Article

An equivalent non-uniform beam-like model for dynamic analysis of multi-storey irregular buildings

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Abstract: Dynamic analyses and seismic assessments of multi-storey buildings at urban level require large-scale simulations and computational procedures based on simplified but accurate numerical models. At this aim the present paper proposes an equivalent non-uniform beam-like model, suitable for the dynamic analysis of buildings with asymmetric plan and non-uniform vertical distribution of mass and stiffness. The equations of motion of this beam-like model, which presents only shear and torsional deformability, are derived through the application of Hamilton's principle. The linear dynamic behaviour is evaluated by discretizing the continuous non-uniform model according to a Rayleigh-Ritz approach based on a suitable number of modal shapes of the uniform shear-torsional beam. In spite of its simplicity, the model is able to reproduce the dynamic behaviour of low- and mid-rise buildings with a significant reduction of the computational burden with respect to that required by more general models. The efficacy of the proposed approach has been tested, by means of comparisons with linear FEM simulations, on three multi-storey buildings characterized by different irregularities. The satisfactory agreement, in terms of natural frequencies, modes of vibration and seismic response, proves the capability of the proposed approach to reproduce the dynamic response of complex spatial multi storey frames.

Keywords: beam-like model; dynamic analysis; Rayleigh-Ritz; shear torsional beam; FEM models.

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1. Introduction

The advancements on computational procedures and parallel processing of the last few years enhance accurate dynamic analysis and seismic assessments of multi-storey buildings. The analyses can be carried out at building or urban level and high fidelity models or sophisticated large-scale simulations can be adopted, respectively. In particular, the dynamic analysis of entire urban area needs optimized computation procedures and simplified although accurate numerical models. Thus, a new and renovated interest has grown on the beam-like models that were introduced in the last century. Beam-like models, which are based on the equivalence of multi-level structures to flexural-shear coupling continuum beams, aim to simulate the dynamic behaviour of multi-level buildings by drastically reducing the computational burden. Several authors demonstrated interest in beam-like models and proposed suitable simplified approaches for the dynamic analysis of multi-level structures.

With regard to high-rise buildings, the discretisation of multi-floor frames with coupled beams was introduced in 1982 [1]. The proposed approach was limited to the principal three modes and in the plane behaviour.

In 1993 other authors [2] studied two-dimensional frames whose members where composed by reticular elements and proposed an approximate method, based on the definition of an equivalent continuous model to replace the repetitive cells, for determining displacements and stress

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characteristics. The equivalence between the lattice structure and the continuous model was established in terms of deformation energies; the continuous model was then analysed in a traditional finite element approach [3, 4].

Later, an approximate method for estimating the maximum lateral displacement in multi-storey buildings, based on an equivalent continuous model, that linearly combines a flexural and a shear deformable cantilever beam, was introduced in 1996 [5]. The lateral displacement of the buildings was given by the combination of shear and bending deformations and the maximum displacement was evaluated. The formulation was successively generalized [6] to take into account the non-uniform lateral stiffness in multi-storey buildings which, when subjected to seismic excitation, assumes the shape of the fundamental mode of vibration. The uniform continuous model was subsequently used to find closed form solutions capable of approximating the dynamic characteristics of buildings (for example modal deformations, periods and modal participation factors) [6]. An estimate of the ground acceleration request on the structures that respond linearly to seismic motions was determined in [7] for planar models.

Applying the concept of "equivalent column", simple formulas were proposed in 2001 [8] for evaluating natural three-dimensional frequencies of buildings. The method considered local bending of single vertical elements, global bending of frames/shear walls (associated with the axial deformation of vertical elements) and shear deformations of frames/shear walls. Approximate formulas were provided to take into account the interaction between translational and torsional modes for non-symmetrical buildings. The same author proposed approximated closed-form solutions for studying tall buildings subjected to horizontal loads for both symmetrical [9] and non-symmetrical [10] systems, in 2009 and 2014 respectively.

In 2003, other authors [11] replaced the structures of the buildings with an equivalent sandwich beam that was defined by three types of stiffness deriving from the resistant elements: global bending stiffness, local bending rigidity and shear stiffness. The deformation energy of the equivalent beam was deduced from the generalization of Timoshenko's theory [12] for spatial problems, introducing separate contributions between global flexural stiffness and shear stiffness, on one hand, and between global flexural stiffness and local flexural stiffness on the other. In any case, the shape of the displacements had to be fixed in order to obtain the rigidity of the equivalent system, leaving the choice of a certain length (a sort of free length of element inflection) correspondent to the best equivalence. Using the obtained equivalent stiffness, an approximate expression was proposed for estimating the natural frequencies of symmetrical structures, while the lateral-torsional vibration modes were determined by an eigenvalue problem. Later, the proposed model was applied to estimate the basic internal forces [13]. Unfortunately, the procedure for calculating the stiffness of the equivalent model was rather complicated and not entirely automatic.

Always in 2003, the Homogenization Method of Discrete Media (HMDM) to repetitive reticular structures composed of interconnected elements (beams or plates) was adopted in other researches with the purpose of deducing the modal characteristics of repetitive framed buildings [14].

In 2010, a continuous model in which the distribution of stresses and the profile of displacements for a combined system of different structural elements, based on previous research [15], was proposed [16]. This model was applied to estimate natural frequencies and modal forms of tall buildings [17, 18], obtaining acceptable errors if compared to Finite Element models.

More recently, in 2014, an equivalent beam model, deformable in shear and torsion, capable of approximately reproducing the dynamic behaviour of three-dimensional shear-type structures was introduced [19]. The model has been adopted for studying the linear and nonlinear aero-elastic behaviour of tower buildings under the assumption that the beam is internally constrained, so that it is capable to experience shear strains and torsion only. The elasto-geometric and inertial characteristics of the beam are directly identified from a discrete model of three-dimensional frame, via a homogenization process.

As this excursus shows, the definition of an equivalent beam was not always simple and immediate, especially in case of three-dimensional with torsional coupling behaviour.

The reduction of complex structural systems to equivalent beam models is still an open challenge of great interest.

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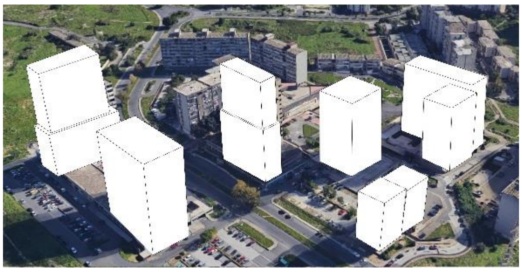
In this paper a non-uniform beam-like model, which is able to take into account two different irregularities, is proposed. In particular, this model is suitable for the schematization of real buildings that do not have uniform mass and stiffness distribution along the height and are characterized by unsymmetrical plans. The equation of motion of the proposed beam-like model is derived through the application of Hamilton's principle. The linear dynamic behaviour of the non-uniform beam-like element is then evaluated by discretizing the continuous model according to a Rayleigh-Ritz approach based on an appropriate number of modal shapes of an uniform beam having only shear and torsional deformability [20]. The proposed approach allows an accurate description of the dynamic behaviour of low- and mid-rise buildings with a limited computational effort. Furthermore, eventual uncertainties in the parameters of the structure, can be taken into account by means of an opportune calibration strategy, based on the introduction of appropriate correction stiffness coefficients.

Aiming at investigating the capabilities of the proposed approach to provide reliable dynamic responses of existing multi-storey reinforced concrete (RC) frame buildings designed to resist only to gravity loading, three representative buildings are taken into account. In particular, the first is characterized only by a horizontal irregularity, the second one only by a vertical irregularity while horizontal as well as vertical irregularities are present in the last case study. Real accelerograms of earthquakes that recently stroke the Italian peninsula have been selected for comparing the seismic responses of the beam-like models to those obtained by 3D FEM models. The comparison showed a good accuracy of the non-uniform beam-like model and encourages the authors to consider the proposed procedure as a very useful and powerful tool for the seismic assessment at urban level. It is worth noting that several residential buildings not designed to withstand to earthquakes are spread throughout Italy and their seismic assessment is still argued at political and academic levels. The use of reliable numerical models based on a reduced number of structural or dynamic information, such as the one here proposed, represents a powerful tool with economic and decision-making advantages. In this light, Figure 1 conceptually shows how entire residential areas can be interpreted with beam-like models facilitating their seismic assessment.

(a)



(b)

Figure 1 Aerial view of (a) residential blocks and (b) their beam-like idealization

2. The Beam-Like Model

The proposed model consists in a 3D shear-torsional cantilever beam able to reproduce the dynamic behaviour of multi-storey buildings. In particular, when, as usual in structures that were designed to gravity loading only, the building columns cross sections decrease along the height of the building, the equivalent beam has non-uniform virtual cross section. Furthermore, planar irregularities due to unsymmetrical distribution of columns or shear walls can be taken into account. This kind of irregularity must be properly considered since it induces eccentricity between the centres of stiffness and mass thus causing not neglectable torsional effects. Each portion of the beam represents a building inter-storey, whose shear and torsional stiffness are initially approximately evaluated according to a geometric reference model. Once the beam-like model of the entire building is defined, it can be calibrated on a reduced number of modal properties that can be easily identified through structural dynamic identification techniques such as Operational Modal Analysis OMA [21].

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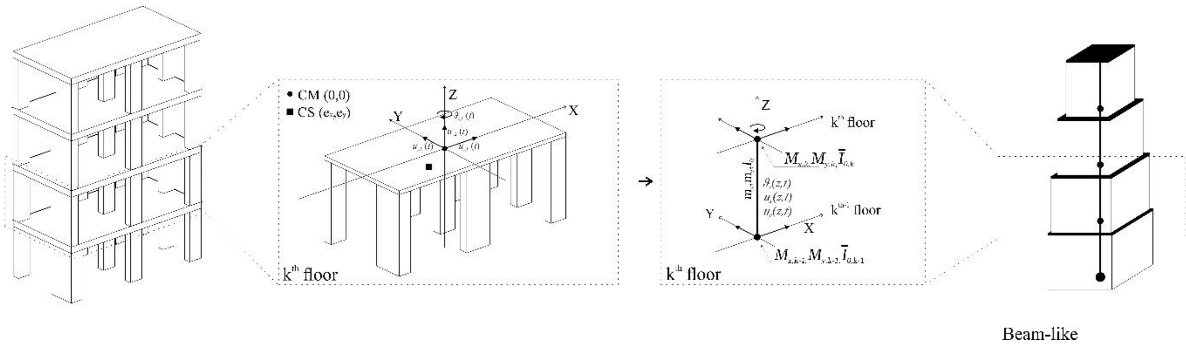
2.1. *Kinematic of frames and beams*

The proposed beam-like model (BLM) is able to represent 3D structures (Figure 2.a) by means of a 1D beam element (Figure 2.d). With reference to the *k*-th storey of the building in the X,Y plane shown in Figure 2.b the centres of mass (CM) and stiffness (CS) will be assumed not coincident to each other thus inducing a torsional behaviour of the structure. It is worth noting that, at each floor, the CM point is placed on the global vertical axis (Z) and, consequently, the CS coordinates coincide with the CS-CM eccentricity (e_x, e_y). Each inter-storey is modelled by a beam segment with uniform stiffness properties, distributed masses $m_{x,k}$, $m_{y,k}$ and second order moment $I_{o,k}$. Concentrated values for the masses $M_{x,k}$ and $M_{y,k}$ and the second order moment $\bar{I}_{o,k}$ are applied at the end of each beam segment in order to simulate the presence of beam and floor masses and applied loadings.



(a) (b) (c) (d)

Figure 2 Concept of representation starting from (a) the 3D structure and (b) the generic kth floor (or interstorey) up to the (c) sub-beam element and the (d) proposed beam-like model.

In order to simplify the description of the kinematics of the multi-storey frame, the hypothesis of rigid floors and inextensible columns have been assumed. Figure 2.b sketches the *k*-th storey and indicates the displacements $u_{x,k}(t)$, $u_{y,k}(t)$ and the rotation $\vartheta_{z,k}(t)$ of the CM of the of the *k*-th rigid floor. According to the hypothesis of rigid floors, very often adopted in literature, the end of each column, which has coordinates ($x_{i,k}$ $y_{i,k}$) at each *k*-th floor level, has the following displacement components:

$$u_{xi,k}(t) = u_{x,k}(t) + \vartheta_{z,k}(t) y_{i,k}$$
$$u_{yi,k}(t) = u_{y,k}(t) - \vartheta_{z,k}(t) x_{i,k} \tag{1}$$

At this stage it could be anticipated that an original strategy to take into account also the out of plane floor deformability is proposed and discussed in the next sub-section.

Each building inter-storey *k* is modelled by means of an equivalent beam segment having the same length (h_k,) and uniform properties (Figure 2.c).

Displacements in *x* and *y* direction and rotations of the beam-like are assumed as continuous functions of the abscissa *z* and denoted respectively as $u_x(z,t)$, $u_y(z,t)$, $\vartheta_z(z,t)$.

The complete non-uniform beam-like model is obtained by the sequence of all the beam segments with uniform properties each, as shown in (Figure 2.d).

2.2. *The inter-storey shear and torsional stiffness*

The equivalent beam shear stiffness in the two principal directions, denoted as $\hat{S}_{xk}$, $\hat{S}_{yk}$, and the torsional stiffness, denoted as $\hat{C}_k$, are obtained, for the *k*-th inter-storey, by considering the contributes of all the n_E columns of the building whose base points have planar coordinates ($x_{i,k}$,$y_{i,k}$). The adopted inter-storey stiffnesses are:

$$\hat{S}_{x,k} = \sum_{i=1}^{n_E} S_{xi,k} \qquad \hat{S}_{y,k} = \sum_{i=1}^{n_E} S_{yi,k} \qquad \hat{C}_k = \left(\sum_{i=1}^{n_E} C_{i,k} + \sum_{i=1}^{n_E} S_{xi,k} y_{i,k}^2 + \sum_{i=1}^{n_E} S_{yi,k} x_{i.k}^2 \right) \tag{2}$$

In case of concrete columns the flexural $S_{xi,k}$, $S_{yi,k}$ and torsional $C_{i,k}$ stiffness are:

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$$S_{xi,k} = \frac{12EJ_{yi,k}}{h_k^3} \qquad S_{yi,k} = \frac{12EJ_{xi,k}}{h_k^3} \qquad C_{i,k} = \frac{GJ_{zi,k}}{h_k} \tag{3}$$

where E is the Young modulus, $J_{xi,k}, J_{yi,k}$ the moment of inertia with respect to the x, y axis, and $J_{zi,k}$ the rotational inertia moment of the *i*-th column of the *k*-th inter-storey.

In order to take into account unknown or not properly identified structural properties (floor out-of-plane deformability, beam-column stiffness ratios, uncertainty on the position of the centre of mass at each floor) three correction coefficients are introduced. These coefficients, denoted as k_x, k_y, k_c, correct the shear stiffness along the x and y directions and the torsional one that have been listed in Equation (2). The actual stiffnesses are therefore reported as follows:

$$S_{x,k} = k_x \hat{S}_{x,k} \qquad S_{y,k} = k_y \hat{S}_{y,k} \qquad C_k = k_c \hat{C}_k \tag{4}$$

As it will be shown in the following, the three correction coefficients can be evaluated by means of an iterative optimization procedure.

2.3. *The equations of motion of the proposed non-uniform beam-like model*

In order to evaluate the response of the non-uniform equivalent beam by introducing a limited number of degrees of freedom, a Rayleigh-Ritz discretization has been performed. The discretization is based on the choice of an appropriate *N* number of modal shapes of a uniform shear cantilever beam, as displacement shape functions of the-non uniform beam, defined as follows:

$$\psi_m(\zeta) = \sin\left(\frac{\pi}{2}(2m-1)\zeta\right) \qquad m = 1, 2, \ldots, \infty \tag{5}$$

where $\zeta = z/h$ is the dimensionless abscissa of the beam, being z the along-axis abscissa and h the beam length.

All the introduced displacements can therefore be expressed as:

$$\begin{aligned} u_x(\zeta,t) &= \sum_{i=1}^{N} \psi_i(\zeta) q_{i_x}(t) \\ u_y(\zeta,t) &= \sum_{i=1}^{N} \psi_i(\zeta) q_{i_y}(t) \\ \vartheta_z(\zeta,t) &= \sum_{i=1}^{N} \psi_i(\zeta) q_{i_\vartheta}(t) \end{aligned} \tag{6}$$

being $q_{i_x}(t), q_{i_y}(t), q_{i_\vartheta}(t)$ the generalized *i*-th coordinates along the x, y, ϑ directions, that represent the contribution of the single shape function to the total response.

The equations of motion of the proposed beam-like model are derived through the application of Hamilton's principle. Displacements $u_{gx}(t) = u_x(\zeta = 0, t)$, $u_{gy}(t) = u_y(\zeta = 0, t)$ at the base ($\zeta = 0$) are considered in the formulation in order to take into account seismic excitations. In the present formulation, primes and dots denote differentiation with respect to the normalized abscissa ζ and time t, respectively. In absence of non-conservative forces, the Hamilton's principle can be reduced to the contribute of kinematic T and elastic energy U only as follows:

$$\int_{t_0}^{t_1} \left[\delta(T-U)\right] dt = 0 \qquad \forall t_0, t_1 \tag{7}$$

where $\delta(T-U)$ indicates the variation of the of the kinetic energy T and the elastic energy U whose relevant expressions are given as follows:

$$T=\left(\frac{1}{2}h\sum_{k=1}^{N_f}m_{x,k}\int_{\zeta_{k-1}}^{\zeta_k}\left[\sum_{i=1}^{N}\psi_i(\zeta)\dot{q}_{i_x}(t)+\dot{u}_{gx}(t)\right]^2d\zeta+\frac{1}{2}h\sum_{k=1}^{N_f}m_{y,k}\int_{\zeta_{k-1}}^{\zeta_k}\left[\sum_{i=1}^{N}\psi_i(\zeta)\dot{q}_{i_y}(t)+\dot{u}_{gy}(t)\right]^2d\zeta+\right.$$
$$+\frac{1}{2}h\sum_{k=1}^{N_f}I_{o,k}\int_{\zeta_{k-1}}^{\zeta_k}\left[\sum_{i=1}^{N}\psi_i(\zeta)\dot{q}_{i_g}(t)\right]^2d\zeta+\frac{1}{2}\sum_{k=1}^{N_f}M_{xk}\left[\sum_{i=1}^{N}\psi_i(\zeta=\frac{z_k}{h})\dot{q}_{i_x}(t)+\dot{u}_{gx}(t)\right]^2+$$
$$\left.+\frac{1}{2}\sum_{k=1}^{N_f}M_{yk}\left[\sum_{i=1}^{N}\psi_i(\zeta=\frac{z_k}{h})\dot{q}_{i_y}(t)+\dot{u}_{gy}(t)\right]^2+\frac{1}{2}\sum_{k=1}^{N_f}\overline{I}_{o,k}\left[\sum_{i=1}^{N}\psi_i(\zeta=\frac{z_k}{h})\dot{q}_{i_g}(t)\right]^2\right) \tag{8}$$

$$U=\left(\frac{1}{2}\frac{1}{h}\sum_{k=1}^{N_f}GA_{x,k}\int_{\zeta_{k-1}}^{\zeta_k}\left[\sum_{i=1}^{N}\psi'_i(\zeta)q_{i_x}(t)+e_y\sum_{i=1}^{N}\psi'_i(\zeta)q_{i_g}(t)\right]^2d\zeta+\right.$$
$$+\frac{1}{2}\frac{1}{h}\sum_{k=1}^{N_f}GA_{y,k}\int_{\zeta_{k-1}}^{\zeta_k}\left[\sum_{i=1}^{N}\psi'_i(\zeta)q_{i_y}(t)-e_x\sum_{i=1}^{N}\psi'_i(\zeta)q_{i_g}(t)\right]^2d\zeta+$$
$$\left.+\frac{1}{2}\frac{1}{h}\sum_{k=1}^{N_f}GJ_{z,k}\int_{\zeta_{k-1}}^{\zeta_k}\left[\sum_{i=1}^{N}\psi'_i(\zeta)q_{i_g}(t)\right]^2d\zeta\right) \tag{9}$$

respectively, where N_f is the number of floors and $GA_x(\zeta), GA_y(\zeta), GJ_z(\zeta)$ represent the along axis variable shear and torsional stiffnesses that can be substituted with a step-wise distribution, constant for each inter-storey k, in accordance to the equivalent beam model adopted in Figure 2.d, as follows:

$$GA_{x,k}=S_{x,k}\cdot h_k \qquad GA_{y,k}=S_{y,k}\cdot h_k \qquad GJ_{z,k}=C_k\cdot h_k \tag{10}$$

In view of the expressions of the kinetic energy, given by Equation (8), and the elastic energy, given by Equation (9), the variation of the difference $\delta(T-U)$, according to the Hamilton's principle, as in Equation (7), and integrating by parts, leads to the following result:

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$$
\begin{aligned}
\int_{t_0}^{t_1} \Bigg\{ & h\sum_{k=1}^{N_f} m_{x,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi_i(\zeta)\psi_j(\zeta)\, d\zeta\, \ddot{q}_{i_x}(t)\,\delta q_{j_x} + \sum_{k=1}^{N_f} M_{x,k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi_i(\zeta_k)\psi_j(\zeta_k)\ddot{q}_{i_x}(t)\,\delta q_{j_x} + \\
& + h\sum_{k=1}^{N_f} m_{y,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi_i(\zeta)\psi_j(\zeta)\, d\zeta\, \ddot{q}_{i_y}(t)\,\delta q_{j_y} + \sum_{k=1}^{N_f} M_{y,k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi_i(\zeta_k)\psi_j(\zeta_k)\ddot{q}_{i_y}(t)\,\delta q_{j_y} + \\
& + h\sum_{k=1}^{N_f} I_{o,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi_i(\zeta)\psi_j(\zeta)\, d\zeta\, \ddot{q}_{i_\vartheta}(t)\,\delta q_{j_\vartheta} + \sum_{k=1}^{N_f} \bar{I}_{o,k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi_i(\zeta_k)\psi_j(\zeta_k)\ddot{q}_{i_\vartheta}(t)\,\delta q_{j_\vartheta} + \\
& + \frac{1}{h}\sum_{k=1}^{N_f} GA_{x,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_j(\zeta)\psi'_i(\zeta)\, d\zeta\, q_{i_x}(t)\,\delta q_{j_x} + \frac{1}{h}\sum_{k=1}^{N_f} e_y GA_{x,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_j(\zeta)\psi'_i(\zeta)\, d\zeta\, q_{i_\vartheta}(t)\,\delta q_{j_x} + \\
& + \frac{1}{h}\sum_{k=1}^{N_f} e_y GA_{x,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_i(\zeta)\psi'_j(\zeta)\, d\zeta\, q_{i_x}(t)\,\delta q_{j_\vartheta} + \frac{1}{h}\sum_{k=1}^{N_f} GA_{y,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_j(\zeta)\psi'_i(\zeta)\, d\zeta\, q_{i_y}(t)\,\delta q_{j_y} + \\
& - \frac{1}{h}\sum_{k=1}^{N_f} e_x GA_{y,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_j(\zeta)\psi'_i(\zeta)\, d\zeta\, q_{i_\vartheta}(t)\,\delta q_{j_y} - \frac{1}{h}\sum_{k=1}^{N_f} e_x GA_{y,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_i(\zeta)\psi'_j(\zeta)\, d\zeta\, q_{iy}(t)\,\delta q_{j_\vartheta} + \\
& + \frac{1}{h}\sum_{k=1}^{N_f} GJ_{z,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_i(\zeta)\psi'_j(\zeta)\, d\zeta\, q_{i_\vartheta}(t)\,\delta q_{j_\vartheta} + \frac{1}{h}\sum_{k=1}^{N_f} e_y^2 GA_{x,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_i(\zeta)\psi'_j(\zeta)\, d\zeta\, q_{i_\vartheta}(t)\,\delta q_{j_\vartheta} + \\
& + \frac{1}{h}\sum_{k=1}^{N_f} e_x^2 GA_{y,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{i=1}^{N}\sum_{j=1}^{N} \psi'_i(\zeta)\psi'_j(\zeta)\, d\zeta\, q_{i_\vartheta}(t)\,\delta q_{j_\vartheta} + \\
& + \ddot{u}_{gx}(t)\, h\sum_{k=1}^{N_f} m_{x,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{j=1}^{N} \psi_j(\zeta)\, d\zeta\, \delta q_{j_x} + \ddot{u}_{gx}(t)\sum_{k=1}^{N_f} M_{x,k} \sum_{j=1}^{N} \psi_j(\zeta_k)\,\delta q_{j_x} + \\
& + \ddot{u}_{gy}(t)\, h\sum_{k=1}^{N_f} m_{y,k} \int_{\zeta_{k-1}}^{\zeta_k} \sum_{j=1}^{N} \psi_j(\zeta)\, d\zeta\, \delta q_{j_y} + \ddot{u}_{gy}(t)\sum_{k=1}^{N_f} M_{y,k} \sum_{j=1}^{N} \psi_j(\zeta_k)\,\delta q_{j_y} \Bigg\} dt = 0
\end{aligned}
$$

$$\forall\, t_0, t_1, \delta q_{j_x}, \delta q_{j_y}, \delta q_{j_\vartheta} \qquad \text{where} \qquad j = 1, 2, \ldots N \tag{11}$$

where $\zeta_k = \dfrac{z_k}{h}$.

The equations of motion have been obtained imposing the appropriate variation $\delta q_{j_x}, \delta q_{j_y}, \delta q_{j_\vartheta}$ of the generalized coordinates. For example, the first equation is obtained setting in Equation (11) only the first variation different from zero ($\delta q_{1_x} \neq 0$).

The mathematical notation of the Hamilton's principle provided by Equation (11) can be simplified by omitting for convenience the dependency on t and ζ, and, furthermore, by rearranging all the terms in a more compact manner as follows:

$$\boldsymbol{\delta}\mathbf{q}^T\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\delta}\mathbf{q}^T\mathbf{K}\mathbf{q} = \boldsymbol{\delta}\mathbf{q}^T\mathbf{P} \tag{12}$$

The generalised mass matrix $\mathbf{M}$, the stiffness matrix $\mathbf{K}$, the load vector $\mathbf{P}$ and the vector $\mathbf{q}$, appearing in Equation (13), are defined as follows:

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$$\mathbf{M}=\begin{bmatrix}\mathbf{M}_x & & \\ & \mathbf{M}_y & \\ & & \mathbf{M}_\vartheta\end{bmatrix}\quad \mathbf{K}=\begin{bmatrix}\mathbf{K}_x & & \mathbf{K}_{x\vartheta} \\ & \mathbf{K}_y & \mathbf{K}_{y\vartheta} \\ \mathbf{K}_{\vartheta x} & \mathbf{K}_{\vartheta y} & \mathbf{K}_\vartheta\end{bmatrix}\quad \mathbf{P}=\begin{bmatrix}\mathbf{P}_x \\ \mathbf{P}_y \\ \mathbf{P}_\vartheta\end{bmatrix}\quad \mathbf{q}=\begin{bmatrix}\mathbf{q}_x \\ \mathbf{q}_y \\ \mathbf{q}_\vartheta\end{bmatrix} \tag{13}$$

where the relevant elements are given as:

$$\begin{aligned}
\mathbf{K}_{j_x i_x} &= \frac{1}{h}\sum_{k=1}^{N_f} GA_{x,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi'_i\psi'_j\,d\zeta \\
\mathbf{K}_{j_y i_y} &= \frac{1}{h}\sum_{k=1}^{N_f} GA_{y,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi'_i\psi'_j\,d\zeta \\
\mathbf{K}_{j_\vartheta i_\vartheta} &= \frac{1}{h}\sum_{k=1}^{N_f}\left(GJ_{z,k}+e_y^2GA_{x,k}+e_x^2GA_{y,k}\right)\int_{\zeta_{k-1}}^{\zeta_k}\psi'_i\psi'_j\,d\zeta \\
\mathbf{K}_{j_x i_\vartheta} &= \mathbf{K}_{j_\vartheta i_x} = \frac{1}{h}\sum_{k=1}^{N_f} e_y GA_{x,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi'_i\psi'_j\,d\zeta \\
\mathbf{K}_{j_y i_\vartheta} &= \mathbf{K}_{j_\vartheta i_y} = -\frac{1}{h}\sum_{k=1}^{N_f} e_x GA_{y,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi'_i\psi'_j\,d\zeta
\end{aligned} \tag{14}$$

$$\begin{aligned}
\mathbf{M}_{j_x i_x} &= h\sum_{k=1}^{N_f} m_{x,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi_i\psi_j\,d\zeta + \sum_{k=1}^{N_f} M_{x,k}\psi_{i,k}\psi_{j,k} \\
\mathbf{M}_{j_y i_y} &= h\sum_{k=1}^{N_f} m_{y,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi_i\psi_j\,d\zeta + \sum_{k=1}^{N_f} M_{y,k}\psi_{i,k}\psi_{j,k} \\
\mathbf{M}_{j_\vartheta i_\vartheta} &= h\sum_{k=1}^{N_f} I_{0,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi_i\psi_j\,d\zeta + \sum_{k=1}^{N_f} \overline{I}_{o,k}\psi_{i,k}\psi_{j,k}
\end{aligned} \tag{15}$$

$$\begin{aligned}
\mathbf{P}_{j_x} &= -\ddot{u}_{gx}h\sum_{k=1}^{N_f} m_{x,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi_j\,d\zeta - \ddot{u}_{gx}\sum_{k=1}^{N_f} M_{x,k}\psi_{j,k} \\
\mathbf{P}_{j_y} &= -\ddot{u}_{gy}h\sum_{k=1}^{N_f} m_{y,k}\int_{\zeta_{k-1}}^{\zeta_k}\psi_j\,d\zeta - \ddot{u}_{gy}\sum_{k=1}^{N_f} M_{y,k}\psi_{j,k} \\
\mathbf{P}_{j_\vartheta} &= 0
\end{aligned} \tag{16}$$

where $\psi_{i,k}=\psi_i(\zeta_k)$ is the shape function evaluated at the floor level.

Finally, the equations of motion of the proposed equivalent multi-stepped beam in the generalised space are derived by Equation (13) in matrix notation as follows:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{P} \tag{17}$$

It is worth noting that the structural damping will be introduced, in terms of modal damping ratios, in the uncoupled equations of motions that will be derived in sub-section 2.6.

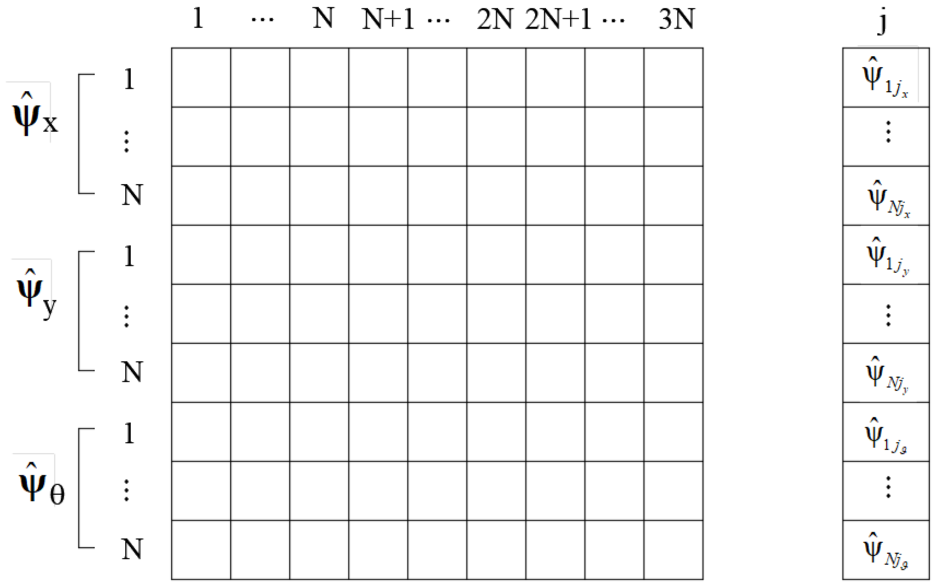
2.4. *Eigen-problem of the non-uniform beam-like model*

As anticipated in sub-section 2.2, the stiffness matrix terms, given by Equation (15), are affected, in view of Equation (4), by three correction coefficients k_x, k_y, k_c that are evaluated by taking into account modal properties such as frequencies and modal displacements. In this view the modes of vibration in the generalized space, denoted as $\hat{\boldsymbol{\psi}}$, and the vibration frequencies ω of the equivalent beam are obtained solving the following standard eigen-problem:

$$\left[\mathbf{K}-\omega^2\mathbf{M}\right]\hat{\boldsymbol{\Psi}}=\mathbf{0} \tag{18}$$

Consequently, the modal shape in the geometric space can be obtained by means of Equation (6) where N displacement shape functions of a uniform beam, of the type given in Equation (5), are considered.

Therefore, since the solution of the multi-stepped beam is approximated with N shape functions, the mass and stiffness matrices have $(3\times N)\times(3\times N)$ dimension and the solution of the eigen-problem, represented by Equation (18), is a $(3\times N)\times 1$ vector of natural frequencies and a $(3\times N)\times(3\times N)$ matrix of eigen-vectors. Figure 3 depicts the eigen-vector matrix in the generalized space and the components of the generic *j*-th eigen-vector.



(**a**) (**b**)

Figure 3 Scheme of: a) the eigen-vector matrix; b) the j-th eigen-vector.

Consequently, the *j*-th modal shape in the geometric space of the equivalent non uniform multi-stepped beam, denoted as vector function $\boldsymbol{\varphi}_j(\zeta)$, is obtained, in view of Equation (6), by multiplying each displacement shape function of the uniform beam $\psi_i(\zeta), i=1,2,\ldots,N$, considered for the discretization, times the corresponding eigen-vector components .

$$\begin{aligned}
\varphi_{u,j}(\zeta)&=\psi_1(\zeta)\hat{\Psi}_{1j_x}+\psi_2(\zeta)\hat{\Psi}_{2j_x}+\ldots+\psi_N(\zeta)\hat{\Psi}_{Nj_x}\\
\varphi_{v,j}(\zeta)&=\psi_1(\zeta)\hat{\Psi}_{1j_y}+\psi_2(\zeta)\hat{\Psi}_{2j_y}+\ldots+\psi_N(\zeta)\hat{\Psi}_{Nj_y}\\
\varphi_{\vartheta,j}(\zeta)&=\psi_1(\zeta)\hat{\Psi}_{1j_\vartheta}+\psi_2(\zeta)\hat{\Psi}_{2j_\vartheta}+\ldots+\psi_N(\zeta)\hat{\Psi}_{Nj_\vartheta}
\end{aligned} \tag{19}$$

Equation (19) provides the three spatial components in the actual geometric space (Figure 2) of the *j*-th modal shape to be adopted to perform a suitable dynamic analysis of the equivalent non-uniform beam by means of a suitable modal superposition procedure as shown later.

2.5. *Shear and torsional beam stiffness optimization*

The proposed non-uniform beam-like approach, in spite of its simplicity, can be enhanced to reproduce accurately the dynamic behaviour of 3D frame structures by coupling an iterative procedure aiming at optimizing the calibration of the shear and torsional stiffness of the equivalent beam as described in what follows. Precisely, in order to consider all the unknown or not properly identified structural properties, three coefficients k_x, k_y, k_c, are applied to correct the shear and the torsional stiffnesses as Equation (4) shows. It is worth noting that the eigen-properties of the non-uniform beam-like model are strictly dependent on the values of the correction coefficients k_x, k_y, k_c.

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In order to identify the best values of k_x, k_y, k_c, appearing in Equation (4), the following objective function O is introduced:

$$O\left(k_x,k_y,k_c\right)=\alpha\underbrace{\sqrt{\sum_{j=1}^{n}\left(\frac{\omega_j\left(k_x,k_y,k_c\right)-\omega_j}{\omega_j}\right)^2}}_{frequencies}+\beta\underbrace{\left[1-\sum_{j=1}^{n}\sum_{k=1}^{m}\frac{\boldsymbol{\varphi}_{j;k}\cdot\boldsymbol{\varphi}_{j;k}^{T}}{\sqrt{\left(\boldsymbol{\varphi}_{j;k}\cdot\boldsymbol{\varphi}_{j;k}^{T}\right)\left(\boldsymbol{\varphi}_{j;k}\cdot\boldsymbol{\varphi}_{j;k}^{T}\right)}}\right]}_{\text{mod}\,al\;shapes}$$

$$\text{where } 0\le\alpha\le1\ ;\ 0\le\beta\le1\ ;\ n<3N \tag{20}$$

The objective function defined in Equation (21), dependent on the stiffness coefficients k_x,k_y,k_c, represents a measure of the deviation with respect to target values of a given number n of natural frequencies ω_j and modal displacement vector $\boldsymbol{\varphi}_{j;k}=\left[\varphi_{u,j}(\zeta_k;k_x,k_y,k_c)\quad\varphi_{v,j}(\zeta_k;k_x,k_y,k_c)\quad\varphi_{\vartheta,j}(\zeta_k;k_x,k_y,k_c)\right]$, evaluated at suitably chosen m floor levels ζ_k, $k=1,\dots,m$. The target values, denoted in Equation (20) with a superimposed tilde "~", represent experimental values that can be obtained by means of dynamic identification methods. Bearing in mind that the present work aims at assessing the reliability of the proposed beam-like model and the suitability of the of selected values for the stiffness coefficients k_x, k_y, k_c, rather than testing a proper execution of dynamic identification methods, the target frequencies ω_j and modal shapes $\boldsymbol{\varphi}_j$ have been numerically obtained by means of detailed FEM models.

The optimal values of the stiffness coefficients k_x, k_y, k_c, to be adopted for the non-uniform beam-like model equivalent to the given 3D frame structure are those that minimize the objective function defined in Equation (21). The latter optimization problem is solved with an iterative solution procedure by means of the Linear Constrained Minimization Pattern search algorithm [22]. At each step of the iterative procedure the trial value of the objective function is calculated by solving the eigen-problem in Equation (19) of the non-uniform equivalent beam discretized as shown in sub-section 2.3 by assuming trial values of the stiffness coefficients k_x, k_y, k_c.

It is worth noting that, in order to make appropriate choices of the objective function O able to account for different types of building irregularities, in Equation (21) two weight coefficients α and β are introduced and are applied to the terms that measure the deviation of frequencies and modal shapes, respectively. In particular, in the applicative section it is shown how information on frequencies and modal shapes must be used in the minimization procedure according to the building complexity.

2.6. *Dynamic response*

The dynamic response of the BLM subjected to external loading, in the generalized space, can be expressed as a combination of $S\le3N$ modes of vibration multiplied by the time dependent functions $z_j(t)$:

$$\mathbf{q}(t)=\sum_{j=1}^{S}\hat{\boldsymbol{\psi}}_j\cdot z_j(t) \tag{21}$$

where S indicates the number of modes of vibration. Substitution of Equation (21) in the equations of motion (17) leads to:

$$\mathbf{M}\sum_{j=1}^{S}\hat{\boldsymbol{\psi}}_j\cdot\ddot{z}_j(t)+\mathbf{K}\sum_{j=1}^{S}\hat{\boldsymbol{\psi}}_j\cdot z_j(t)=\mathbf{P} \tag{22}$$

Furthermore, in view of the orthogonality conditions of the vibration modes, Equation (22) provides:

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$$\underbrace{\hat{\boldsymbol{\psi}}_j^T \mathbf{M} \hat{\boldsymbol{\psi}}_j}_{M_{mod,j}} \cdot \ddot{z}_j(t) + \underbrace{\hat{\boldsymbol{\psi}}_j^T \mathbf{K} \hat{\boldsymbol{\psi}}_j}_{K_{mod,j}} \cdot z_j(t) = \underbrace{\hat{\boldsymbol{\psi}}_j^T \mathbf{P}}_{P_{mod,j}} \tag{23}$$

So the equations of motions are simplified as follows:

$$M_{mod,j} \cdot \ddot{z}_j(t) + C_{mod,j} \cdot \dot{z}_j(t) + K_{mod,j} \cdot z_j(t) = P_{mod,j} \tag{25}$$

It is worth noting that the terms $C_{mod,j}$ are representative of the generalized modal damping for all the modes of vibration and are related to the corresponding modal damping ratios as follows $\dfrac{C_{mod,j}}{M_{mod,j}} = 2\xi_j \omega_j$.

Lastly, the dynamic response in the geometric space is obtained by means of the following expressions:

$$u_x(z,t) = \sum_{j=1}^{S} \sum_{i=1}^{N} \psi_i(z) \hat{\psi}_{ij_x} \cdot z_j(t)$$
$$u_y(z,t) = \sum_{j=1}^{S} \sum_{i=1}^{N} \psi_i(z) \hat{\psi}_{ij_y} \cdot z_j(t) \tag{26}$$
$$\vartheta(z,t) = \sum_{j=1}^{S} \sum_{i=1}^{N} \psi_i(z) \hat{\psi}_{ij_\vartheta} \cdot z_j(t)$$

where S is the number of modes of vibration and N is the number of shape functions.

3. Applications

In order to apply the proposed procedure to real structures with different irregularities, namely planar and vertical ones, in the present paper three multi-storey RC frames representative of residential buildings designed to resist only to gravity loading have been considered. The considered buildings have been chosen taking into account the results of an extensive survey on existing multi-storey RC buildings built in Catania, Italy, before the introduction of the seismic code [23, 24, 25]. The geometrical details of the typical building can be found in [25], where further information on the loading and the material properties are reported.

A synthetic description of the proposed method, for a demonstrative purpose only, together with applications to simple spatial frames have been presented by the authors in previous works [26, 27].

The three models of realistic buildings here considered will be denoted as Model Type 1, Type 2 and Type 3 in the following.

Type 1, refers to a building with planar asymmetry and vertical uniform distribution of mass and stiffness. Due to the peripheral staircase, which is located on one side of the building, the centre of stiffness does not coincide with the centre of mass. The eccentricity determinates torsional effects that are correctly identified by the proposed beam-like model. Due to the presence of the planar asymmetry only (no vertical irregularity is introduced), the objective function O introduced in Equation (5) to determine the stiffness correction coefficients of the equivalent beam model for Type 1 building accounting for the natural frequencies only (i.e. weight coefficients α=1, β=0) provided accurate results.

Type 2, presents a planar symmetry and vertical non-uniform distribution of mass and stiffness due to the reduction of the cross sections of the columns along the height. Once again, due to the existence of only one of the considered irregularities (i.e. the vertical one) the minimization of objective function O in Equation (5) has been performed by considering only the frequencies of vibration by setting the weight coefficients α=1, β=0.

The structural plans of Type 1 and Type 2 are reported in Figure 4 and the inter-storey height of both models is set equal to 3.30 meters at each level.

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In order to take into account the contemporary occurrence of both planar and vertical irregularities Type 3 has been considered. The latter has been obtained by adding vertical asymmetries to the already described Type 1. In this view, the height of the first floor has been incremented up to 4.3 meters, a reduction along the height of the building in the column cross-section has been introduced and a total number of six floors have been considered. In this case, due to the contemporary presence of the two types of irregularities the contribution of the modal shape vector term in the objective function has been also considered, hence, both weight coefficients in Equation (5) have been used ($\alpha \neq 0$, $\beta \neq 0$). Precisely, it is worth to notice that only the modal displacements of the top floor have been introduced in the objective function (5).

In all the three building types the dimensions of the cross sections of the beams on the edge are assumed to be 30x50 cm^2, while all the remaining beams have size 110x23 cm^2. The columns have been assumed fully fixed at the base. An equivalent floor thickness of 9.32 cm at each level is considered. The material properties are characterized by a Young's modulus of 29962 MPa, a Poisson ratio equal to 0.2 and a mass density of 25 kN/m^3.

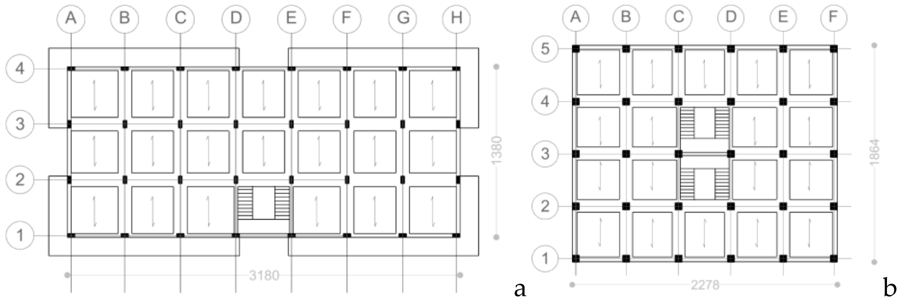


Figure 4 Plan Archetypes: (a) Type 1 low- or mid-rise and (b) Plan B mid- or high-rise buildings

The results, in terms of seismic response, have been compared to those obtained by means of a conventional FEM model developed with SAP2000 [28]. The reliability of the beam-like model has been evaluated by comparing the time histories of the x and y displacements of some control points. For the sake of brevity in the following only the results related to a control point located at the upper left corner of the highest floor are reported. Furthermore, the maximum displacements of the same corner located at each floor have been calculated. In order to make an appropriate comparison between the two models the in-plane rigid floor, no live loads and inextensible concrete columns have been considered in the FEM model.

Aiming to simulate representative seismic inputs that may occur on the Italian peninsula, the linear dynamic analyses have been performed by considering two real seismic records [29] in x and y directions occurred in Santa Venerina (2018) and L'Aquila (2009) plotted respectively in Figure 5a and 5b. Some characteristic data of the two records (PGA, PGV, PGD, distance for epicentre) are reported in Table 1. In all the time-histories constant modal damping ratios, for all considered modes, ξ_j =0.05 have been assumed.

Table 1 Records adopted in numerical simulation

Record	State	Date	Hour	Mw	Dist. [km]	PGA [cm/s^2]	PGV [cm/s]	PGD [cm]	Lat. [°]	Long. [°]
Santa Venerina	Italy	2018-12-26	02:19:17	4.9	4.5	-547.932 (N)	37.128 (N)	5.696 (N)	37.644	15.116
L'Aquila	Italy	2009-04-06	01:32:40	6.1	4.9	644.246 (E)	-42.720 (N)	6.789 (E)	42.342	13.380

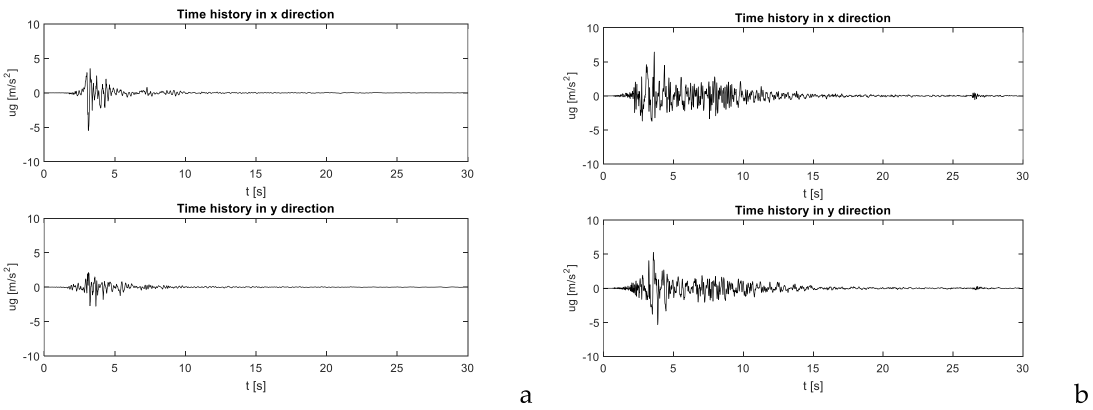


Figure 5 Accelerograms of a) Santa Venerina 2018 and b) L'Aquila 2009

3.1. *Type 1: building with planar asymmetry and vertical uniform distribution of mass and stiffness*

The four-storey model is characterized by uniform column cross-sections along the whole height. The planar distribution of the cross section of the columns is reported in Figure 6.a while Figure 6.b shows a scheme of the FEM model

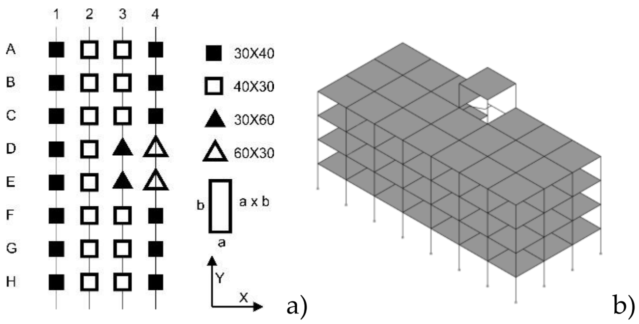
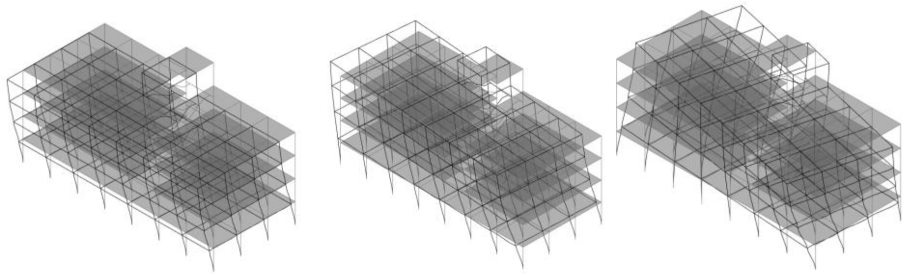


Figure 6 Four storey building benchmark (Type 1): a) column cross section plan and b) FEM model [28].

Table 2 and Figure 7 compare three periods and modal shapes, respectively, calculated through the FEM approach to those obtained by means of the proposed beam like model.

Table 2 Modal Period comparison

Type 1	Mode 1	Mode 2	Mode 3
FEM	0.571	0.564	0.514
PROPOSED BEAM-LIKE MODEL	0.571	0.564	0.514



a)

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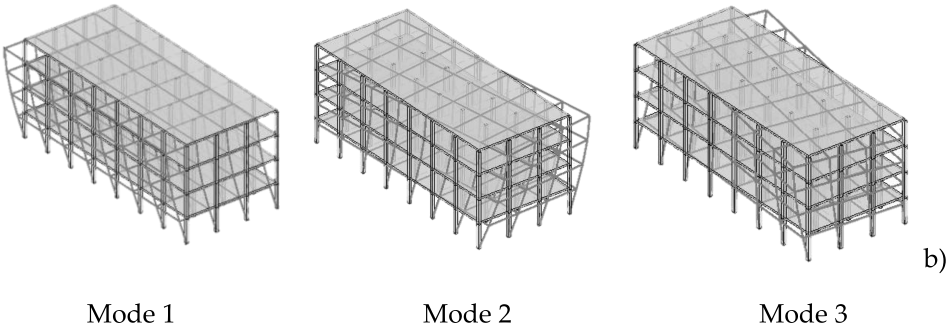

Figure 7 Four-storey benchmark. Modal shapes comparison.

It can be easily observed the satisfactory agreement between the results for all the modal properties related to the first two translational modes and to the third torsional one.

Figure 8 reports the displacement time histories of the control point in both x and y directions, calculated by means of the beam like and the FEM models, during the considered seismic excitation. In particular Figure 8.a and b report respectively the x and y displacements due to Santa Venerina earthquake loading. The correspondent results for L'Aquila earthquake are reported in Figure 9.

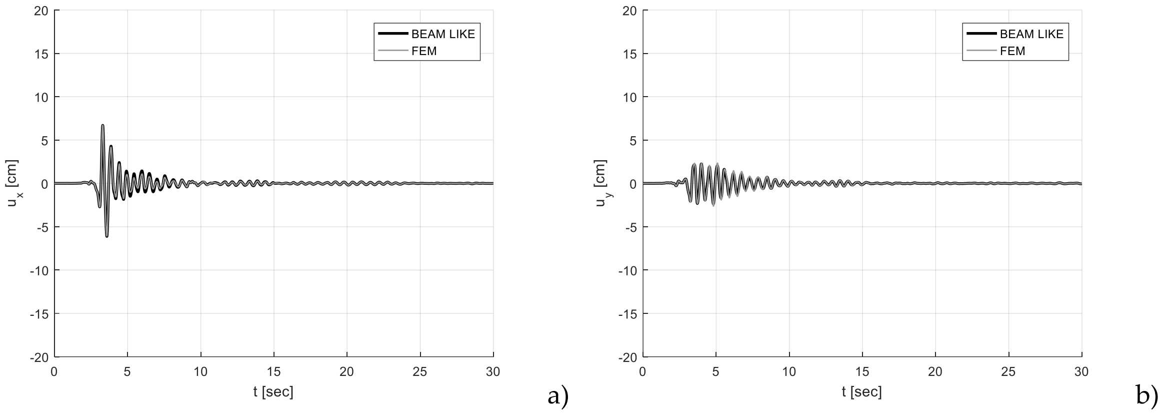

Figure 8 Four-storey benchmark. Santa Venerina earthquake. Displacements time history.

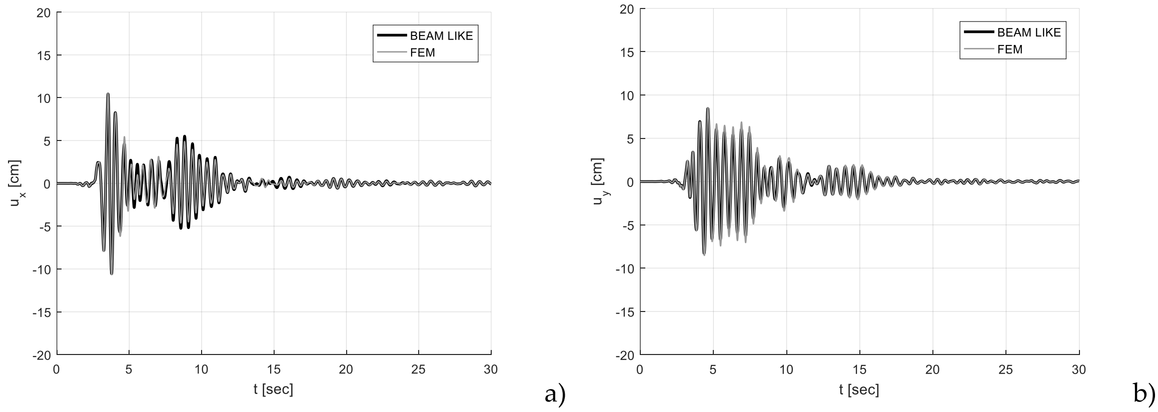

Figure 9 Four-storey benchmark. L'Aquila earthquake. Displacements time history.

Once again, it must be remarked how the proposed approach is able to correctly reproduce the time history of a chosen point on the building. Finally as it can be observed in Figure 10 also the maxima displacements of the same corner located at each floor, calculated by means of the beam like model, give reliable results.

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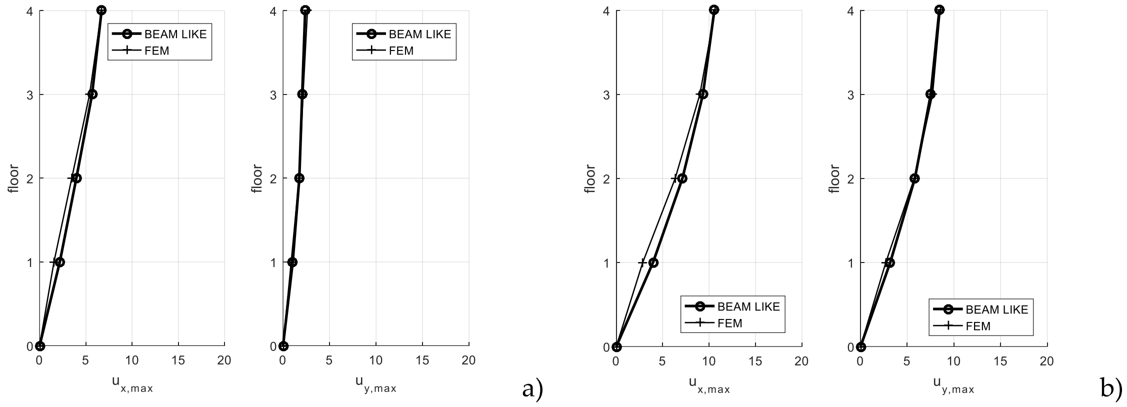

Figure 10 Four-storey benchmark. Maxima floor displacements in a) SantaVenerina and b) L'Aquila earthquakes.

3.2. *Type 2, building with planar symmetry and vertical non-uniform distribution of mass and stiffness*

The second benchmark regards Type 2, an eight-storey frame characterised by a reduction of the cross sections of the columns along the height. The planar distribution of the cross section of the columns is reported for each floor in Figure 11.a while Figure 11.b shows a scheme of the FEM model.

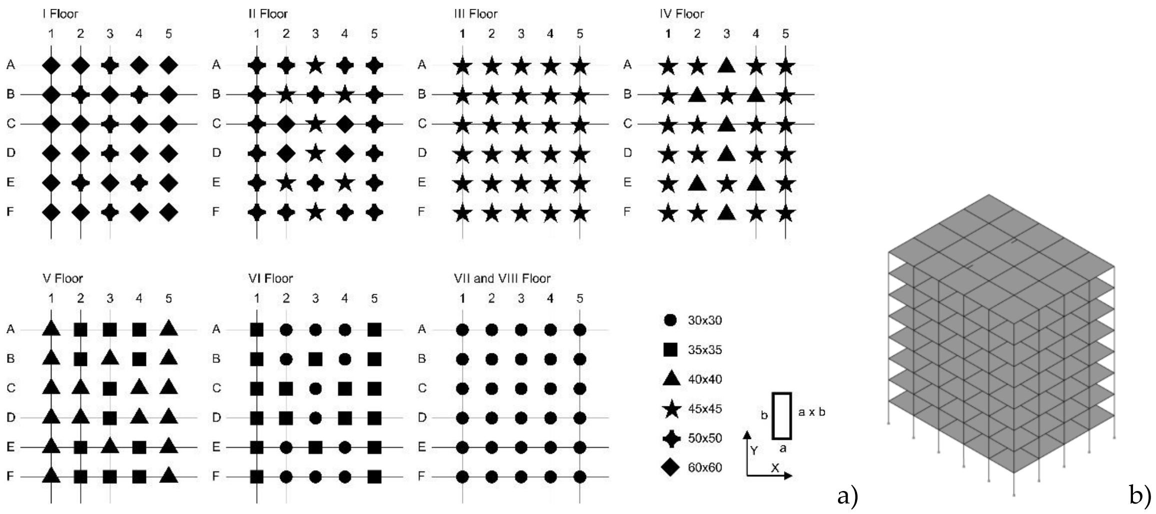

Figure 11 Eight storey building benchmark (Type 2): a) column cross section plan and b) FEM model.

The first three modal properties of Type 2 evaluated by means of the proposed approach have been compared to those obtained by conventional FEM model and reported in Table 3 and Figure 12.

Table 3 Modal Period comparison

Type 2	Mode 1	Mode 2	Mode 3
FEM	0.943	0.917	0.813
PROPOSED BEAM-LIKE MODEL	0.943	0.917	0.813

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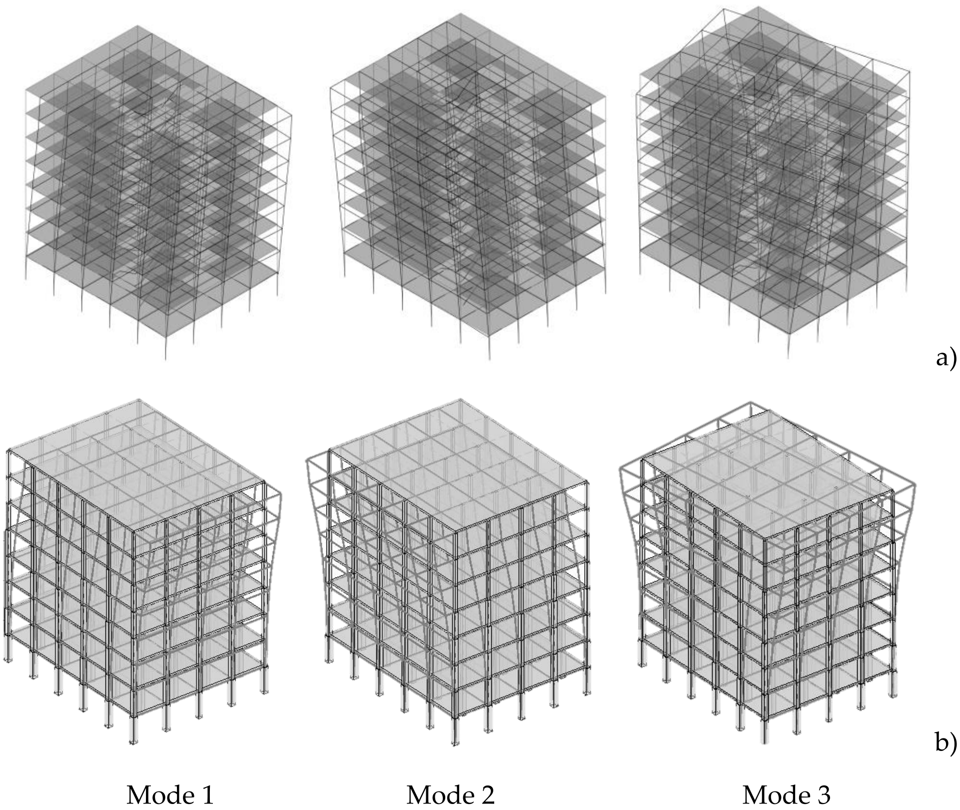

Figure 12 Eight -storey benchmark. Modal shapes comparison.

The first and second mode of vibration are translational in the two directions while the third one is torsional. A good correspondence can be observed between the results obtained by means of the two approaches.

Figure 13 and 14 report the displacement time histories in x and y directions of the considered control point during Santa Venerina and L'Aquila earthquake, respectively. Minor differences can be noticed between the responses obtained by means of the two considered models but they do not affect sensibly the global response.

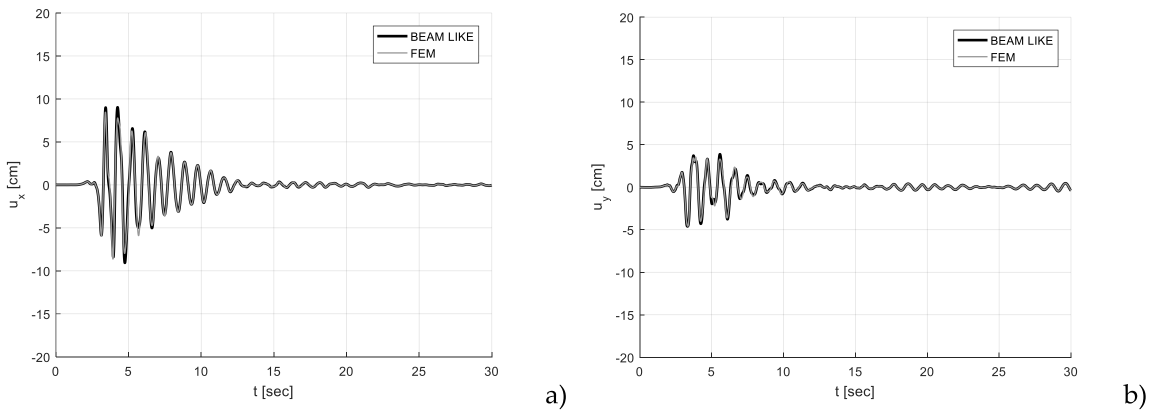

Figure 13 Eight-storey benchmark. Santa Venerina earthquake. Displacements time history.

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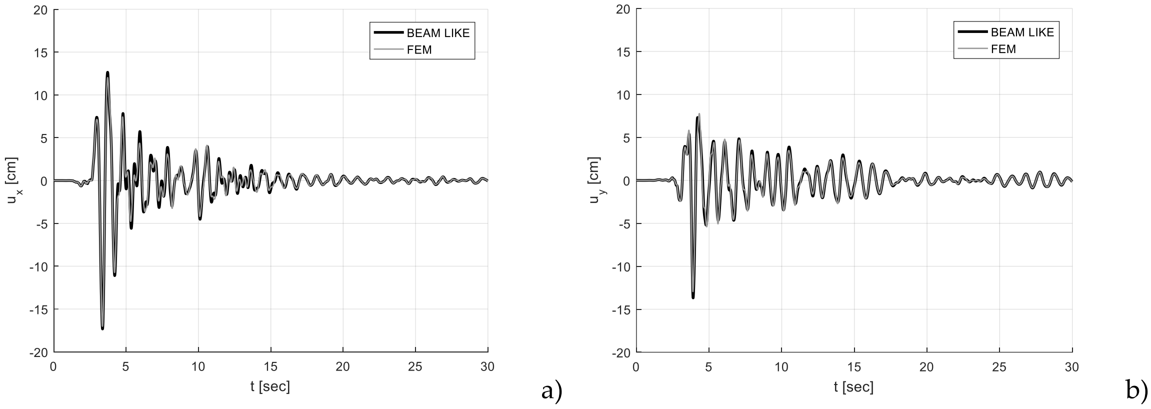

Figure 14 Eight -storey benchmark. L'Aquila earthquake. Displacements time history.

Figure 15 reports the maxima displacements of the same corner located at each floor level. Some small differences can be noticed in this case with reference to the intermediate floors but, globally, the proposed beam-like model reproduces the dynamic behaviour of this benchmark with a satisfactory agreement.

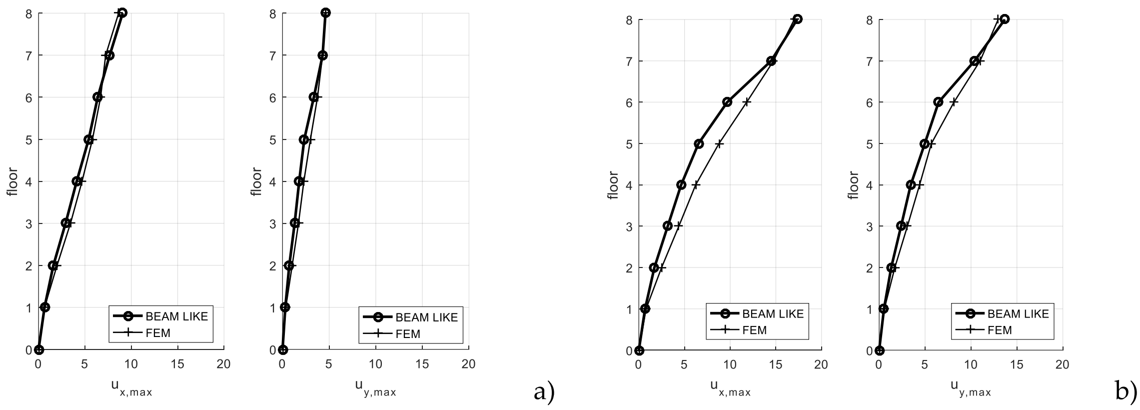

Figure 15 Eight -storey benchmark. Maxima floor displacements in a) SantaVenerina and b) L'Aquila earthquakes.

3.3. *Type 3: building with planar asymmetry and vertical non-uniform distribution of mass and stiffness*

Type 3, is represented by a six-storey building, with decreasing size of the column cross-sections along the height and eccentricity between CS and CM at each floor. The planar distribution of the cross section of the columns is reported for each floor in Figure 16.a while Figure 16.b shows a scheme of the FEM model.

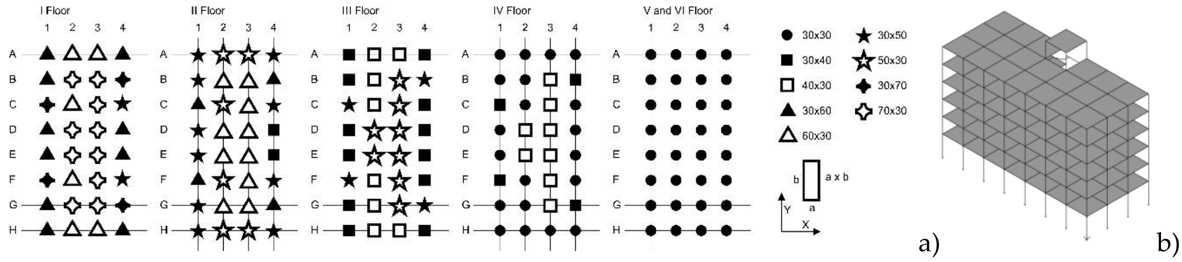

Figure 16 Six storey building benchmark (Model 2): a) column cross section plan and b) FEM model.

Table 4 and Figure 17 compare the modal properties, periods and modal shapes respectively, for the conventional FEM and the proposed beam-like model. It is worth to remember that in this case, due to the contemporary presence of planar and vertical irregularities, both natural frequencies and modes of vibration need to be introduced in the minimization of the objective function (Eq.5). It can

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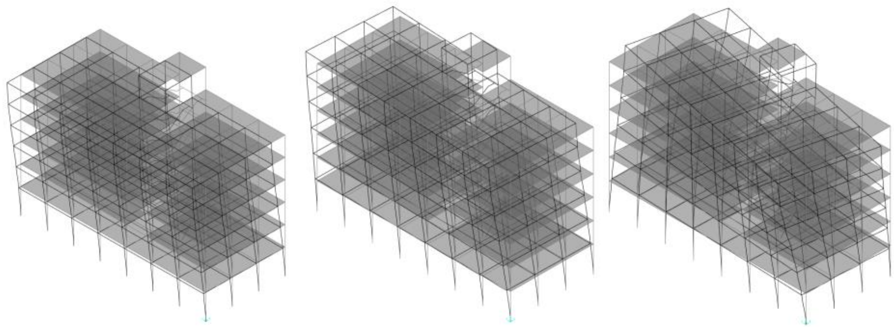
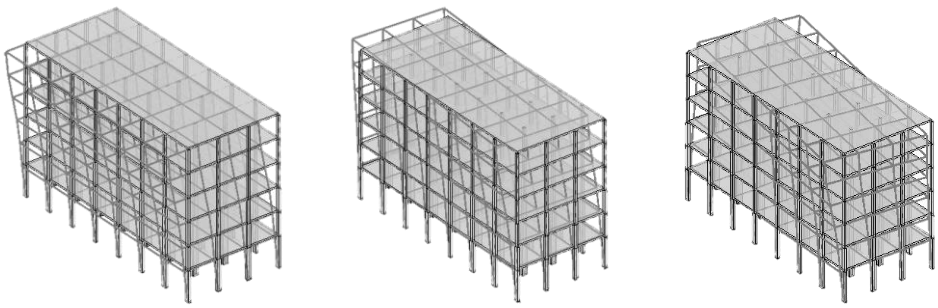
be observed that the results for the two considered models, reported in Table 4 and Figure 17, show a good agreement and confirm the reliability of the proposed approach .

Table 4 Modal Period comparison

Type 3	Mode 1	Mode 2	Mode 3
FEM	0.854	0.836	0.766
PROPOSED BEAM-LIKE MODEL	0.854	0.797	0.781



a)

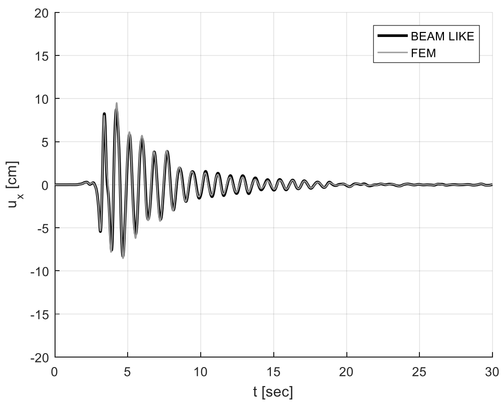


b)

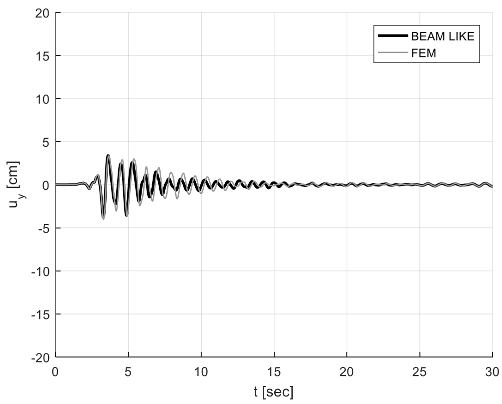
Mode 1 Mode 2 Mode 3

Figure 17 Six-storey benchmark. Modal shapes comparison.

For this irregular building type few minor differences between the two models can be noticed in the time histories of the displacement components of the considered control point shown in Figure 18 and Figure 19. Nevertheless, despite the complexity of the benchmark, a satisfactory agreement has been obtained also in terms of maxima displacements at each floor as reported in Figure 20.



a)



b)

Figure 18 Six-storey benchmark. Santa Venerina earthquake. Displacements time history.

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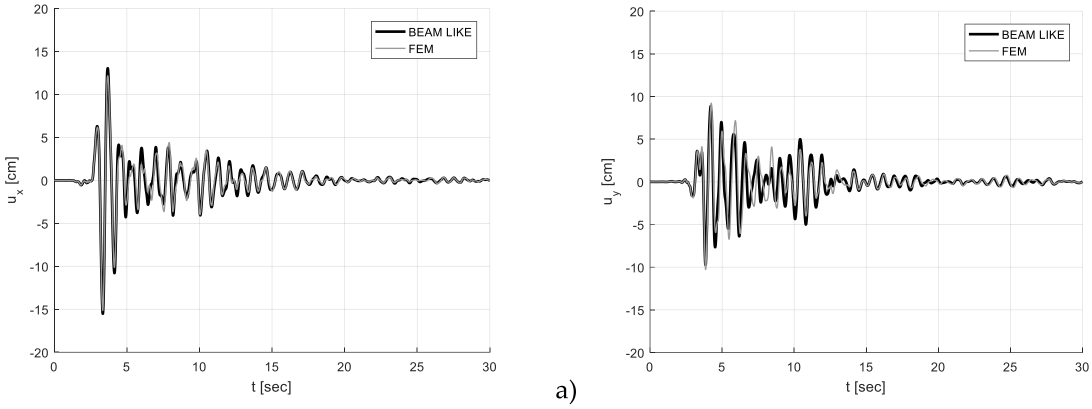

Figure 19 Six-storey benchmark. L'Aquila earthquake. Displacements time history.

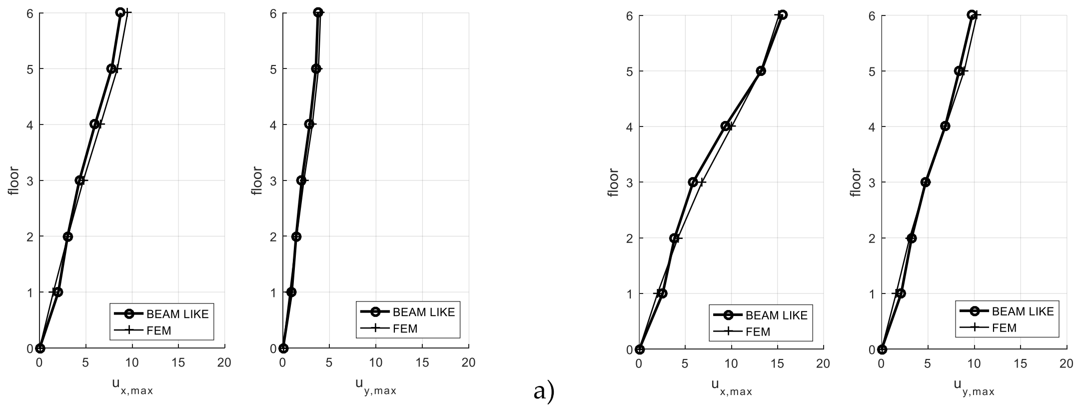

Figure 20 Six-storey benchmark. Maxima floor displacements in a) SantaVenerina and b) L'Aquila earthquakes.

4. Conclusions

The study proposes a beam-like model suitable for the schematization of real buildings that do not have uniform mass and stiffness distribution along the height and are characterized by asymmetric plans.

The linear dynamic behaviour of the beam-like element has been evaluated by discretizing the continuous model according to a Rayleigh-Ritz approach based on a suitable number of modal shapes of the uniform shear torsional beam clamped at the base. An optimization procedure has been adopted for estimating appropriate stiffness correction coefficients in order to take into account possible uncertainties in the structural parameters and the out of plane floor deformability. These correction coefficients have been identified by minimizing an objective function which measures the difference between a fixed number of eigen-properties of the beam like model and correspondent target values. The latter represent the natural frequencies and the modes of vibration of the real structure and could be directly measured by means of dynamic tests. In the present paper however, without limiting the general applicability of the proposed procedure, these target values have been calculated by means of three-dimensional linear FEM models.

The proposed procedure has been applied to the evaluation of the dynamic response of three multi-storey RC frames representative of residential buildings designed for vertical loads. Only the first three modes have been considered for calibrating the correction factors. The first two case studies report only one irregularity either in the planar or in the vertical distribution of mass and stiffness. In the last analysed building both planar and vertical irregularities are present. It has been highlighted that the increase in the number of irregularities require the knowledge of more data in

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order to seek an appropriate set of correction coefficients for the stiffness of the building. The results reported in the applicative section show that, in spite of the simplicity of the proposed beam-like model and the small computational burden, satisfactory results have been obtained. In particular, frequencies and modes of vibration, time histories of the displacements of some control points and maxima displacement at each floor level of the 3D FEM models have been compared to those calculated by means of the proposed beam-like model showing, for all the considered buildings, good agreements.

The obtained results show the good accuracy and the potentialities of the proposed approach in modelling real buildings and therefore encourages the authors to consider it as a very useful and powerful tool for the seismic assessment of urban areas with an extremely low computational burden.

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5. References

1. A. Basu and G. Dar, "Dynamic characteristics of coupled wall-frame systems," *Earthquake Engineering and Structural Dynamics*, pp. 615-631, 1982.
2. M. Chajes, K. Romstad and D. McCallen, "Analysis of multiple-bay frames using continuum model," *Journal of Structural Engineering ASCE*, pp. 522-546, 1993.
3. M. Chajes, W. Finch and J. Kirby, "Dynamic analysis of a ten-story reinforced concrete building using a continuum model," *Computers & Structures*, pp. 487-498, 1996.
4. M. Chajes, L. Zhang and J. Kirby, "Dynamic analysis of tall building using reduced-order continuum mode," *Journal of Structural Engineering ASCE*, pp. 1284-1291, 1996.
5. E. Miranda, "Approximate seismic lateral deformation demands in multistory buildings.," *Journal of Structural Engineering ASCE*, pp. 417-425, 1999.
6. E. Miranda and S. Taghavi, "Approximate floor acceleration demands in multistory buildings. I. Formulation," *Journal of Structural Engineering ASCE*, pp. 203-211, 2005.
7. S. Taghavi and E. Miranda, "Approximate floor acceleration demands in multistory buildings. II. Applications," *Journal od Structural Engineering ASCE*, pp. 212-220, 2005.
8. K. Zalka, "A simplified method for calculation of the natural frequencies of wall-frame buildings," *Engineering Structures*, pp. 1544-1555, 2001.
9. K. Zalka, "A simple method for the deflection analysis of tall wall-frame building structures under horizontal load," *The Structural Design of Tall and Special Buildings*, pp. 291-311, 2009.
10. K. Zalka, "Maximum deflection of asymmetric wall-frame buildings under horizontal load," *Periodica Polytechnica: Civil Engineering*, pp. 387-396, 2014.
11. G. Potza and L. Kollar, "Analysis of building structures by replacement sandwich beams," *International Journal of Solids and Structures*, pp. 535-553, 2003.
12. S. Timoshenko and D. Young, Theory of Structures, McGraw-Hill, 1965.
13. G. Tarjan and L. Kollar, "Approximate analysis of buildings with identical stories subjected to earthquakes," *International Journal of Solids and Structures*, pp. 1411-1433, 2004.
14. C. Boutin and S. Hans, "Homogenisation of periodic discrete medium: Application to dynamics of framed structures," *Computers and Geotechnics*, pp. 303-320, 2003.
15. A. Kwan, "Simple method for approximate analysis of framed tube structures," *Journal of Structural Engineering ASCE*, pp. 1221-1239, 1994.
16. R. Rahgozar, A. Ahmadi and Y. Sharifi, "A simple mathematical model for approximate analysis of tall buildings," *Applied Mathematical Modelling*, pp. 2437-2451, 2010.
17. M. Malekinejad and R. Rahgozar, "A simple analytic method for computing the natural frequencies and mode shapes of tall buildings," *Applied Mathematical Modelling*, pp. 3419-3432, 2012.

18. M. Malekinejad and R. Rahgozar, "An analytical model for dynamic response analysis of tubular tall buildings," *The Structural Design of Tall and Special Buildings,* pp. 67-80, 2014.
19. G. Piccardo, F. Tubino and A. Luongo, "A shear-shear torsional beam model for nonlinear aeroelastic analysis of tower buildings," *Journal of Applied Mathematics and Physics,* pp. 1895-1913, 2014.
20. A. K. Chopra, "Dynamics of Structures, Theory and Application to Earthquake Engineering," in *Dynamics of Structures, Theory and Application to Earthquake Engineering*, Upper Saddle River, Prentice Hall, 2001.
21. D. Spina, G. Acunzo, N. Fiorini, F. Mori and M. Dolce, "A probabilistic simplified seismic model of masonry buildings based on ambient vibrations," *Bulletin of Earthquake Engineering,* vol. 17, no. 2, pp. 985-1007, 2019.
22. Mathworks., "Global Optimization Toolbox: User's Guide (R2019b).," [Online]. Available: www.mathworks.com/help/pdf_doc/gads/gads_tb.pdf. [Accessed 03 January 2020].
23. G.Occhipinti, B.Izzuddin, L.Macorini and I.Caliò, "Realistic 3D Nonlinear Dynamic Analysis Of Existing And Retrofitted Multi-Storey RC Buildings Subject To Earthquake Loading," in *COMPDYN 2017*, Rhodes Island, Greece, 2017.
24. G.Occhipinti, B.Izzuddin, L.Macorini and I.Caliò, "Realistic Seismic Assessment Of RC Buildings With Masonry Infills Using 3D High-Fidelity Simulations," in *6th European Conference on Computational Mechanics (ECCM 6)*, Glasgow, 2018.
25. I. Caliò, A. Ghersi, M. Edoardo , P. P. Rossi, F. Contrafatto, F. Barbagallo, M. Bosco and G. Occhipinti, "Definition of the pilot building (ANCE|Catania Project 2016)," ANCE | Catania, Catania, 2017.
26. A. Greco, I. Fiore, G. Occhipinti and S. Caddemi, "Inhomogeneous beam-like models for the dynamic analysis of," in *SEMC2019 - The Seventh International Conference On Structural Engineering, Mechanics And Computation*, Cape Town, 2019.
27. A. Greco, I. Fiore, G. Occhipinti, S. Caddemi and I. Caliò, "A simplified inhomogeneous beam-like model for the dynamic analysis of multi-storey buildings," in *AIMETA2019*, Roma, 2019.
28. CSI, *CSI Analysis Reference Manual for SAP2000,* Computers and Structures Inc., 2007.
29. Ingv, "ITACA - Italian Accelerometric Archieve," [Online]. Available: itaca.mi.ingv.it.