie wavelength

$$\beta_p = \frac{\lambda_c}{\lambda}.$$

The elementary forms of the Compton and de Broglie wavelength formulas in 9 and 10 give a dual meaning to the Compton wavelength. For particles without rest mass, including photons, the Compton wavelength gives a quantity of energy equal to the rest mass of a massive particle through inelastic scattering [10–12]. But the Compton wavelength is also the wavelength of a *massive* particle at the limit of its velocity potential  $c$ . The two wavelengths coincide at this limit as shown in equations 9 and 10.

The equations show that the effect of rest mass is to dramatically reduce an electron's potential for producing momentum and energy compared to a photon whose limit is the Planck length. The Compton wavelength defines the electron's greatest wavelength *potential*—its shortest possible wavelength as its velocity approaches the speed of light.

In terms of the New Foundations Model, the presence of rest mass re-defines a particle's maximum potential, replacing the Planck scale with the Compton scale. In non-relativistic terms, quantities of mass, momentum, and energy are reached at the Compton scale and not the Planck scale.

Calculating momentum is as simple as multiplying the Planck momentum by the inverse wavelength operator *whether a particle has rest mass or not*:

$$p = \left(\frac{l_p}{\lambda}\right) p_p. \quad (12)$$

A consistent definition of momentum arises for both types of particles when we treat momentum as quantifying a particle's potential due to its wavelength and *not* its velocity. Each of these two factors—wavelength and velocity—contribute to a particle's kinetic energy, but only wavelength gives a consistent quantification of momentum for particles with and without rest mass.

The New Foundations Model clarifies the roles played by particle wavelength and velocity in determining momentum and energy. Momentum is strictly a measure of a particle's strength due to its *spatial* configuration, or wavelength, where shorter wavelengths yield greater potential for momentum and energy. It is the particle's kinetic energy that incorporates velocity, producing energy that is proportional to its rate of displacement. Momentum is therefore analogous to a payload reflected in the concentration of the particle's wavelength, while velocity delivers the payload.

The misleading formula  $p = mv$  used for particles with rest mass produces the correct answer because the ratio of a particle's displacement  $v/c$  is proportional to the change in its wavelength  $\lambda_c/\lambda$ . The concept of momentum pre-dates quantum theory when wave-like attributes of matter first became known. For Newton and others who described momentum prior to the 20th century, the quantity  $v^2$  simply matched the observational data. But Louis de Broglie's introduction of the quantum mechanical formula for momentum clarifies the separate roles played by wavelength and velocity in determining momentum and energy. Consequently, only the second  $v$  in the kinetic energy formula  $E = 1/2mv^2$  represents velocity. The first  $v$  quantifies the wavelength.

A simpler model of physics might apply the dimensionless wavelength operator to the Planck mass to quantify momentum in kilograms rather than introducing unnecessary unit dimensions.

While the ratio  $l_p/\lambda$  determines momentum for all types of particles, the wavelength operator is split into rest mass and momentum operators to characterize rest mass in Fermions. The relationship between the three operators is

$$\begin{aligned}\beta_\lambda &= \beta_m \beta_p \\ &= \left(\frac{l_p}{\lambda_C}\right) \left(\frac{\lambda_C}{\lambda}\right) = \frac{l_p}{\lambda}\end{aligned}$$

Separating the wavelength operator into two components allows the model to account for the change in potential from the Planck scale to the Compton scale. While momentum and energy are proportional to wavelength up to the Planck scale for photons, the presence of rest mass defines a new scale for momentum and energy between 0 and the Compton scale. The rest mass operator sets the new scale and the momentum operator quantifies momentum on that scale.

The equivalence of operators across unit types gives a mass form of the momentum operator

$$\beta_p = \frac{m}{m_0}$$

where  $m$  is a quantity of mass defined as  $m = m_0(v/c)$  and  $m = m_0(\lambda_C/\lambda)$ . This form of *inertial* mass defined by the New Foundations Model and quantified in kilograms is equal to the particle's momentum in units of  $kgm/s$ . Inertial mass is distinct from rest mass and therefore not generally acknowledged as mass. However, there are compelling reasons to treat inertial mass as a genuine form of mass, including

1. The concept of *effective mass* is often cited in the context of massless particles, [10, 13] and gives a quantitative measure of photon strength in kilograms that is consistent with momentum in units of  $kgm/s$ .
2. The elementary formula for Planck's constant (1) reveals the presence of mass in  $\hbar$ . The value and dimension of mass included in formulas with  $\hbar$  are determinants of the physical quantities calculated using the constant, even when there is no rest mass.
3. Applying the wavelength operator  $l_p/\lambda$  to the Planck mass embedded in  $\hbar$  produces a quantity of mass that, when inserted into Einstein's mass energy equivalence formula  $E = mc^2$ , produces the correct energy of a photon.
4. The New Foundations Model demonstrates in section 3.7 that an operator applied to units of length must have corresponding values in units of mass and time.

The wavelength, rest mass, and momentum operators stated in terms of inertial mass are therefore

$$\begin{aligned}\beta_\lambda &= \beta_m \beta_p \\ &= \left(\frac{m_0}{m_p}\right) \left(\frac{m}{m_0}\right) = \frac{m}{m_p}.\end{aligned}$$

### 3.4.4 Velocity operator



The velocity operator gives the ratio of a particle's displacement rate to the maximum rate of displacement  $c$ . The velocity operator is defined as

$$\beta_v = \frac{v}{c} = \frac{\Delta x}{\Delta t} = \left(\frac{\lambda}{l_p}\right) \left(\frac{t_p}{T}\right)$$

where  $T$  is a particle's oscillation period.

For particles with rest mass, the velocity and momentum operators are equal, giving the same ratio of wavelength and displacement to their maximum potentials. The equivalence of these two operators is shown in the ratios

$$\beta_v = \frac{v}{c} = \beta_p = \frac{\lambda_C}{\lambda} = \frac{m}{m_0}.$$

The momentum and velocity operators normalize different unit dimensions of mass and momentum so that a measurement of a particle's velocity also gives the change in its wavelength and inertial mass. These two operators create a combined reduction of  $\beta^2$ , or  $v^2/c^2$ .

Taken together, the momentum and velocity operators constitute a 2-part mechanism governing the kinetics of elementary particles. The two operators act together, modifying a particle's wavelength at the same time changing its velocity in a unitary fashion. Because the operators are equal in magnitude, they produce a squared quantity of kinetic energy relative to either the particle's wavelength or its velocity. This 2-part mechanism gives a beautifully simple explanation for the kinetic energies of particles with or without rest mass in the combined contributions of wavelength and velocity.

The relationship between the four operators—wavelength, rest mass, momentum, and velocity—are illustrated in figure 12. Only the wavelength operator is needed to describe the attributes of photons which have no rest mass. Quantities

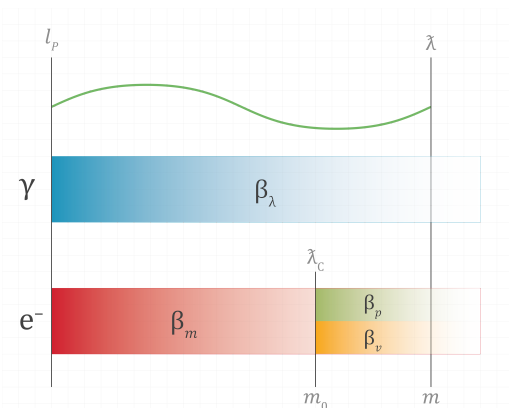


Figure 12: The relationship between wavelength, rest mass, momentum, and velocity operators are shown for photons and electrons. Due to rest mass, the wavelength operator is split into rest mass and momentum operators. The momentum and velocity operators are equal giving a combined ratio of  $v^2/c^2$ .

of wavelength, inertial mass, momentum, and energy are all determined by applying the wavelength operator to the maximum potentials of length, mass, momentum, and energy.

For particles with rest mass, the wavelength operator is split into rest mass and momentum operators. The rest mass operator quantifies the particle's rest mass and also defines its maximum wavelength and inertial mass potentials at the Compton scale.

### 3.4.5 Spin operator



The four operators for wavelength, rest mass, momentum, and velocity explain the basic mechanical properties of particles that have rest mass except for a one-half reduction in kinetic energy. The New Foundations Model offers a physical explanation for this reduction in the form of an elementary particle's intrinsic spin attribute, where two half-spin rotations complete an oscillation cycle. This 1/2 reduction in kinetic energy is described as the dilution of energy across an extended cycle and remains present in composite particles that have aggregate spin attributes.

The spin operator accounts for quantum mechanical spin and is defined for half-spin particles as

$$\beta_s = \frac{1}{2}$$

and for spin one particles as

$$\beta_s = 1.$$

### 3.4.6 Time Operators

The equivalence of operators across unit dimensions means that operators expressed in terms of length and mass can also be stated as ratios of the Planck time. The equivalence of these ratios can be shown in the derivation of a time operator quantifying kinetic energy. Multiplying the four operators that give the kinetic energy of particles with rest mass yields

$$\begin{aligned}\beta &= \beta_s \beta_m \beta_p \beta_v \\ &= \frac{1}{2} \left( \frac{l_p}{\lambda_C} \right) \left( \frac{\lambda_C}{\lambda} \right) \left( \frac{\lambda}{T} \frac{t_p}{l_p} \right)\end{aligned}$$

which can be reduced to the simple operator

$$\beta = \frac{t_p}{2T} \quad (13)$$

This single, dimensionless operator representing the particle's oscillation period can be applied to the Planck energy to give the correct kinetic energy

$$E_k = \frac{t_p}{2T} E_p$$

Using a spin operator of 1 gives the following formula for particles with no rest mass

$$E_\gamma = \frac{t_p}{T} E_p.$$

The simplicity of this kinetic energy time operator reflects the power of natural symmetries that will be introduced in section 3.7.

### 3.4.7 Summary of mechanical operators

Table 6 summarizes the set of mechanical operators defined by the New Foundations Model.

Table 6: A summary of mechanical operators: the wavelength, rest mass, momentum, velocity, and spin operators.

Operator Name	Symbol	Photon	Charged Leptons
Wavelength operator	$\beta_\lambda$	$\frac{l_p}{\lambda}$	$\frac{l_p}{\lambda}$
Rest mass operator	$\beta_m$	-	$\frac{l_p}{\lambda_C}, \frac{m_0}{m_p}$
Momentum operator	$\beta_p$	$\frac{l_p}{\lambda}$	$\frac{\lambda_C}{\lambda}, \frac{m}{m_0}, \alpha$
Velocity operator	$\beta_v$	1	$\frac{v}{c}, \frac{\lambda t_p}{l_p T}$
Spin operator	$\beta_s$	1	$\frac{1}{2}$

$\alpha$  = fine-structure constant;  $l_p$  = Planck length;  $\lambda_C$  = reduced Compton wavelength;  $\lambda$  = particle wavelength;  $m_p$  = Planck mass;  $m_0$  = rest mass;  $m$  = inertial mass;  $T$  = particle oscillation period  $t_p$  = Planck time

The New Foundations Model also defines a pair of gravitational operators based on the physical attributes of gravitational bodies including mass, radius, and distance from the center of the body. The ratios of these physical properties to their unit potentials determine field properties in the vicinity of a gravitational body. The gravitational operators were introduced in equations 4, 5, 6 and table 2. The New Foundations Model defines the operators as follows.

### 3.4.8 Mass density operator

$$\begin{array}{|c|c|} \hline l_p & M \\ \hline r & m_p \\ \hline \end{array}$$

The principal operator describing gravitational bodies is produced by the ratios of a body's mass and radius. The mass density operator is defined as

$$\beta_\rho = \left(\frac{l_p}{r}\right)\left(\frac{M}{m_p}\right)$$

where  $r$  is the radius of the gravitational body and  $M$  is its mass.

The signature characteristic of equations describing gravitational field potentials is a mass input in the numerator and a length input in the denominator. The mass density operator is formed out of the ratios produced when these two inputs are paired with the Planck length and mass contained in the gravitational constant. The ratio of Planck length to the body's radius produces one part of the operator, and the ratio of the body's mass to the Planck mass produces the second part. The mass density operator requires the product of both ratios to accurately characterize the intensive property of mass density.

The maximum potential for mass density produces an important physical constant in the ratio of Planck length to mass. This potential is equal to  $7.43 \times 10^{-28} m/kg$ . The significance of this ratio is seen in the attributes of black holes where the relationship between the Schwarzschild radius and the mass is defined by the quantity  $7.43 \times 10^{-28} m/kg$ , according to equation 3. A limit to the density of mass in a black hole suggests that black holes are not singularities and that a better description of black hole geometry is needed.

### 3.4.9 Distance operator

$$\begin{array}{|c|} \hline l_p \\ \hline r \\ \hline \end{array}$$

The mass density operator determines gravitational field properties surrounding a gravitational body, but calculating a specific potential requires a coordinate from the distance operator. The distance operator quantifies the radial dilution of the gravitational field as the ratio of Planck length to the radial distance

$$\beta_r = \frac{l_p}{r}$$

where  $r$  is the distance from the center of the gravitational body.

### 3.4.10 Summary of gravitational operators

Table 7 summarizes the two gravitational operators defined by the New Foundations Model.

Table 7: A summary of the gravitational operators: the mass density and distance operators.

Operator Name	Symbol	Formula
Mass density operator	$\beta_\rho$	$\left(\frac{l_p}{r}\right)\left(\frac{M}{m_p}\right)$
Distance operator	$\beta_r$	$\frac{l_p}{r}$

### 3.5 Demonstrating the New Foundations Model operators

The New Foundations Model accurately describes the Bohr hydrogen atom in terms of maximum potentials and proportionality operators. In the hydrogen atom, the kinetic energy of a ground state electron is given by the Rydberg energy formula. The Rydberg energy is calculated using the Rydberg constant

$$R_{\infty} = \frac{1}{4\pi} \left( \frac{m_e}{m_p} \right) \frac{\alpha^2}{l_p}$$

where  $m_p$  is the Planck mass. The Rydberg constant contains the operators necessary for calculating the electron's ground state energy, including the electron rest mass operator in the ratio of  $m_e/m_p$ , the spin operator in the ratio  $1/2$ , the momentum and velocity operators in two instances of the fine-structure constant, and a quantity of Planck length in the denominator. The Planck length is required in the denominator of the Rydberg constant simply to remove an instance of Planck length from the historical constant  $hc$ , equal to  $2\pi l_p E_p$ . The extraneous use of  $l_p$  in each of these constants favors the Planck unit expression as the fundamental description, as the composite constants cannot produce the Planck energy without the fundamental units.

Removing the Planck length from the denominator of the Rydberg constant and the numerator of the Rydberg energy gives the four required operators and the maximum energy potential  $E_p$ . An additional quantity of  $2\pi$  is included in the denominator of the Rydberg constant because the Rydberg energy formula does not use the reduced form of Planck's constant.

Multiplying the Rydberg constant by  $hc$  gives

$$\frac{1}{2} \frac{1}{2\pi} \left( \frac{m_e}{m_p} \right) \frac{\alpha^2}{l_p} 2\pi E_p l_p$$

The simplified formula

$$\frac{1}{2} \left( \frac{m_e}{m_p} \right) \alpha^2 E_p \quad (14)$$

gives the correct operators and potentials which have the following quantities in the electron ground state

- **Rest mass operator:**  $4.185 \times 10^{-23}$
- **Spin operator:**  $1/2$
- **Momentum operator:**  $0.0073$
- **Velocity operator:**  $0.0073$

Applying the operators to the Planck energy potential produces the ground state energy

$$\begin{aligned} E_p &= 1.2209 \times 10^{19} \text{ GeV} \\ \times 4.185 \times 10^{-23} &= 510,999 \text{ eV} \\ \times 1/2 &= 255,500 \text{ eV} \\ \times 0.0073^2 &= 13.6 \text{ eV} \end{aligned}$$

We can also calculate the electron's wavelength in the ground state using  $10$ , the de Broglie wavelength formula

$$\lambda = \frac{1.616 \times 10^{-35} \text{ m}}{(4.185 \times 10^{-23})(0.0073)} = 5.292 \times 10^{-11} \text{ m}$$

which is equal to the Bohr radius.

### 3.6 Historical constants

Historical constants are comprised of maximum unit potentials, proportionality operators, and combinations of both. Restating the constants in fundamental units reveals the role of operators and potentials in each constant. Table 8 restates several important historical constants in terms of operators and potentials.

Table 8: Traditional constants are comprised of potentials and proportionality operators. Restating historical constants in fundamental Planck units reveals their compositions.

Constant	Symbol	Standard form	Elementary form	Alternate forms	Value
Reduced Planck const	$\hbar$	-	$\frac{l_p^2 m_p}{t_p}$	$l_p p_p, E_p t_p$	$1.054572 \times 10^{-34} \frac{kgm^2}{s}$
Gravitational const	$G$	-	$\frac{l_p^3}{m_p t_p^2}$	$\frac{l_p}{m_p} c^2$	$6.67430 \times 10^{-11} \frac{m^3}{kg s^2}$
Speed of light	$c$	$\frac{l_p}{t_p}$	$\frac{l_p}{t_p}$	-	$299,792,458 \frac{m}{s}$
Fine-structure	$\alpha$	$\frac{e^2}{4\pi\epsilon_0 \hbar c}$ [14]	-	-	.0072973525693
Rydberg constant	$R_\infty$	$\frac{m_e \alpha^2 c}{2h}$ [14]	$\frac{1}{2} \frac{1}{2\pi} \frac{m_e}{m_p} \alpha^2 \frac{1}{l_p}$	-	$10973733 \frac{1}{m}$
Rydberg energy	$hcR_\infty$	$\frac{1}{2} m_e \alpha^2 c^2$ [14]	$\frac{1}{2} m_e \alpha^2 \frac{l_p^2}{t_p^2}$	$\frac{1}{2} \frac{m_e}{m_p} \alpha^2 E_p$	$2.179872 \times 10^{-18} \frac{kgm^2}{s^2}$

$\alpha$ =Fine-structure constant;  $E_p$ =Planck energy;  $l_p$ =Planck length;  $m_e$ =electron mass;  $m_p$ =Planck mass;  $N$ =newton;  $p_p$ =Planck momentum;  $t_p$ =Planck time

### 3.7 Symmetries

The relationships between fundamental quantities of length, mass, and time are governed by natural symmetries such that changes in one unit dimension correspond with predictable changes in the other two dimensions.

The New Foundations Model identifies three symmetries in these relationships giving rise to quantifiable physical dynamics. Two symmetries govern the dynamics of elementary particles with or without rest mass, and a third symmetry defines the impact of rest mass. All three symmetries are based on the simple premise that a change in one unit dimension is accompanied by comparable changes in the other two.

Throughout this section I'll reference the following table of photon properties. The table shows the attributes of six familiar photons by applying the wavelength operator,  $l_p/\lambda$ , to potentials of length, mass, time, momentum, and energy. Values are equivalent to those calculated using traditional formulas in which  $\hbar = l_p m_p c$ .

Table 9: Photon properties are calculated by applying the wavelength operator  $l_p/\lambda$  to Planck units of length, mass, time, momentum, and energy. Proportional and conserved quantities in the table reflect natural symmetries in their physical properties.

Photon	$\beta_\lambda(l_p/\lambda)$	$\lambda$ (m)	$m$ (kg)	$T$ (s)	$p$ (kgm/s)	$E$ (kgm <sup>2</sup> /s <sup>2</sup> )	$\lambda m, pT$ (kgm)
$\lambda_p$	1	$1.62 \times 10^{-35}$	$2.18 \times 10^{-8}$	$5.39 \times 10^{-44}$	6.52	$1.96 \times 10^9$	$3.52 \times 10^{-43}$
$\lambda_C, \tau$	$1.46 \times 10^{-19}$	$1.11 \times 10^{-16}$	$3.17 \times 10^{-27}$	$3.70 \times 10^{-25}$	$9.50 \times 10^{-19}$	$2.85 \times 10^{-10}$	$3.52 \times 10^{-43}$
$\lambda_C, \mu$	$8.65 \times 10^{-21}$	$1.87 \times 10^{-15}$	$1.88 \times 10^{-28}$	$6.23 \times 10^{-24}$	$5.65 \times 10^{-20}$	$1.69 \times 10^{-11}$	$3.52 \times 10^{-43}$
$\lambda_C$	$4.19 \times 10^{-23}$	$3.86 \times 10^{-13}$	$9.11 \times 10^{-31}$	$1.29 \times 10^{-21}$	$2.73 \times 10^{-22}$	$8.19 \times 10^{-14}$	$3.52 \times 10^{-43}$
$\lambda_{H,1s-2p}$	$1.33 \times 10^{-28}$	$1.22 \times 10^{-7}$	$2.89 \times 10^{-36}$	$4.06 \times 10^{-16}$	$8.67 \times 10^{-28}$	$2.60 \times 10^{-19}$	$3.52 \times 10^{-43}$
$\Delta\nu_{Cs}$	$4.96 \times 10^{-34}$	$3.26 \times 10^{-2}$	$1.08 \times 10^{-41}$	$1.09 \times 10^{-10}$	$3.23 \times 10^{-33}$	$9.69 \times 10^{-25}$	$3.52 \times 10^{-43}$

#### 3.7.1 Length-mass Symmetry

The first symmetry defined by the New Foundations Model is the relationship between length and mass. It says that the product of an elementary particle's wavelength and inertial mass is equal to the constant  $3.52 \times 10^{-43} kgm$ . The maximum length-mass potential at the Planck scale is given by

$$l_p m_p = 3.52 \times 10^{-43} kgm.$$



According to the symmetry, an increase in wavelength from the Planck scale is accompanied by an inverse change in mass. This relationship can be shown by re-arranging 10 into a length-mass equality

$$\lambda m_0 \left( \frac{v}{c} \right) = l_P m_P.$$

Substituting inertial mass  $m$  for  $m_0(v/c)$  as proposed in section 3.4.3 gives a general form of the symmetry for particles *irrespective of rest mass and in any state of inertia*

$$\lambda m = l_P m_P = 3.52 \times 10^{-43} \text{ kgm}. \quad (15)$$

The New Foundations Model explains the length-mass symmetry as a correlation between unit dimensions of length, mass, and time. For particles without rest mass, table 9 gives the correlated values—all of which are determined using the same wavelength operator shown in column 2. Quantities of wavelength, oscillation period, momentum, and energy all agree with traditional formulas and the quantity of mass agrees with the proposed definition of inertial mass. Furthermore, the quantity  $\lambda m$  is conserved for each photon.

For particles that have rest mass, the conservation of wavelength and inertial mass is shown in table 11 and will be discussed more in section 3.7.3.

A conserved quantity of length-mass is portrayed as an inverse function in figure 13. The curve reflects possible

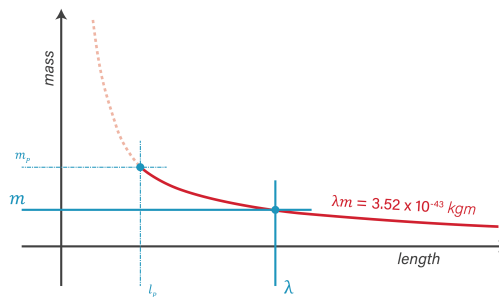


Figure 13: A standard model elementary particle's wavelength and inertial mass are inversely related, conserving the product of wavelength and mass as  $3.52 \times 10^{-43} \text{ kgm}$ .

combinations of wavelength and inertial mass up to the Planck scale, shown at the natural unit coordinate (1,1). From the Planck scale, as units of length increase, units of mass decrease reciprocally so that  $m = 1/\lambda$ .

The quantity  $3.52 \times 10^{-43} \text{ kgm}$  is an important physical constant in the conservation of length-mass. The consistency of this relationship across the different elementary particles of the standard model—demonstrated for charged leptons in table 10—suggests that a standard particle unit may underlie each of the elementary particles of the standard model. Because the length-mass relationship is conserved for individual particles—reflecting the quantization of particle fields—the New Foundations model refers to this constant as the *quantum constant*.

Table 10: The Compton wavelengths and rest masses of the charged leptons conserve the quantity  $3.52 \times 10^{-43}$

Lepton	$\lambda_C$ (m)	$m_0$ (kg)	$\lambda_C m_0$ (kgm)
e	$3.8616 \times 10^{-13}$	$9.1094 \times 10^{-31}$	$3.5177 \times 10^{-43}$
$\mu$	$1.8676 \times 10^{-15}$	$1.8835 \times 10^{-28}$	$3.5177 \times 10^{-43}$
$\tau$	$1.1105 \times 10^{-16}$	$3.1675 \times 10^{-27}$	$3.5177 \times 10^{-43}$

### 3.7.2 Mass-momentum Symmetry

The second symmetry is an alternative expression of the first, but stated in terms of traditional dimensions of momentum. At the Planck scale, momentum is defined as

$$p_P = m_P \frac{l_P}{t_P}.$$

The formula can be re-stated as an equality

$$l_P m_P = p_P t_P \quad (16)$$

in which the product of length-mass is equal to the product of momentum-time. The relationship is illustrated in figure 14 which also highlights the ratios between units.

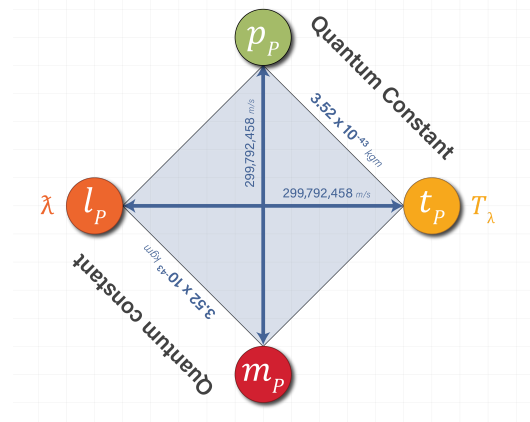


Figure 14: The ratio between Planck mass and momentum is equal to the ratio between Planck length and time. Each is equal to the speed of light. Products of momentum-time, and mass-length are conserved by the quantum constant.

According to the symmetry, there is a correlation between unit dimensions of length, mass, and time such that

$$\lambda m = p T_{\lambda}. \quad (17)$$

The value of each term in the equality is determined by applying the wavelength operator to the Planck length, mass, time, and momentum. Furthermore, each side of the equality gives the conserved quantum constant as shown in table 9.

Traditional dimensions of momentum require a time component which is defined as

$$T_{\lambda} = \frac{\lambda}{c} = \frac{\lambda_C}{v}.$$

The time component is designated  $T_{\lambda}$  to distinguish it from  $T$ , the particle's oscillation period.  $T_{\lambda}$  is equal to  $T$  for particles with *no* rest mass but diverges for particles with rest mass. The divergence between  $T_{\lambda}$  and  $T$  is addressed in the third symmetry.

The  $\lambda$  subscript conveys the physical meaning of momentum explained in section 3.4.3, which showed that momentum represents the inertial strength of a particle due to the concentration of its wavelength. This physical property can be represented simply as inertial mass in units of kilograms but is converted into traditional units of momentum using  $T_{\lambda}$  and the second symmetry.

Re-arranging 17 gives an equation for momentum in terms of the three unit dimensions of length, mass, and time

$$p = \frac{\lambda m}{T_{\lambda}} \text{ kgm/s}. \quad (18)$$

Because  $\lambda m$  is constant, a particle's momentum can be determined simply from the time component  $T_{\lambda}$ . Equation 18 gives the same result as traditional momentum formulas  $p = mv$  and  $p = \hbar/\lambda$ , but has the advantage of giving physical quantities in all three unit dimensions. The New Foundations Model also identifies the time component required to convert inertial mass into traditional units of momentum.

Figure 15 illustrates the proportional transformation of Planck length, mass, and time by applying the wavelength operator to the maximum potentials.

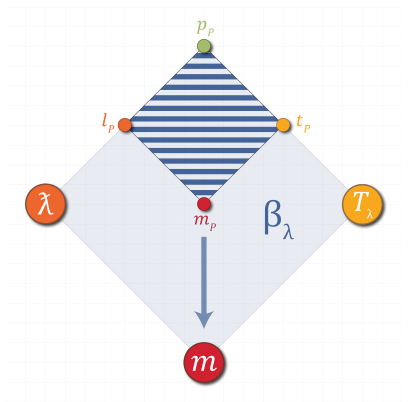


Figure 15: The wavelength operator changes quantities of length, mass, and time in equal proportions.

The operator modifies each of the terms by the same ratio, increasing the photon’s wavelength, decreasing its inertial mass, and increasing the time component  $T_\lambda$ . Only one instance of the operator is needed to calculate the correct quantity of momentum because of the inverse relationship between mass potential and comparable potentials for wavelength and time. However, the operator must be applied individually to the three terms to obtain the correct value in each unit dimension. This is easy to see in table 9 where the period changes in proportion to the wavelength while inertial mass is inversely proportional.

Figure 16 gives a conceptual illustration of the physical attributes of a photon according to 17.

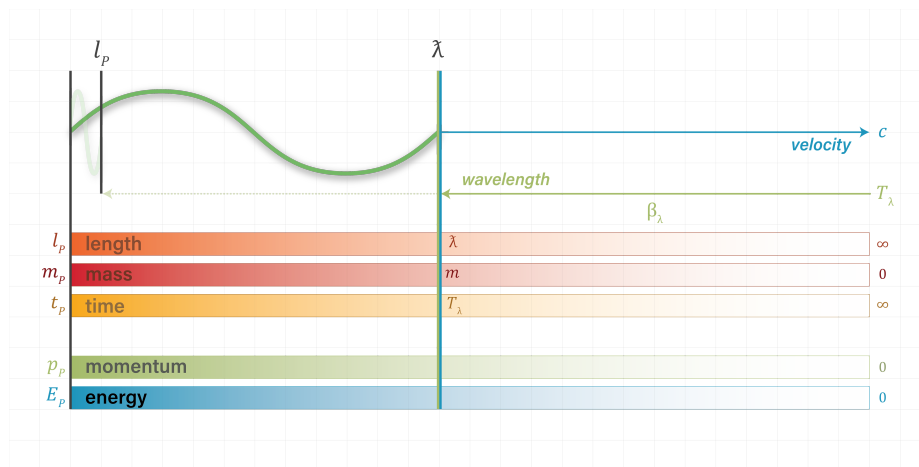


Figure 16: Photon properties are determined using the wavelength operator. An increase in wavelength is proportional to decreases in inertial mass, momentum, and energy.

The wavelength operator  $l_p/\lambda$  quantifies an increase in wavelength from the minimum Planck length. As the illustration shows, the resulting wavelength corresponds to reduced quantities of mass, momentum, time, and energy from their maximum potentials. Meanwhile, the photon’s velocity remains at a constant rate of  $c$ .

### 3.7.3 Momentum-energy symmetry

The New Foundations Model defines a third symmetry in the relationship between momentum and energy. It says that the product of length and momentum is equal to the product of energy and time. At the Planck scale, energy is defined as

$$E_p = p_p \frac{l_p}{t_p}.$$

This second factor of  $l_p/t_p$  adds the dynamic property of motion that is often mistakenly attributed to momentum. The ratio of Planck length to time represents the maximum rate of displacement in a particle’s position. Energy combines

the spatial distribution of the particle’s wavelength—described by momentum—with the rate of change in its displacement described by velocity. Energy is the result of these two factors, dynamically redistributing the particle over space and time.

From the definition of Planck energy comes an equality between Planck momentum and energy in the form

$$l_P p_P = E_P t_P. \tag{19}$$

The third symmetry is represented in figure 17 including important ratios between the units. As the figure shows,

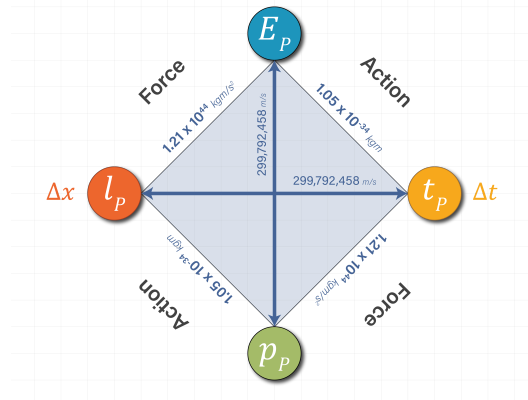


Figure 17: The ratio between Planck energy and momentum is equal to the ratio between Planck length and time. Each is equal to the speed of light. The relationships between the Planck units give rise to the physical dynamics of action and force.

the Planck ratios of energy to momentum and length to time are equal to  $c$ . The figure also shows the equivalent relationships between the product of Planck energy & time, and the product of Planck momentum & length. Each of these conserved quantities is equal to Planck’s constant,  $1.05 \times 10^{-34} \text{kgm}^2/\text{s}$ .

Equation 19 expresses two important physical dynamics. Each side of 19 represents a form of *action* in dimensions of momentum-length and energy-time, both in units of  $\text{kgm}^2/\text{s}$ . The second physical dynamic arises by re-arranging 19 as

$$\frac{p_P}{t_P} = \frac{E_P}{l_P} \tag{20}$$

to produce two equivalent forms of *force* in dimensions of momentum per time and energy per length, both in units of  $\text{kgm}/\text{s}^2$ .

According to the third symmetry, quantities of length, mass, and time are correlated such that

$$\Delta x p = \Delta t E_k \tag{21}$$

where  $\Delta x$  represents a change in position and  $\Delta t$  represents the corresponding time. By selecting the particle’s wavelength  $\lambda$  as the change in position, we get the particle’s oscillation period  $T$  as the change in time. Restating the equation in terms of a single oscillation period gives

$$\lambda p = T E_k. \tag{22}$$

Just as  $\lambda m$  and  $p T_\lambda$  are conserved by the mass-momentum symmetry, the pairs of values  $\lambda p$  and  $T E_k$  are conserved by the momentum-energy symmetry. The former symmetry conserves the quantum constant  $3.52 \times 10^{-43} \text{kgm}$  and the latter symmetry conserves Planck’s constant  $1.05 \times 10^{-34} \text{kgm}^2/\text{s}$ .

All three symmetries are illustrated in figure 18.



The second section applies the rest mass operator  $\beta_m$  which reduces the electron’s maximum mechanical potentials from the Planck scale down to the Compton scale.

The third section further reduces the kinetic energy potential by applying the spin operator, reducing the electron’s kinetic energy in half.

The final section applies the wavelength and velocity operators. Each row represents a different value of the two equivalent operators on a scale of 0 to 1, where 1 is the maximum wavelength potential (the shortest unit wavelength) and the greatest velocity. Demonstrated values of the operator include  $\alpha$ , the fine-structure constant, which produces the ground state properties of an electron in the hydrogen atom.

For every value of the wavelength and velocity operators, the quantum constant and Planck’s constant are conserved by the  $v^2$  currents.

Figure 19 is a conceptual illustration of the massive particle mechanics described by the New Foundations Model. As

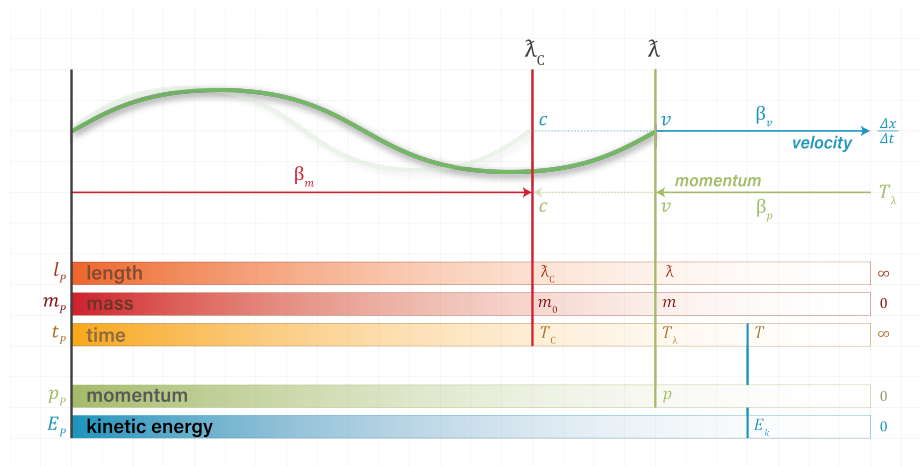


Figure 19: The mechanics of particles with rest mass are conceptually illustrated. The rest mass, momentum, and velocity operators are applied to the maximum Planck potentials yielding observed properties of the particle. The spin operator is not shown.

shown in the illustration, the  $\beta_m$  operator determines the particle’s Compton wavelength, which has a dual meaning. It is both the measure of a particle’s rest mass in terms of a photon’s wavelength, and also the limit of the particle’s own wavelength as it approaches the speed of light. The spin operator for particles with rest mass is not shown in the illustration but reduces the maximum kinetic energy potential by half.

Given the application of rest mass, the illustration shows that a particle’s wavelength is further refined by the  $\beta_p$  operator, and its velocity is determined by the equivalent  $\beta_v$  operator. The maximum potential of these operators is shown as  $c$  at the Compton wavelength. The two operators approach zero as the particle’s wavelength increases.

The illustration also shows how different quantities of length, mass, and time relate to each other. For a given wavelength there is a corresponding quantity of mass and time, where rest mass is also present but does not affect the kinetic energy. The time unit  $T_\lambda$  gives the particle’s wavelength in units of seconds, normalizing the wavelength and velocity operators as  $v^2/c^2$ . The second time component is the particle’s oscillation period  $T$ , which increases with the combined effects of longer wavelength and slower rate of displacement.

Figure 19 gives physical meaning to Einstein’s iconic formula  $E = mc^2$ . The formulaic expression  $c^2$  is shown to represent the maximum potential of a particle’s wavelength to decrease in length and its velocity to increase. Both of these dynamics reach a limit quantified by the particle’s rest mass.

The New Foundations model also gives physical meaning to the Lorentz factor

$$\sqrt{1 - \frac{v^2}{c^2}}$$

The value 1 represents the particle’s maximum potential at the Compton scale, while  $v^2$  represents the degree to which the particle’s wavelength approaches the Compton wavelength and its velocity approaches the speed of light. The square root of the term gives the dilation of length and time.

### 3.7.4 Summary of symmetries

Table 12 summarizes the New Foundations Model symmetries. Three equalities express the conservation of mass, momentum, and energy, and give rise to the mechanical properties of force and action.

Table 12: A summary of symmetries accounting for the conservation of mass, momentum, and energy.

Name	Symmetry	Current	Max potential	Invariant	Conserved quantity
Length-mass	$\lambda m = l_P m_P$		$l_P m_P$	$\lambda m$	$3.52 \times 10^{-43} \text{ kgm}$
Mass-momentum	$\lambda m = p T_\lambda$		$l_P m_P$	$\lambda m$	$3.52 \times 10^{-43} \text{ kgm}$
			$p_P t_P$	$p T_\lambda$	$3.52 \times 10^{-43} \text{ kgm}$
		$\lambda_C / T_\lambda$	$c$		
Momentum-energy	$\lambda p = T E_k$		$l_P p_P$	$\lambda p$	$1.05 \times 10^{-34} \text{ kgm}^2/s$
			$t_P E_P$	$T E_k$	$1.05 \times 10^{-34} \text{ kgm}^2/s$
		$\Delta x / \Delta t$	$c$		

The presence of rest mass generates two currents in a particle's spatial and temporal distribution. The spatial current  $\lambda_C / T_\lambda$  dilutes the particle's potential over a larger region of space, while the temporal current  $\Delta x / \Delta t$  dilutes its potential over longer timeframes.

### 3.8 Natural units

Table 13 demonstrates the proposed operators and equations using two scenarios. The first scenario treats the Planck units as natural units of length, mass, and time, where units of length are integer multiples of the Planck length. The example demonstrates in simple ratios the physical transformations made by each operator and equation. The second example calculates the same properties for a ground state electron in the hydrogen atom.

Table 13: The New Foundations Model is summarized in two examples: one using natural units and another using the ground state electron of a hydrogen atom.

Name	Symbol	Operators & Potentials	Formula	e.g. natural units	e.g. H1 1s electron
Planck length	$l_P$	-	-	1	$1.616255 \times 10^{-35} m$
Planck mass	$m_P$	-	-	1	$2.176434 \times 10^{-8} kg$
Planck time	$t_P$	-	-	1	$5.391247 \times 10^{-44} s$
Rest mass operator	$\beta_m$	-	$\frac{m_0}{m_P}, \frac{l_P}{\lambda_C}$	1/10	$4.185463 \times 10^{-23}$
Momentum operator	$\beta_p$	-	$\frac{\lambda_C}{\lambda}, \frac{m}{m_0}$	10/25	0.007297353
Velocity operator	$\beta_v$	-	$\frac{\lambda t_P}{T l_P}, \frac{v}{c}$	10/25	0.007297353
Spin operator	$\beta_s$	-	$\frac{1}{2}$	1/2	0.5
Compton wavelength	$\lambda_C$	$\frac{l_P}{\beta_m}$	$\frac{l_P m_P}{m_0}$	10	$3.861593 \times 10^{-13} m$
Wavelength	$\lambda$	$\frac{l_P}{\beta_m \beta_p}$	$\frac{l_P m_P}{m}$	25	$5.291772 \times 10^{-11} m$
Rest mass	$m_0$	$\beta_m m_P$	$\frac{l_P m_P}{\lambda_C}$	1/10	$9.109384 \times 10^{-31} kg$
Inertial mass	$m$	$\beta_m \beta_p m_P$	$\frac{l_P m_P}{\lambda}$	1/25	$6.647438 \times 10^{-33} kg$
T Lambda	$T_\lambda$	$\frac{t_P}{\beta_m \beta_p}$	$\frac{\lambda}{c}$	25	$1.765145 \times 10^{-19} s$
Momentum	$p$	$\beta_m \beta_p p_P$	$\frac{m_0 \lambda_C}{T_\lambda}, \frac{m \lambda}{T_\lambda}$	1/25	$1.992852 \times 10^{-24} kgm/s$
Velocity	$v$	$\beta_v \frac{l_P}{t_P}$	$\frac{\lambda}{T}, \frac{\Delta x}{\Delta t}$	25/62.5	2,187,691m/s
Period	$T$	$\frac{t_P}{\beta_m \beta_p \beta_v}$	$t_P \frac{\lambda}{l_P \lambda_C}$	25 <sup>2</sup> /10	$2.418884 \times 10^{-17} s$
Kinetic Energy	$E_k$	$\beta_m \beta_s \beta_p \beta_v E_P$	$\frac{m \lambda^2}{2 T_\lambda T}$	$\frac{(1/25)(25^2)}{(2)(25)(62.5)}$	$2.179872 \times 10^{-18} J$

## 4 Electromagnetism

The New Foundations Model shows that particle mechanics and gravitational potentials can be described using proportionality operators and potentials constructed from the fundamental units. It might be expected, therefore, to find a similar description of electromagnetic potentials—and this is the case. Current descriptions of the electromagnetic interaction use a unique set of units based on the abstract property of electric charge. But even this abstract quality produces measurable phenomena in the same four dimensions of space and time used to describe the other interactions. The mechanical properties induced by electric charge must have commensurate values of length, mass, and time.

The New Foundations Model reformulates electromagnetic units and constants into equivalent meter-kilogram-second (MKS) units. It is easy to show that the model reproduces the same results as traditional electromagnetic units; while the consistency of operators, potentials, and unit intervals also provide compelling evidence for a fundamental description of electromagnetism in MKS units.

The key to converting electromagnetic units into MKS units lies in identifying the maximum potentials of physical quantities which coincide across the different unit types. The derived system of Planck units offers a number of such potentials. In particular, translating the maximum charge potential from units of Coulomb into MKS units enables the subsequent translation of the remaining units and electromagnetic constants.



One method of deriving the MKS equivalent of the electromagnetic units requires formulas for Planck charge [4,8,15,16] and Planck inductance [16]

$$q_P = I_P t_P = \sqrt{4\pi\epsilon_0 \hbar c}. \quad (24)$$

$$L_P = \frac{E_P}{I_P} = \frac{1}{4\pi\epsilon_0} \sqrt{\frac{\hbar G}{c^7}} \quad (25)$$

where  $q_P$  is the Planck charge and  $L_P$  is Planck inductance. Equation 25 can be simplified by substituting Planck units for  $\hbar$ ,  $G$ , and  $c$  according to 1 and 2

$$\sqrt{\frac{\hbar G}{c^7}} = \sqrt{\frac{m_P l_P^5 t_P^7}{m_P l_P^7 t_P^3}} = \frac{t_P^2}{l_P} = \frac{1}{a_P}$$

giving

$$L_P = \frac{E_P}{I_P} = \frac{t_P^2}{4\pi\epsilon_0 l_P}. \quad (26)$$

Equation 24 can be simplified by substituting Planck units for  $\hbar$  and  $c$

$$q_P^2 = 4\pi\epsilon_0 \hbar c = 4\pi\epsilon_0 l_P E_P$$

and arranged into a formula for electric permittivity

$$\epsilon_0 = \frac{q_P^2}{4\pi l_P E_P}. \quad (27)$$

Inserting this form of electric permittivity into 26 gives a relationship between  $q_P^2$  and  $I_P$  in terms of fundamental units

$$\frac{E_P}{I_P} = \frac{m_P l_P^2}{q_P^2}.$$

Rearranging the formula in terms of Planck current gives

$$I_P = \frac{E_P q_P^2}{m_P l_P^2}$$

which can be substituted into 24 to produce a formula for Planck charge in terms of fundamental Planck units

$$q_P = \frac{E_P q_P^2 t_P}{m_P l_P^2}.$$

Simplifying the equation gives

$$\begin{aligned} q_P &= \frac{m_P l_P^2 t_P}{m_P l_P^2 t_P} \\ &= t_P. \end{aligned}$$

Given the long history of quantifying electric charge in coulomb, one may feel inclined to dismiss this result out-of-hand. But that would be a mistake. Substituting Planck time for Planck charge produces consistent quantities and dimensions in formula after formula. Furthermore, the proportionality of electromagnetic potentials given in MKS units are equivalent across the fundamental unit dimensions—just as the New Foundations Model demonstrates for quantum mechanical and gravitational phenomena. The consistency of this substitution warrants a closer look.

Deeper reflection on the use of time in mechanical and gravitational formulas shows that time is a natural way of quantifying force and force potential. The unit dimensions of force include a time value quantifying the phenomenon as a rate of change in momentum. In this regard, the electromagnetic interaction is no different from the other interactions. For dynamical processes, time is the factor that determines rates of change.

Substituting Planck time for Planck charge produces a coherent set of electromagnetic units, constants, and formulas in MKS units. This translation gives new meaning to historical constants whose values and dimensions are ambiguous. It shows, for example, that the Planck force is the maximum potential embedded in constants for electric permittivity and magnetic permeability. The same potential will be shown in the stress-energy tensor of the Einstein field equations using only 1 and 2.

#### 4.1 Electric permittivity

Substituting Planck time for Planck charge in equation 27 gives

$$\begin{aligned}\epsilon_0 &= \frac{t_P^4}{4\pi l_P^3 m_P} \\ &= \frac{1}{4\pi F_P c^2}\end{aligned}\quad (28)$$

where  $F_P$  is the Planck force. The corresponding formula for the Coulomb constant is therefore

$$k_e = F_P c^2.$$

Formulas using electric permittivity and the Coulomb constant show that the embedded quantity  $c^2$  becomes dimensionless as it pairs up with formula inputs to create proportionality operators. This leaves the Planck force as the constants' maximum potential.

#### 4.2 Coulomb's law

Coulomb's law demonstrates the electrostatic force in terms of operators and potentials. The traditional formula for the electrostatic force

$$F = k_e \frac{q_1 q_2}{r^2}$$

can be written in natural units as

$$F = (F_P c^2) \frac{t_1 t_2}{r^2}. \quad (29)$$

Using the coulomb unit conversion in table 15, charge is entered in seconds and distance in meters. These inputs pair up with the Planck unit values  $l_P^2/t_P^2$  embedded in the constant, yielding four operators acting on the Planck force potential

$$F = \left(\frac{l_P}{r}\right) \left(\frac{t}{t_P}\right) \left(\frac{l_P}{r}\right) \left(\frac{t}{t_P}\right) F_P.$$

The four operators can be summarized using the relationship

$$\frac{c}{v} = \frac{\frac{l_P}{r}}{\frac{t}{t_P}} = \left(\frac{l_P}{r}\right) \left(\frac{t}{t_P}\right)$$

and simplified as

$$F = \left(\frac{c^2}{v^2}\right) F_P.$$

The New Foundations Model shows that electrostatic potential is naturally described in the same terms as mechanical and gravitational forces. The operators  $c^2/v^2$  and the maximum force potential represent a magnificent correspondence between the interactions. The inverse square relation found in gravitational and electromagnetic field potentials reflects the 2-part mechanism described in section 3 which affects particle wavelengths and velocities equally.

Unfortunately, the Planck units do not reveal what physical attribute distinguishes one type of electric charge from another. For now, the Planck unit formulation, like other descriptions of electromagnetism, can only say what electric charge *does* and not what it *is*.

The maximum potential of the Einstein field equations can be shown by converting  $\hbar$  and  $G$  into fundamental Planck units. The stress-energy tensor

$$\frac{8\pi G}{c^4} T_{uv}$$

can be restated in elementary form as

$$8\pi \left(\frac{t_P}{p_P}\right) T_{uv} \quad (30)$$

where  $t_P/p_P$  is the Planck force appearing in the formula's denominator

$$F = \frac{8\pi T_{uv}}{F_P}.$$

### 4.3 Elementary charge

The elementary charge can be converted into MKS units and dimensions given the electric permittivity. Equation 28 gives a formula for magnetic permeability of

$$\mu_0 = 4\pi F_P.$$

Substituting magnetic permeability into the formula for elementary charge [14]

$$e = \sqrt{\frac{4\pi\hbar\alpha}{\mu_0 c}}$$

gives the elementary form of the constant

$$\begin{aligned} e &= \sqrt{\frac{4\pi\hbar\alpha t_P t_P}{4\pi\hbar\alpha t_P t_P}} \\ &= \sqrt{t_P^2 \alpha} \\ &= t_P \sqrt{\alpha}. \end{aligned}$$

The formula describes elementary charge as a reduction from the maximum Planck charge potential by a factor of  $\sqrt{\alpha}$ , and the reduction between a pair of charges as  $\alpha$ .

At a glance, it might appear as though the elementary charge of  $4.6 \times 10^{-45} s$  represents a shorter time duration than the Planck time. Examination shows, however, that the square root of the fine-structure constant embedded in the elementary charge *increases* the time input that is paired with the Planck time in formulas—effectively reducing the potential and not increasing it.

The New Foundations Model treats  $\sqrt{\alpha}$  as a proportionality operator applied to the maximum Planck charge. The operator is defined as

$$\beta_q = \sqrt{\alpha}$$

### 4.4 Electromagnetic constants

Table 14 gives the New Foundations Model equivalents of electromagnetic constants in fundamental quantities of length, mass, and time. Each constant is defined in elementary form using the three fundamental Planck units and alternative forms are given in units of Planck energy, Planck momentum, and the speed of light.

Table 14: Electromagnetic constants can be expressed in units of length, mass, and time. Like other constants, they are combinations of maximum potentials and proportionality operators.

Constant	Sym	Std form	Elem form	Alternate forms	MKS value	EM value
Planck charge	$q_p$	$\sqrt{4\pi\epsilon_0\hbar c}$ [4, 8, 15]	$t_p$	-	$5.391247 \times 10^{-44} s$	$1.876 \times 10^{-18} C$
Elementary charge	$e, q$	$\sqrt{\frac{4\pi\hbar\alpha}{\mu_0 c}}$ [14]	$t_p \sqrt{\alpha}$	-	$4.605448 \times 10^{-45} s$	$1.602 \times 10^{-19} C$
Electric permittivity	$\epsilon_0$	$\frac{1}{\mu_0 c^2}$ [14]	$\frac{t_p^4}{4\pi l_p^3 m_p}$	$\frac{t_p}{4\pi p_p c^2}, \frac{1}{4\pi F_p c^2}$	$7.315968 \times 10^{-63} \frac{s^4}{kgm^3}$	$8.854 \times 10^{-12} \frac{F}{m}$
Coulomb const	$k_e$	$\frac{1}{4\pi\epsilon_0}$	$\frac{l_p^3 m_p}{t_p^4}$	$\frac{p_p}{t_p} c^2, F_p c^2$	$1.087723 \times 10^{61} \frac{kgm^3}{s^4}$	$8.988 \times 10^9 \frac{Nm^2}{C^2}$
Magnetic permeability	$\mu_0$	$\frac{4\pi\alpha\hbar}{e^2 c}$ [14]	$4\pi \frac{l_p m_p}{t_p^2}$	$4\pi \frac{p_p}{t_p}, 4\pi F_p$	$1.520851 \times 10^{45} \frac{kgm}{s^2}$	$1.256 \times 10^{-6} \frac{N}{A^2}$
Voltage	$V_p$	$\sqrt{\frac{c^4}{4\pi G \epsilon_0}}$ [16]	$\frac{l_p^2 m_p}{t_p^3}$	$\frac{E_p}{t_p}, \frac{E_p}{q_p}$	$3.628253 \times 10^{52} \frac{kgm^2}{s^3}$	$1.043 \times 10^{27} V$
Current	$I_p$	$\sqrt{\frac{4\pi\epsilon_0 c^6}{G}}$ [16]	$\frac{t_p}{t_p}$	$\frac{q_p}{t_p}, \frac{t_p}{q_p}$	1	$3.479 \times 10^{25} A$
Inductance	$L_p$	$\frac{1}{4\pi\epsilon_0} \sqrt{\frac{\hbar G}{c^7}}$ [16]	$\frac{l_p^2 m_p}{t_p^2}$	$E_p$	$1,956,081,000 \frac{kgm^2}{s^2}$	$1.616 \times 10^{-42} H$
Magnetic inductance	$B_p$	$\sqrt{\frac{c^5}{4\pi\epsilon_0 \hbar G^2}}$ [16]	$\frac{m_p}{t_p^2}$	$\frac{p_p}{l_p t_p}, \frac{F_p}{l_p}$	$7.488021 \times 10^{78} \frac{kg}{s^2}$	$2.152 \times 10^{53} T$
Conductance	$G_p$	$4\pi\epsilon_0 c$	$\frac{t_p^3}{l_p^3 m_p}$	$\frac{t_p}{E_p}, \frac{q_p}{E_p}$	$2.756147 \times 10^{-53} \frac{s^3}{kgm^2}$	0.03336S
Impedance	$Z_p$	$\frac{1}{4\pi\epsilon_0 c}$ [16]	$\frac{l_p^2 m_p}{t_p^3}$	$\frac{E_p}{t_p}, \frac{E_p}{q_p}$	$3.628253 \times 10^{52} \frac{kgm^2}{s^3}$	29.98Ω
Impedance of vacuum	$Z_0$	$\mu_0 c$ [14]	$4\pi \frac{l_p^2 m_p}{t_p^3}$	$4\pi \frac{E_p}{t_p}$	$4.559397 \times 10^{53} \frac{kgm^2}{s^3}$	376.7Ω
Capacitance	$C_p$	$4\pi\epsilon_0 \sqrt{\frac{\hbar G}{c^3}}$ [16]	$\frac{t_p^4}{l_p^2 m_p}$	$\frac{t_p^2}{E_p}$	$1.485907 \times 10^{-96} \frac{s^4}{kgm^2}$	$1.798326 \times 10^{-45} F$
Magnetic flux quantum	$\phi_0$	$\frac{2\pi\hbar}{2e}$ [14]	$\frac{\pi l_p^2 m_p}{\sqrt{\alpha} t_p^2}$	$\frac{\pi}{\sqrt{\alpha}} E_p$	$7.193730 \times 10^{10} \frac{kgm^2}{s^2}$	$2.067834 \times 10^{-15} Wb$
von Klitzing	$R_K$	$\frac{2\pi\hbar}{e^2}$ [14]	$\frac{2\pi m_p l_p^2}{\alpha t_p^3}$	$\frac{2\pi E_p}{\alpha t_p}$	$3.124008 \times 10^{55} \frac{kgm^2}{s^3}$	25,812.81Ω
Josephson	$K_J$	$\frac{2e}{h}$ [14]	$\frac{\sqrt{\alpha} t_p^2}{\pi l_p^2 m_p}$	$\frac{\sqrt{\alpha}}{\pi E_p}$	$1.390100 \times 10^{-11} \frac{s^2}{kgm^2}$	$4.835978 \times 10^{14} \frac{Hz}{V}$
Bohr magneton	$\mu_B$	$\frac{e\hbar}{2m_e}$ [14]	$\frac{\sqrt{\alpha} m_p}{2 m_e} l_p^2$	-	$2.665808 \times 10^{-49} m^2$	$9.274010 \times 10^{-24} \frac{J}{T}$
Conductance quantum	$G_0$	$\frac{2e^2}{2\pi\hbar}$ [14]	$\frac{\alpha t_p^3}{\pi l_p^2 m_p}$	$\frac{\alpha t_p}{\pi E_p}$	$6.402033 \times 10^{-56} \frac{s^3}{kgm^2}$	$7.748092 \times 10^{-5} S$

$\alpha$ =Fine-structure constant; A=ampere; C=coulomb;  $E_p$ =Planck energy; F=farad; H=henry;  $l_p$ =Planck length;  $m_e$ =electron mass;  $m_p$ =Planck mass;  $p_p$ =Planck momentum; N=newton;  $t_p$ =Planck time; S=siemens; T=tesla; W=weber; V=volt; Ω=ohm

Converting maximum potentials from electromagnetic units into MKS units produces a conversion factor for each electromagnetic unit type. Table 15 gives a conversion factor for each unit type and the formula used to determine its value.

Table 15: Electromagnetic units have equivalent values in kilograms, meters, and seconds. The table provides a conversion factor for each unit type derived from the Planck unit definitions of the electromagnetic constants.

Em Unit	Symbol	Formula	Conversion	MKS units
coulomb	q, e	$q_P/q_P$	$2.874495 \times 10^{-26}$	s
ampere	A	$I_P/I_P$	$2.874495 \times 10^{-26}$	dimensionless
volt	V	$V_P/V_P$	$3.478872 \times 10^{25}$	$kgm^2/s^3$
tesla	T	$B_P/B_P$	$3.478872 \times 10^{25}$	$kg/s^2$
weber	Wb	$Vt$	$3.478871 \times 10^{25}$	$kgm^2/s^2$
ohm	$\Omega$	$Z_P/Z_P$	$1.210255 \times 10^{51}$	$kgm^2/s^3$
henry	H	$L_P/L_P$	$1.210255 \times 10^{51}$	$kgm^2/s^2$
farad	F	$C_P/C_P$	$8.262723 \times 10^{-52}$	$s^4/kgm^2$
siemen	S	$G_P/G_P$	$8.262720 \times 10^{-52}$	$s^3/kgm^2$

The conversion of electromagnetic constants into Planck units in table 14 allows the electromagnetic interaction to be interpreted entirely in units of length, mass, and time.

#### 4.5 Demonstrating the Electromagnetic operators and potentials

The New Foundations Model translation of electromagnetic dimensions and values into MKS equivalents accurately reproduces the Bohr model of the hydrogen atom. Furthermore, it explains the Bohr model in terms of New Foundations Model potentials and operators, and gives new insights into the meaning of quantized orbital angular momentum.

Bohr gave a formula for calculating the allowed electron orbitals using the electrostatic force [17]

$$r_n = \frac{n^2 \hbar^2}{Z k_e e^2 m_e}$$

where  $n$  is the orbital number and  $Z$  is the integer number of protons in the nucleus. Replacing the traditional constants with fundamental Planck units, including the Coulomb constant and elementary charge from table 14, gives

$$\begin{aligned} r_n &= \left( \frac{n^2}{Z} \right) \left( \frac{l_P^2 m_P m_P}{t_P^2} \right) \left( \frac{t_P^4}{l_P^2 m_P^2} \right) \left( \frac{1}{t_P^2 \alpha} \right) \frac{1}{m_e} \\ &= \left( \frac{n^2}{Z} \right) \left( \frac{m_P}{m_e} \right) \left( \frac{1}{\alpha} \right) l_P. \end{aligned}$$

The term  $n^2/Z$  gives an index of atomic numbers and orbitals. The ratio of electron rest mass to Planck mass is the familiar rest mass operator, and the fine-structure constant is the momentum operator. While  $\alpha$  also represents the velocity operator, the formula is solving for *distance* which is consistent with the definition of momentum and not velocity.

We can compare the electrostatic formula result with the generally accepted formula for orbital angular momentum

$$L = m_e v r_n = n \hbar.$$

Writing the equation in terms of  $r_n$  and replacing  $\hbar$  with  $l_P m_P c$  shows that the two formulas are equivalent with  $Z = 1$  for the hydrogen atom

$$r_n = n \left( \frac{m_P}{m_e} \right) \left( \frac{c}{v} \right) l_P. \quad (31)$$

Comparing the two formulas with 10 demonstrates that the atomic orbitals are calculated in intervals of the electron's wavelength.

In the elementary form of the equation, there is nothing to suggest that atomic orbitals are quantized in *units* of  $\hbar$ . In fact, the New Foundations Model shows that  $\hbar$  is not a unit, but a conserved quantity generated from the ratios of more granular units and dimensions. The interpretation that  $\hbar$  quantizes orbitals in intervals of  $n$  does not follow necessarily

from the term  $n\hbar$ . The formula simply says that intervals of  $n$  are accounted for by the particle's velocity since the rest mass is fixed. Planck's constant would remain conserved for any sized orbital, including for fine and hyperfine transitions of the electron, whether or not the orbital radius is an integer  $n$ . The conservation of  $\hbar$  was demonstrated in section 3.7 and table 11.

We can show the relationship between orbital radii and the electron's wavelength in two forms. Multiplying the rest mass operator by the Planck length gives the electron's Compton wavelength, and showing  $\alpha$  as the momentum operator gives

$$r_n = \lambda_C \left( \frac{n^2}{\alpha} \right).$$

This form of the equation might suggest that the fine-structure constant is quantizing the orbitals. Multiplying the Compton wavelength by the inverse momentum operator gives the electron's wavelength for any state of inertia

$$r_n = \lambda_n n = a_0 n^2. \quad (32)$$

where  $\lambda_n$  is the electron wavelength in the  $n^{\text{th}}$  orbital, and  $a_0$  is the Bohr radius. This form of the equation could be used as evidence supporting the standing wave hypothesis. The motivation for integer wavelengths is subject to interpretation but the relationship between radii and wavelength is clear from the formulas; wavelength increases in units of  $n$  while the radius increases in units of  $n^2$ .

## 5 Comparing forces

The New Foundations Model provides a method for explaining the similarities and differences between mechanical, electromagnetic, and gravitational forces. Table 16 compares three forces under the common scenario of an electron in the hydrogen atom ground state.

The mechanical force of the electron is compared with the electrostatic and gravitational potentials between the electron and proton. The table begins with the maximum force potential in all three columns and then applies the proper proportionality operators to produce the known forces. In the ground state, the electron's wavelength and radius are equal, making the mechanical and electromagnetic forces comparable. The comparison thus presumes that distance represented by an electron's wavelength is analogous to the distance between electric charges in the electron and proton.

Table 16: Mechanical, electromagnetic, and gravitational forces are compared using maximum potentials and proportionality operators. Small rest mass in relation to the Planck mass makes the gravitational force much weaker than the other forces.

Reduction	Operator(s)	Formula	Mechanical	Electrostatic	Gravitational
<b>Planck Force</b>			<b><math>1.21 \times 10^{44}</math></b>	<b><math>1.21 \times 10^{44}</math></b>	<b><math>1.21 \times 10^{44}</math></b>
Electron rest mass	$\beta_m$	$m_0/m_P$	$4.19 \times 10^{-23}$		$4.19 \times 10^{-23}$
Proton rest mass	$\beta_m$	$m_0/m_P$			$7.69 \times 10^{-20}$
Electron inertial mass	$\beta_p$	$m_0/m$	$7.30 \times 10^{-3}$		
Electron period	$\beta_m, \beta_p, \beta_v$	$T_\lambda \lambda / \lambda_C$	$2.23 \times 10^{-27}$		
Distance	$\beta_r$	$l_P/r$		$3.05 \times 10^{-25}$	$3.05 \times 10^{-25}$
Distance	$\beta_r$	$l_P/r$		$3.05 \times 10^{-25}$	$3.05 \times 10^{-25}$
Electron charge	$\beta_q$	$\sqrt{\alpha}$		$8.54 \times 10^{-02}$	
Proton charge	$\beta_q$	$\sqrt{\alpha}$		$8.54 \times 10^{-02}$	
<b>Total Reductions</b>			<b><math>6.81 \times 10^{-52}</math></b>	<b><math>6.81 \times 10^{-52}</math></b>	<b><math>3.00 \times 10^{-91}</math></b>
<b>Force</b>			<b><math>8.24 \times 10^{-8}</math></b>	<b><math>8.24 \times 10^{-8}</math></b>	<b><math>3.63 \times 10^{-47}</math></b>

The proportionality operators in table 16 show why we experience a weaker force of gravity than electromagnetism. The potential of both interactions is the Planck force which is embedded in the Coulomb and gravitational constants. Both interactions also share the same reductions in strength due to the distance between proton and electron. It is the remaining operators which account for the vast difference between electromagnetic and gravitational potentials. The electrostatic force is only reduced further in magnitude by the fine-structure constant. The gravitational potential,

on the other hand, is reduced by the ratios of particle masses to their maximum mass potentials. The comparable reductions are about 0.7 percent in electric charge versus 42 orders of magnitude in mass. The effect of rest mass is to substantially reduce a particle's inertial mass and energy, which in turn reduces the particle's impact on the gravitational field. Particles approaching the maximum mass and energy potentials at the Planck scale would produce only small reductions in the strength of gravity.

The electromagnetic and gravitational forces in table 16 were calculated using the New Foundations Model potentials and operators described in sections 3 and 4. The mechanical force was also calculated by applying the correct proportionality operators to the maximum force potential

$$F_P = \frac{l_P m_P}{t_P^2} \text{kgm/s}^2.$$

To transform the Planck force into the correct quantity of mechanical force in the electron, we need to apply operators quantifying the electron's length, mass, and time in the ground state. The physical quantities we need to produce are

$$F = \frac{\lambda m}{T_\lambda T} \text{kgm/s}^2. \quad (33)$$

The quantity of length-mass in the numerator is invariant for all combinations of  $\lambda m$  due to length-mass symmetry shown in 15 and explained in section 3.7. In other words, the operators transforming Planck length and mass into the electron's wavelength and inertial mass will always offset each other. The quantity of force we need can therefore be determined simply from the time variables in the denominator, which was also shown for energy in section 3.4.6. The operators shown in table 13 that are required to produce  $T_\lambda$  and  $T$  are

$$T_\lambda = \frac{t_P}{\beta_m \beta_p}$$

$$T = \frac{t_P}{\beta_m \beta_p \beta_v}.$$

$T_\lambda$  gives the dilution of the electron's potential in terms of its spatial configuration, or wavelength.  $T$  gives the dilution of the electron's potential due to its temporal configuration, or period. The rest mass, momentum, and velocity operators applied to the Planck time give the correct oscillation period. Table 16 groups the operators for comparison with the other forces.

## 6 Fine tuning the constants

Physical constants are comprised of natural quantities of length, mass, and time. It is therefore important to obtain the most accurate values possible for these fundamental units.

The New Foundations Model provides a framework for improving the values of physical constants and the Planck units they embody. With the understanding that many traditional constants are comprised of the Planck units, measured values of constants like the speed of light and Planck's constant serve as accurate measurements of the *ratios* between Planck units. These constants demonstrate that while individual Planck units lie outside the range of direct measurement [10], the ratios between Planck units can be measured accurately.

Each measured ratio between Planck units provides a constraint on the possible values of the Planck units. Applying these constraints collectively through the science of metrology may improve the values of all derived constants.

Under the redefinition of the International System of Units in May 2019, Planck's constant and the speed of light are given exact values while CODATA values of the Planck units have a relative standard uncertainty of  $1.1 \times 10^{-5}$ . Using the defined values of Planck's constant and the speed of light as target values, and the Planck unit formulas for  $\hbar$  and  $c$  in 1 and 2, we can assess the degree to which the Planck units produce targeted values. Table 17 compares the defined values with values calculated using the Planck length, mass, and time. Note that calculations performed using CODATA Planck units *do* fall within published ranges of uncertainty.

Table 17: Deviations between exactly defined constants and values calculated with Planck units provide information for improving the accuracy of the fundamental units.

Constant	Exact value	Planck unit formula	Planck unit value	Deviation
$c$	299,792,458 m/s	$l_P/t_P$	299,792,423 m/s	0.999999 88
$\hbar$	$1.054571818 \times 10^{-34} \text{ kgm}^2/\text{s}$	$l_P^2 m_P/t_P$	$1.05457151 \times 10^{-34} \text{ kgm}^2/\text{s}$	0.999999 71
$\hbar/c$	$3.51767294 \times 10^{-43} \text{ kgm}$	$l_P m_P$	$3.51767233 \times 10^{-43} \text{ kgm}$	0.999999 83
$\hbar/c^2$	$1.17336939 \times 10^{-51} \text{ kgs}$	$m_P t_P$	$1.17336933 \times 10^{-51} \text{ kgs}$	0.999999 94

The table shows that  $m_P t_P$  is closest to its target value while  $l_P^2 m_P/t_P$  is furthest. Ratios of the three Planck unit values need to change in predictable ways as their values improve.

We can also use the symmetries and equations presented in section 3.7 to evaluate the accuracy of measured values compared to theoretical values. For example, equation 15 requires that each value in table 18 be equal to the quantum constant (values are shown in non-reduced form).

Table 18: New Foundations Model symmetries offer theoretical constraints on values of physical quantities. Each of the quantities in table 18 should be the same.

Formula	Value
$h/c$	$2.21021\ 90943 \times 10^{-42}$
$\lambda_C m_e$	$2.21021\ 90943 \times 10^{-42}$
$\lambda_{C,\mu} m_\mu$	$2.21021\ 90937 \times 10^{-42}$
$\lambda_{C,\tau} m_\tau$	$2.21021\ 75533 \times 10^{-42}$
$2\pi l_P m_P$	$2.21021\ 87129 \times 10^{-42}$

The ratio of  $h/c$  in the first row is exactly defined under the 2019 redefinition of units. Rows two, three, and four show values of the constant calculated using the measured mass and Compton wavelength of the electron, muon, and tau. Row five calculates the quantum constant using CODATA values of Planck length and Planck mass. Again, the values fall within published ranges of uncertainty, but the comparison shows that we can use theoretical target values to evaluate measurement results.

Of the six paired relationships between the fundamental units (three quotient and three product relationships), three of them inherit exactly defined values from the definitions of  $\hbar$  and  $c$ . These are shown with exact values in table 17. The other three paired relationships,  $l_P t_P$ ,  $l_P/m_P$ , and  $m_P/t_P$  are accurate only to the level of certainty in the Planck unit values.

Given current information, an improved measurement of *any one of the three remaining paired relationships* would enable the calculation of all three Planck unit values with the same level of precision. For example, a more accurate measurement of  $l_P t_P$  produces a comparably accurate value of Planck mass using the formula

$$m_P = \sqrt{\frac{\hbar^2}{c^3 l_P t_P}}$$

With the Planck mass, a value of Planck length could be determined using the value of the quantum constant ( $\hbar/c$ ). Planck time could then be determined using  $l_P/c$ . It is easy to show that more accurate values of  $l_P/m_P$  and  $m_P/t_P$  will also produce improved values of all three fundamental Planck units.

The pursuit of a more accurate gravitational constant can be superseded by the goal of accurately measuring  $l_P/m_P$ . This would give a more accurate value of  $G$  according to equation 2; however, the traditional gravitational constant is not needed in light of the New Foundations Model formulas in section 2. The gravitational constant does not improve on the uncertainty of the Planck units themselves.

Although Planck's constant and the speed of light do not provide enough information to derive precise values of the fundamental Planck units, they do provide constraints on their collective values. That is, given a value for any one of the units, the other two units can be determined with the same precision as  $\hbar$  and  $c$ . Table 19 gives the formulas for calculating the second and third units given a value of the first.



Table 19: Given a value for any one of the fundamental Planck units, the other two can be calculated using the formulas shown.

$l_P$	$m_P$	$t_P$
given	$\frac{\hbar}{l_P c}$	$\frac{l_P}{c}$
$\frac{\hbar}{m_P c}$	given	$\frac{l_P}{c}$
$t_P c$	$\frac{\hbar}{l_P c}$	given

The proportional consistency of the current CODATA Planck unit values is illustrated in figure 20. Each node of the equilateral triangle represents one of the three fundamental Planck units with current CODATA values indexed at 1.0. Three proportionally consistent sets of Planck units are plotted as triangles overlaying the base triangle, where one node of each triangle is the indexed CODATA value and the other two Planck units are calculated using the formulas in table 19. For example, the blue triangle uses the CODATA value of Planck length as the index and plots corresponding values of Planck mass and time in the correct proportions. The image shows that, given the current value of Planck length, the value of Planck mass is too small, and the value of Planck time is too large. Each proportionally consistent set of Planck units produces the defined values of  $\hbar$  and  $c$  exactly.

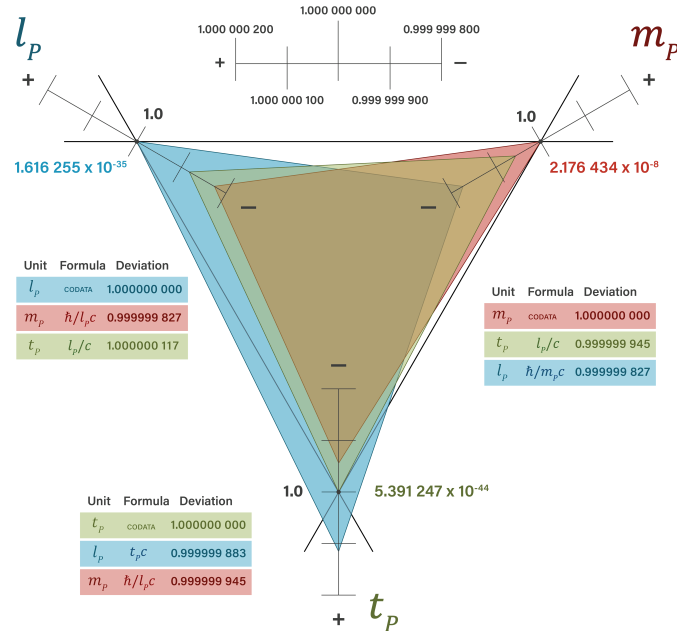


Figure 20: The New Foundations Model provides constraints on possible combinations of Planck length, mass, and time—dictated by exactly defined values of  $\hbar$  and  $c$ .

## 7 Conclusion

More than 120 years after Max Planck proposed natural units of length, mass, and time, the constants used predominantly in physics today are aggregated quantities of the fundamental units, including  $\hbar$  and  $G$ . The physical meaning of these constants is hidden within the larger envelope of their composite values. These quantities fail to explain why Planck's constant gives the momentum and energy of a photon, or why the gravitational constant determines the force generated by a massive body. Replacing these constants with natural units of length, mass, and time offers a refreshing look at physical dynamics encoded in the equations of physics.

The New Foundations Model of physics offers a new description of the natural world based on fundamental units and dimensions. It challenges the presumed incompatibility between an intuitive description of the quantum universe and the abstract mathematical formulations developed over the past century. While it does not address the genuinely,

non-classical phenomenon of wave function collapse, it lifts the veil over many abstruse physical transformations revealing simple ratios of length, mass, and time. It justifies the treatment of quantum field oscillations as more than mathematical symbols, but as veritable, physical dynamics.

The model is appealing because of its simplicity in explaining natural laws. It reduces the long inventory of composite constants down to three fundamental units, and replaces nine electromagnetic units with units of length, mass, and time. From three symmetries it defines momentum, velocity, acceleration, force, action, and energy in the simple, conserved relationships between the three fundamental units.

The model's applicability to quantum mechanics, gravity, and electromagnetism portends a deeper unification in our understanding of the natural universe and may contribute to the search for a grand unified theory. The universal principles of potentiality and proportionality shed new light on existing theories and will catalyze interesting new ideas.

The New Foundations Model will make a positive impact on the field of metrology. Countless experiments have steadily improved the accuracy of physical constants that can now be interpreted as ratios between natural units of length, mass, and time. Incorporating this new model into measurement sciences will improve the Planck unit values as well as the constants they define.

The question may be asked why it has taken so long to recognize the importance of natural units in the construct of physical constants, and to incorporate this information into a description of the natural world. Contemporary theories are deeply abstract, and until the mathematical formulations invoke a better understanding of the universe in physically meaningful ways, these theories will remain on tenuous foundations. Indications that our understanding is improving will appear as theories become less dependent on interpretation, as more theories are ruled out, and as consensus grows within the scientific community on the meaning of preferred theories.

The imperative that physical theories offer more than a mathematical representation of the physical universe was expressed well by David Deutsch

“ Being able to predict things or to describe them, however accurately, is not at all the same thing as understanding them...Facts cannot be understood just by being summarized in a formula, any more than by being listed on paper or committed to memory. They can be understood only by being explained. ” ( [18])

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