Gray Level Image Threshold Using Neutrosophic Shannon Entropy

Vasile Patrascu,

Research Center in Electrical Engineering, Electronics and
Information Technology,
Valahia University,
Targoviste, Romania,
E-mail: patrascu.v@gmail.com

Abstract

This article presents a new method of segmenting grayscale images by minimizing Shannon's neutrosophic entropy. For the proposed segmentation method, the neutrosophic information components, i.e., the degree of truth, the degree of neutrality and the degree of falsity are defined taking into account the belonging to the segmented regions and at the same time to the separation threshold area. The principle of the method is simple and easy to understand and can lead to multiple thresholds. The efficacy of the method is illustrated using some test gray level images. The experimental results show that the proposed method has good performance for segmentation with optimal gray level thresholds.

Keywords: image segmentation; neutrosophic information; Shannon entropy; gray level image threshold
AMS Classification: 68U10, 62H35.

1 Introduction

Image segmentation is a process that divides the image into its component parts. One of the most used methods is the thresholding one. Recent advances in the neutrosophic representation of information allow different possibilities for the development of new image segmentation techniques. Neutrosophic models have the ability to work with data uncertainty and appear as an alternative to improving the threshold selection process so we get the right segmentation. In this paper we use as a thresholding function the Shannon entropy of neutrosophic information.

Next, the structure of the article is the following: Section 2 gives the neutrosophication procedure for the gray level images. The proposed neutrosophication is very specific to the thresholding technique; Section 3 shows how Shannon entropy can be calculated for the neutrosophic information; Section 4 shows the thresholding algorithm; Section 5 shows experimental results; Section 6 shows the conclusions while the last section is the references one.

The 3rd Conference on Recent Advances in Artificial Intelligence, RAAI 2019
Bucharest, România, June 28-30, 2019.

2 The Neutrosophic Information Construction

Neutrosophic representation of information was proposed by Smarandache [2], [7], [8] and [9] as an extension of fuzzy representation proposed by Zadeh [12] and intuitionistic fuzzy representation proposed by Atanassov [1]. Primary neutrosophic information is defined by three parameters: degree of truth T, degree of falsity F and degree of neutrality I. In the next, we will show, how can construct the neutrosophic information for a gray level image, in such a way to be useful for threshold methods.

Consider the gray levels in the range $[0, 1]$. We denote with X the multiset [11] of the gray levels existing in the whole image and take a point $t \in (\min(X), max(X))$. Also, we denote with X_1 the multiset of gray levels existing in the image from the interval $[0, t]$ and with X_2 the multiset of gray levels existing in the image from the interval $[t, 1]$, that is:

$$X_1 = \{x \in X | x \leq t\} \tag{1}$$

$$X_2 = \{x \in X | x \geq t\} \tag{2}$$

The threshold t represents the neutrality point between the two multisets X_1 and X_2. Then, we calculate the average v_1 for gray levels lower than t and the average v_2 for gray levels larger than t with formulas:

$$v_1(t) = \frac{1}{card(X_1)} \sum_{x \in X_1} x \tag{3}$$

$$v_2(t) = \frac{1}{card(X_2)} \sum_{x \in X_2} x \tag{4}$$

For calculation the dissimilarity between two gray levels $x, y \in [0, 1]$ we will use the following metric [5], [6]:
$d : [0, 1] \times [0, 1] \rightarrow [0, 1],$

$$d(x, y) = \frac{2|x - y|}{1 + |x - 0.5| + |y - 0.5|} \tag{5}$$

Using the metric d, we calculate the dissimilarities between each gray level x and the averages v_1, v_2 and the threshold t. It results $d(x, v_1)$, $d(x, v_2)$ and $d(x, t)$.

$$d(x, v_1) = \frac{2|x - v_1(t)|}{1 + |x - 0.5| + |v_1(t) - 0.5|} \tag{6}$$

$$d(x, v_2) = \frac{2|x - v_2(t)|}{1 + |x - 0.5| + |v_2(t) - 0.5|} \tag{7}$$

$$d(x, t) = \frac{2|x - t|}{1 + |x - 0.5| + |t - 0.5|} \tag{8}$$

We denote the minimum between the dissimilarities $d_1(x, v_1)$ and $d_2(x, v_2)$ with:

$$d_v(x) = min(d(x, v_1), d(x, v_2)) \tag{9}$$

The 3rd Conference on Recent Advances in Artificial Intelligence, RAAI 2019
Bucharest, Romånia, June 28-30, 2019.

With these dissimilarities, we will construct the neutrosophic information associated with each gray level x and related to the threshold t: the degree of truth $T(x,t)$, the degree of falsity $F(x,t)$ and the degree of neutrality $I(x,t)$.

The degree of truth:

$$T(x,t) = \frac{d(x,v_2) - d(x,v_1) \cdot d(x,v_2)}{d(x,v_1) + d(x,v_2) - d(x,v_1) \cdot d(x,v_2)} \tag{10}$$

The degree of falsity:

$$F(x,t) = \frac{d(x,v_1) - d(x,v_1) \cdot d(x,v_2)}{d(x,v_1) + d(x,v_2) - d(x,v_1) \cdot d(x,v_2)} \tag{11}$$

The degree of neutrality:

$$I(x,t) = \frac{d_v(x) - d(x,t) \cdot d_v(x)}{d(x,t) + d_v(x) - d(x,t) \cdot d_v(x)} \tag{12}$$

In figure 1, we can see the graphic of the functions $T(x,t)$, $I(x,t)$ and $F(x,t)$ for the particular case: $v_1 = 0.15$, $v_2 = 0.75$ and $t = 0.3$
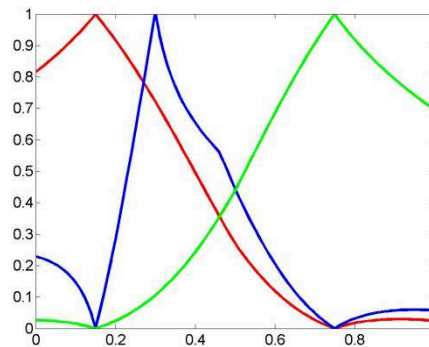

Figure 1: The graphic of the functions $T(red), I(blue), F(green)$ for $v_1 = 0.15$, $v_2 = 0.75$ and $t = 0.3$.

3   The Shannon entropy for neutrosophic information

In this paper, the Shannon function [10] is used as a measure for the neutrosophic information uncertainty. We do the following notations:

The bifuzzy undefinedness U:

$$U(x,t) = \max(0, 1 - T(x,t) - F(x,t)) \tag{13}$$

The bifuzzy contradiction C:

$$C(x,t) = \max(0, T(x,t) + F(x,t) - 1) \tag{14}$$

3

The 3rd Conference on Recent Advances in Artificial Intelligence, RAAI 2019
Bucharest, România, June 28-30, 2019.

The escort fuzzy degree of truth p_T:

$$p_T(x,t) = \frac{T(x,t) + U(x,t) + \dfrac{I(x,t)}{2}}{1 + I(x,t) + U(x,t) + C(x,t)} \quad (15)$$

The escort fuzzy degree of falsity p_F:

$$p_F(x,t) = \frac{F(x,t) + U(x,t) + \dfrac{I(x,t)}{2}}{1 + I(x,t) + U(x,t) + C(x,t)} \quad (16)$$

Then, we calculate the neutrosophic Shannon entropy $e(x,t)$ for each gray level x using formula proposed in [3, 4].

$$e(x,t) = \frac{p_T(x,t) \cdot ln(p_T(x,t)) + p_F(x,t) \cdot ln(p_F(x,t))}{-ln(2)} \quad (17)$$

In the space (T, I, F) the Shannon entropy for neutrosophic information verifies the general conditions of neutrosophic uncertainty [4]:

i) $e(1, 0, 0) = e(0, 0, 1) = 0$

ii) $e(T, I, T) = e(F, I, F) = 1$

iii) $e(T, I, F) = e(F, I, T)$

iv) $e(T_1, I_1, F_1) \leq e(T_2, I_2, F_2)$ if $|T_1 - F_1| \geq |T_2 - F_2|$, $|T_1 + F_1 - 1| \leq |T_2 + F_2 - 1|$ and $I_1 \leq I_2$.

The property (iv) shows that the Shannon entropy decreases with $|T - F|$, increases with $|T + F - 1|$ and increases with I.

From (iv) it results that $e(T, I, F) \in [0, 1]$ because $e(T, I, F) \geq e(1, 0, 0)$ and $e(T, I, F) \leq e(0, 1, 0)$. Also, we must mention that there exists the following equality:

$$|T + F - 1| = C + U \quad (18)$$

4 The algorithm for thresholding operation

We denote by $x_m = min(X)$ and $x_M = max(X)$. For each $t \in (x_m, x_M) \cap \{\frac{1}{Q}, \frac{2}{Q}, \ldots, \frac{Q-1}{Q}\}$, we calculate the entropy average for the three fuzzy sets defined by the neutrosophic components T, I and F. Here, the natural number Q is the step of threshold quantization. Typically, $Q = 255$. It results the following three partial entropy functions: $e_T(t)$, $e_I(t)$, $e_F(t)$.

$$e_T(t) = \frac{\sum_{x \in X} T(x,t) \cdot e(x,t)}{\sum_{x \in X} T(x)} \quad (19)$$

$$e_I(t) = \frac{\sum_{x \in X} I(x,t) \cdot e(x,t)}{\sum_{x \in X} I(x,t)} \quad (20)$$

The 3rd Conference on Recent Advances in Artificial Intelligence, RAAI 2019
Bucharest, România, June 28-30, 2019.

$$e_F(t) = \frac{\sum_{x \in X} F(x,t) \cdot e(x,t)}{\sum_{x \in X} F(x,t)} \quad (21)$$

Then, we define the optimal function that is the total entropy, which is calculated with the average of the three partial entropies calculated above.

$$E(t) = \frac{e_T(t) + e_I(t) + e_F(t)}{3} \quad (22)$$

The segmentation thresholds are the local minimum points of the total entropy E.

5   Experimental results

The proposed method was applied for segmentation of the following four images: ball, block, mammography and spider. The obtained results can be seen in figs. 2, 4, 6 and 8 while the entropy functions can be seen in figs. 3, 5, 7 and 9.
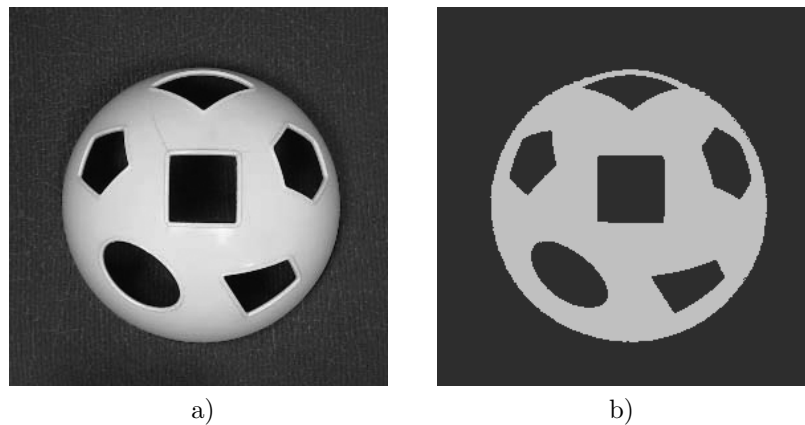

a)   b)

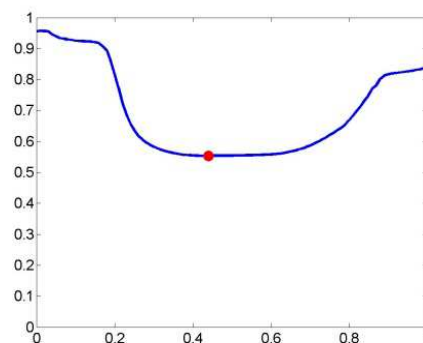Figure 2: The image ball (a). The segmented image with two gray levels (b).

Figure 3: The graphic of neutrosophic Shannon entropy where the red circle represents the gray level threshold (local minimum) for image ball.

a)   b)
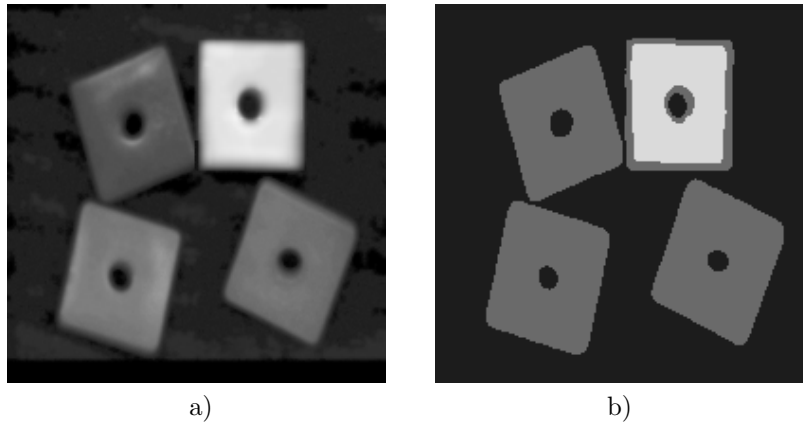
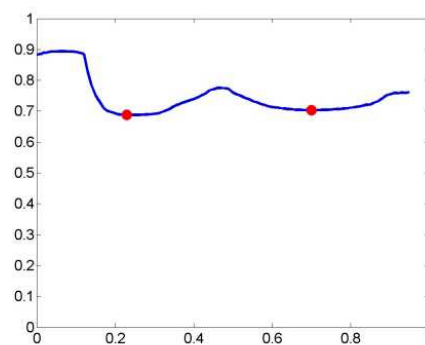Figure 4: The image block (a). The segmented image with three gray levels (b).

Figure 5: The graphic of neutrosophic Shannon entropy where the red circles represent the two gray level thresholds (local minima) for image block.

6   Conclusions

Based on the concept of neutrosophic information and the definition of the degree of truth, degree of neutrality and degree of falsity, a new image thresholding method is proposed. It utilizes the Shannon entropy in order to determine adequate threshold values. It is expected that the absolute value of the difference between degree of truth and degree of falsity for each pixel can be as close to 1 as possible, so that the entropy of each pixel is as minimal as possible. The proposed method which is based on minimizing the neutrosophic entropy of an image has demonstrated performance in multilevel thresholding. At the same time, the experimental results indicate that the proposed method can find appropriate threshold values.

References

1. K. Atanassov. Intuitionistic Fuzzy sets. Fuzzy Sets and Systems 20, pp. 87-96, (1986).

The 3rd Conference on Recent Advances in Artificial Intelligence, RAAI 2019
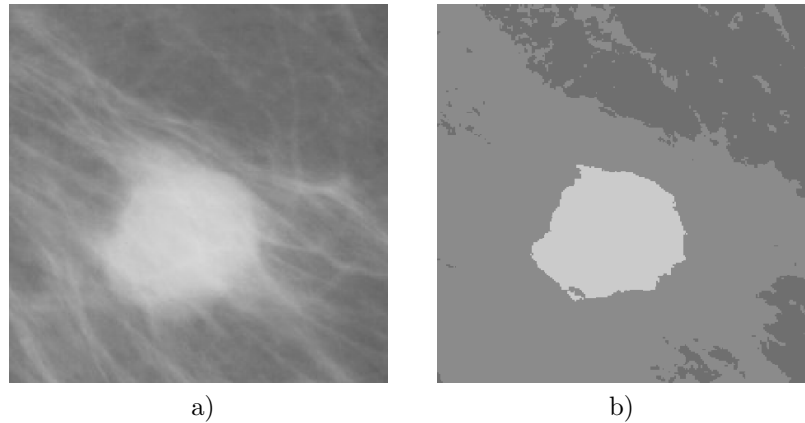Bucharest, România, June 28-30, 2019.

a)   b)

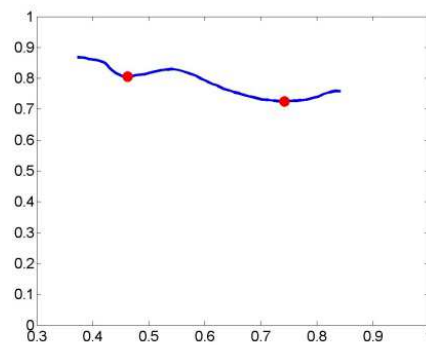Figure 6: The image mammography (a). The segmented image with three gray levels (b).

Figure 7: The graphic of neutrosophic Shannon entropy where the red circles represent the gray level thresholds (local minima) for image mammography.

2. C. Ashbacher, Introduction to Neutrosophic Logic, American Research Press, Rehoboth, NM, (2002).

3. V. Patrascu, Shannon entropy for imprecise and under-defined or over-defined information, The 25th Conference on Applied and Industrial Mathematics, CAIM 2017, Iasi, Romania, ROMAI Journal, vol. 14, no. 1, pp. 169-185, (2018).

4. V. Patrascu, Shannon Entropy for Neutrosophic Information, doi:10.13140 /RG.2.2.32352.74244, arXiv:1810.00748, September, (2018).

5. V. Patrascu, A Novel Penta-Valued Descriptor for Color Clustering, The 6th International Conference on Image and Signal Processing, ICISP 2014, Cherbourg, Normandy, France, June 30 - July 2, 2014, Volume: Image and Signal Processing, Lecture Notes in Computer Science, Volume 8509, Springer International Publishing Switzerland, pp. 173-182, (2014).
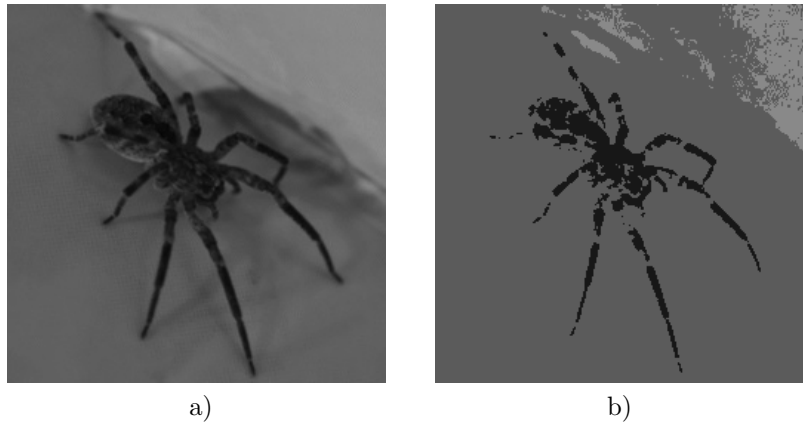
a)   b)

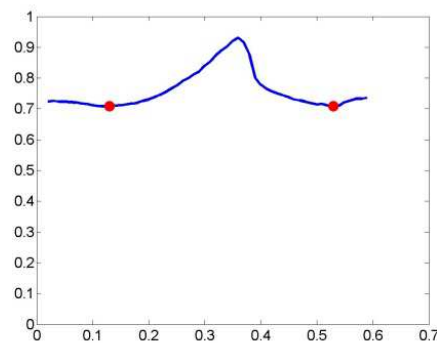Figure 8: The image spider (a). The segmented image with three gray levels (b).

Figure 9: The graphic of neutrosophic Shannon entropy where the red circles represent the gray level thresholds (local minima) for image spider.

6. V. Patrascu, New Framework of HSL System Based Color Clustering Algorithm, The 24th Midwest Artificial Intelligence and Cognitive Sciences Conference, MAICS 2013, April 13-14, 2013, New Albany, Indiana. USA, ceur-ws.org, Vol. 1348, pp. 85-91, (2013).

7. U. Rivieccio, Neutrosophic Logics: Prospects and Problems, Fuzzy Sets and Systems, 159, pp. 1860-1868, (2008).

8. F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, (1999).

9. F. Smarandache, Neutrosophic Set - A Generalization of the Intuitionistic Fuzzy Set, International Journal of Pure and Applied Mathematics, 24, no. 3, pp. 287-297, (2005).

10. C.E. Shannon, A mathematical theory of communication, Bell System Tech., J. 27, pp. 379-423, (1948).

The 3rd Conference on Recent Advances in Artificial Intelligence, RAAI 2019
Bucharest, România, June 28-30, 2019.

11. Wikipedia, https://en.wikipedia.org/wiki/Multiset.

12. A. L. Zadeh. Fuzzy sets, Information and Control, 8, pp. 338-353, (1965).