

Performance Comparing and Analysis for Slot Allocation Model

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Abstract: The purpose of this study is to ascertain the effect of weight setting of objectives on displacement and implementation difficulty in slot allocation model. Therefore, a linear integer programming model including three kinds of evaluation objectives with different weights is designed to compare and analyze displacement and implementation difficulty. Test results show that the difficulty of implementation can be significantly reduced to a certain extent by weight setting with a little increase of displacement under the condition of specific capacity setting. In addition, whether the movements are listed in order of descending priority or not has a great impact on displacement and implementation difficulty in the model. Capacity is surely a key factor affecting displacement and implementation difficulties. This study contributes to proposing an index of implementation difficulty for comparing and selecting suitable weights of evaluation objectives which can help decision makers to upgrade slot allocation policies.

Keywords Slot Allocation; Performance Comparing; Implementation Difficulty; Linear Integer Programming

1 Introduction

Most of the busiest airports worldwide experience serious congestion and delay problems, such as Beijing Capital International Airport (PEK), Shanghai Pudong International Airport (PVG), and Guangzhou Baiyun International Airport (CAN) ([Wu et al., 2018](#)). The existing imbalance between supply and demand for air transport services forces all aviation stakeholders to drastically rethink airport capacity and its utilization meanwhile readdressing the issue of experienced or anticipated capacity shortages ([Zografos et al., 2013](#)). Results of past research have proved that demand management could provide significant benefits at busy airports worldwide by permitting large delay reductions through limited interference with airline competitive scheduling ([Jacquillat and Odoni, 2015b](#)) or by evenly small substantial increase in declared capacity ([Zografos et al., 2012](#)). However, the latter, aiming to build new capacity, are capital intensive solutions which require significant implementation time thus are often subject to heated political debates. The need for an immediate relief to seriously congested airports calls for short to medium-term, demand-side solutions that are based on the optimum allocation and the use of available airport capacity ([Zografos et al., 2017](#)). To control over-capacity scheduling, the most common demanding management schemes fall into two categories: (i) approaches introducing market-driven or pure economic instruments (e.g., slot trading, auctions, congestion pricing), which aim to allocate capacity among competing users by considering real market (or approximations of) valuations of access to congested airport facilities ([Basso and Zhang, 2010](#), [Verhoef, 2010](#), [Zhang and Zhang, 2010](#), [Czerny and Zhang, 2011](#), [Le et al., 2008](#), [Vaze and Barnhart, 2012](#), [Grunewald, 2016](#)). (ii) efforts aiming to improve the efficiency by using administrative allocation mechanism ([Zografos et al., 2012](#), [Jacquillat and Odoni, 2013](#), [Pyrgiotis and Odoni, 2016](#), [Ribeiro et al., 2018](#), [Zografos et al., 2018](#), [Zografos et al., 2017](#), [Jacquillat and Odoni, 2015b](#), [Jacquillat and Odoni, 2015a](#), [Jacquillat and Odoni, 2018](#), [Benlic, 2018](#), [Zografos et al., 2013](#)).

In order to control the excessive demand of airports, Chinese Civil Aviation (CAA) has enacted slot regulation since 2010, but satisfactory effect has not been achieved. Consequently, the new slot regulation of CAA similar to that of International Air Transport Association (IATA) is introduced in April 2018. Although flight delay in Chinese airport has been improved after the implementation of these slot management methods, the unreasonable slot structure still exists (Li et al., 2016, Sun and Su, 2013, Hu et al., 2019). As the analysis of previous articles indicated, the typical characteristics of slot structure in large Chinese airports are that the departure of flights is concentrated in the morning while the arrival of flights in the evening. Because of this unreasonable schedule, the problems of low punctuality rate of flights in the morning and evening rush hours are doomed. The inefficiency of slot allocation also exists in other countries (Picard et al., 2019). So, in order to further improve the arrival and departure punctuality rate, it is still necessary to further reasonably adjust the slot structure while accurately assess and utilize declared capacity.

A practical implication of the current slot allocation process is that it may schedule a slot request far away from its initially requested time interval. This, in turn, may decrease substantially the actual utility realized by the relevant airline in using the allocated slot, hence resulting in poor (if any at all) slot utilization rates. Just as Zografos et al. (2018) pointed out that most existing models of flight scheduling typically do not consider acceptability of slot schedules. The more difficult the slot displacement is, the less likely the flight will accept the displacement. On the contrary, the less difficult the slot displacement is, the more likely the flight will accept it. Therefore, instead of slot allocation acceptance index, implementation difficulty index is proposed in this study, which is constructed into the model and examined together with slot displacement under condition that movements are listed in order of descending priority or not. Another key question is whether there is an opportunity to reduce implementation difficulty while not to increase too much displacement. Is it possible that movements with high priority will have a small displacement (HPSD) by our algorithm? Can priority is considered as the cost of displacing the unit time to ensure that the high priority movement has a low probability of being displaced (HPLP)? Considering priority as the cost of displacing the unit time, what are the differences in performance indicators when priority is batched into a computer program in priority order or in chronological order (morning-to-night final slot-table presentation)? All these questions which have not been investigated in previous articles will be discussed and answered within this study.

The main contribution of this paper is three folds. The first is that the difficulty index of slot displacement for quantifying implementation difficulties of each movement is proposed. Secondly, a multi-objectives linear integer programming model and algorithm are developed to minimize the total compound cost of slot allocation. Thirdly, how to reduce the displacement amount and implementation difficulty while guarantee HPLA is investigated.

The remainder of this paper is organized as follows. Section 2 formulates the model, including the technical aspects of capturing the CAA regulation and difficulty in optimizing the allocation of slots. Section 3 is the iterative linear integer programming algorithms based on data-splitting for proposed model. Section 4 present test and simulation results. Section 5 is the summary of the study with implications for future research.

2 Proposed slot displacement models

Before describing the model, notations are stated as follows:

p_m : Priority of movement m .

f_m^D : Flight implementation difficulties

I_m^D : Flight difficulty index of one displacement unit

τ_m : Interval that movement m required.

t_m : Interval that movement m is scheduled.

n : Seats of flights corresponding movement m .

π_m : The average flight elapse time of movement m , to (arrival flight) or from (departure flight) this airport.

L_a : Coordination level parameters of this airport (main coordinator, auxiliary coordinated airport and uncoordinated airport; 7, 4, 1).

L_b : Coordination level parameters of associated airports.

x_m^t : { 0, 1 }, if movement m is scheduled at t interval.

C_{60} : Hourly capacity constraint;

C_{15} : 15-minute capacity constraint;

C_5 : 5-minute capacity constraint;

b_{me} : Corridor capacity constraints, $e = (1, 2 \dots E)$.

a_m^d : Movement m plans to operate on day d of a series day, usually series day expressed as [1, 2, 3, 4, 5, 6, 7]. a_m^d is set mandatorily as 1, which means the same flight operating in one day has two movement number, such as m_1 and m_2 .

b_{mc} : The amount of the kind of capacity c consumed by movement m . In our model, c may be hourly capacity, 15-minute capacity, 5-minute capacity, or corridor capacity. In our model, the amount of each capacity consumed by a movement is 1, which means all of $b_{mc_{60}}$, $b_{mc_{15}}$, b_{mc_5} and b_{me} are set as 1.

n : Number of seats.

The coordination time interval represents the unit of time (e.g., 5-min, 15-min, 60-min) used as the basis for capacity determination and slot allocation. Usually each time interval contains multiple slots. A movement corresponds to a takeoff or landing activity. A slot refers specifically to the interval occupied by one movement.

We are motivated by the idea that the less difficult it is to adjust slot, the easier it will be accepted; the more difficult it is to adjust slot, the harder it will be accepted. Therefore, the problem of slot displacement is transformed into the problem of minimizing the difficulty of slot displacement subject to the priority rule stipulated by CAA slot regulations and other operational constraints.

2.1 Difficulty index and difficulty of Displacement

Difficulty, f_m^D , is inversely proportional to flight time π_m , and is proportional to slot displacement. It is easier and more possible for passengers to choose other transportation for a shorter flight distance. In addition, the variation of elapse time increases with the range of flight distance. The greater the flight distance, the more time the aircraft can gain or kill by displacing its speed during flight ([Han et al., 2010](#), [Richard et al., 2011](#), [De Smedt and Berz, 2007](#), [Tang and Han, 2012](#), [Wang and Li, 2011](#), [Brooker, 2009](#), [Durand et al., 2010](#), [Han et al., 2017](#), [Rey et al., 2012](#), [Rosenow et al., 2019](#), [Haraldsdottir et al., 2007](#), [Cafieri and D'Ambrosio, 2017](#), [De Smedt and Putz, 2009](#), [Rezaee and Izadpanah, 2009](#)). Therefore, difficulty is inversely proportional to elapse time. Difficulty is subject to the coordination level of the connected airport. The larger the number of the coordination level of the connected airport, the more difficult the slot coordination is. On the contrary, the smaller the number, the easier the slot coordination is.

Flight implementation difficulties index I_m^D and difficulty f_m^D brought by difficulty expressed as follow:

$$I_m^D = \left[\left(\frac{n}{\pi_m} \right)^{\frac{1}{2}} \cdot (L_a L_b)^{\frac{3}{2}} \right] \quad (1)$$

$$f_m^D = |t_m - \tau_m| \cdot I_m^D = |t_m - \tau_m| \cdot \left[\left(\frac{n}{\pi_m} \right)^{\frac{1}{2}} \cdot (L_a L_b)^{\frac{3}{2}} \right] \quad (2)$$

2.2 Standardized priority

The CAA slot regulations enforced the requirement of different priorities assigned to various types of slot requests. This is achieved through a sequential approach that first allocates historic series of slots, then followed by the "change to historic" series of slots, next followed by new entrant slots, and finally by the remaining slots. The CAA slot regulations clearly stipulate that priority should be determined from high to low according to the product of allocation cardinality and time efficiency allocation coefficient of aviation enterprises. We standardize

the calculated priority values, so that the four types of priority are distributed within a clearly distinguishable range showed as in Table 1.

Table 1. Range of standardized priority

| | |
|--------------------------------------|-------------|
| historic series of slots | [1501,2000] |
| “change to historic” series of slots | [1001,1500] |
| new entrant slots | [501,1000] |
| remaining slots | [1,500] |

In reality, priority determines the priority of slot selection. Similarly, in model computing, priority determines which movement can enter the model earlier and get the preferred slot. This is called High-Priority First (HPF).

2.3 Comprehensive displacement cost

As mentioned before, cost of one movement in most of traditional slot allocation models is as following:

$$f_m^O = |t_m - \tau_m| \cdot 1 \quad (3)$$

In expression (3), f_m^O is the interval displacement when required interval τ_m is replaced with scheduled interval t_m .

When priority is considered as the cost of displacing the unit time, cost of one movement can be presented as following:

$$f_m^P = |t_m - \tau_m| \cdot p_m \quad (4)$$

In our model, we integrate cost of displacement, difficulties and priorities as a whole in order to facilitate calculation and comparison. At the same time, three weight factors (w_1, w_2, w_3) are introduced artificially for the same reason. The comprehensive displacement cost of a movement, displaced from required interval τ_m to scheduled interval t_m , could be expressed as following:

$$\delta_m^{ODP} = (w_1 \cdot 1 + w_2 \cdot I_m^D + w_3 \cdot p_m) = (w_1 \cdot 1 + w_2 \cdot \left[\left(\frac{n}{\pi_m} \right)^{\frac{1}{2}} (L_a L_b)^{\frac{3}{2}} \right] + w_3 \cdot p_m) \quad (5)$$

$$f_m^t = w_1 \cdot f_m^O + w_2 \cdot f_m^D + w_3 \cdot f_m^P = |t_m - \tau_m| \delta_m^{ODP} \quad (6)$$

We call δ_m^{ODP} as the comprehensive displacement cost factor with the consideration of implementation difficulties and priority. It is important to find that δ_m^{ODP} is a constant, which makes it possible to solve it by Linear Integer Programming (LIP).

2.4 Displacement model for all flights

$$\text{minimize} \quad \sum_{m \in M} \sum_{t \in T} f_m^t x_m^t = \sum_{m \in M} \sum_{t \in T} |t_m - \tau_m| \delta_m^{ODP} x_m^t \quad (7)$$

$$\text{subject to} \quad \sum_{t \in S} x_m^t = 1, \quad m \in M \quad (8)$$

$$\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}, \quad c \in C, d \in D, s \in T_c \quad (9)$$

$$x_m^t \in \{0,1\}, \quad m \in M, \quad t \in S \quad (10)$$

The objective function (7) minimizes the overall displacement cost of all flights. Constraint (8) stipulates that every movement must be allocated to one interval. Constraint (9) specifies that total movement consumption cannot exceed capacity, for each constraint, day and interval. Constraint (10) ensures that this model can be solved by integer programming method. There is no detailed description of the coefficient matrix in previous papers. Thus, in the next section, we will elaborate proposed approach.

3 LIP for slot displacement models

In order to limit the flow of corridors, it is necessary to determine which corridor should be allocated to each flight. Firstly, the courses of all flights from the airport to the linked airport are calculated. Then, the courses for arrival or departure are sorted descending. If there are m corridors for arrival (or departure), then the arrival (or departure) courses are classified into m categories.

In general, the number of corridors for arrival is equal to the number of corridors for departure. Each corridor entrance must meet the capacity constraints. All movement is classified according to the number of corridors and assigned evenly to corridor before executing LIP. By designing the variable $y_i^e = \{0,1\}, i = 1 \dots M, e = 1 \dots E$, the relationship matrix of movement and corridors is constructed as shown in table 2. The sum of each column must be 1, that is, each movement i must be assigned to a corridor e . The sum of each row is limited by the capacity of the corridor in every interval. This makes it easy to solve the capacity constraints of the corridor with linear programming, but with the increase of the number of flights and the number of corridors, the dimension of the constraints increases rapidly.

Table 2. relationship matrix of movement i and corridors e

| $e \backslash i$ | 1 | 2 | 3 | 4 | ... | M | C_e |
|------------------|-----|-----|-----|-----|-----|-----|-----------|
| 1 | 1 | 0 | 0 | 0 | ... | 0 | C_1 |
| 2 | 0 | 0 | 1 | 0 | ... | 1 | C_2 |
| 3 | 0 | 0 | 0 | 0 | ... | 0 | C_3 |
| ... | ... | ... | ... | ... | ... | ... | ... |
| $E-1$ | 0 | 0 | 0 | 1 | ... | 0 | C_{E-1} |
| E | 0 | 1 | 0 | 0 | ... | 0 | C_E |

Considering that the flight execution cycle is usually at least once a week, and that most flights operate every day, in order to reduce the computing time with the proposed approach, it is possible to determine the calendar days with the same set of requests, and then represent these as a single calendar day. In order to keep the accumulated rolling volume of flights from exceeding the hourly capacity, we consider three kinds of capacities in brackets (hourly capacity, 15-minute capacity and 5-minute capacity) to prevent this from happening. For preventing memory overflow, these three capacities are arranged separately and in the order of hourly capacity, 15-minute capacity and 5-minute capacity sequentially, that is to say, the other two are not active in the arrangement of the third capacity. Because of the fact that 5-minute capacity has greatest impact on the uniform distribution of time, the check of 5-minute capacity is put at the end. When the proposed algorithm is used for 5-minute check, the dimension of the constraint is too large, and sometimes the memory is insufficient to execute. So we activate constraints of runway capacity and corridors capacity in batches according to the order of priority. The corresponding capacity is updated after each batch of arrangement. The following is the procedures of iterative linear integer programming algorithms based on data-splitting.

Iterative linear integer programming algorithms based on data-splitting

Priority calculation f_m^P

For $i=1:7$

Load movement in day 1

Set up hourly capacity c_{60} , 15-minute capacity c_{15} , 5-minute capacity c_5

Calculating the minimum total capacity, $Z = \text{Min}(24 \cdot c_{60}, 96 \cdot c_{15}, 288 \cdot c_5)$

Compared with M and Z , if M is larger than Z , the number of discarded requests is equal to $M-Z$

Use simplex method to arrange all remaining applications into each hourly period

Use simplex method to arrange the 15-minute period on the basis of the hour schedule

Use simplex method to arrange the 5-minute period on the basis of the 15-minute schedule

End

Table 3 is an example of description of the coefficient matrix in detail in the objective function and constrains within hours.

Table 3. Coefficient matrix and detailed expression when interval is based on hour

| | | | | |
|---|---|--|---|----------|
| Execution batch equals $P=[M/Q]$ | $\sum_{m \in M} \sum_{t \in T} f_m^t x_m^t$ $= \sum_{m \in M} \sum_{t \in T} t_m - \tau_m \delta_m^{DP} x_m^t$ | f_m^t | $f_m^t x_m^t = f_1^1 x_{11} + f_1^2 x_{12} + f_1^3 x_{13} + \dots + f_1^{24} x_{124} +$ $f_2^1 x_{21} + f_2^2 x_{22} + f_2^3 x_{23} + \dots + f_2^{24} x_{224} +$ \dots $f_m^1 x_{m1} + f_m^2 x_{m2} + f_m^3 x_{m3} + \dots + f_m^{24} x_{m24}$ | (a.0) |
| | $\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}$ $c \in \text{hour}, d \in [1, 2, \dots, 7],$ $S \in [1, 2, \dots, 24]$ $a_m^d = 1, b_{mc} = 1$ | A_1 | $x_{11} + x_{21} + x_{31} + \dots + x_{m1} \leq u_{hour}^1$ | (a.1) |
| | | | $x_{12} + x_{22} + x_{32} + \dots + x_{m2} \leq u_{hour}^2$ | (a.2) |
| | | | \vdots | \vdots |
| | | | $x_{124} + x_{224} + x_{324} + \dots + x_{m24} \leq u_{hour}^{24}$ | (a.24) |
| | $\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}$ $c \in c1, \text{corridor } 1$ $d \in [1, 2, \dots, 7],$ $S \in [1, 2, \dots, 24]$ $a_m^d = 1, b_{mc} = 1$ | Y | $y_1^1 x_{11} + y_2^1 x_{21} + y_3^1 x_{31} + \dots + y_m^1 x_{m1} \leq u_{c1}^1$ | (b.1) |
| | | | $y_1^1 x_{12} + y_2^1 x_{22} + y_3^1 x_{32} + \dots + y_m^1 x_{m2} \leq u_{c1}^2$ | (b.2) |
| \vdots | | | \vdots | |
| $y_1^1 x_{124} + y_2^1 x_{224} + y_3^1 x_{324} + \dots + y_m^1 x_{m24} \leq u_{c1}^{24}$ | | | (b.24) | |
| $\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}$ $c \in c8, \text{corridor } 8$ $d \in [1, 2, \dots, 7],$ $S \in [1, 2, \dots, 24]$ $a_m^d = 1, b_{mc} = 1$ | Y | $y_1^8 x_{11} + y_2^8 x_{21} + y_3^8 x_{31} + \dots + y_m^8 x_{m1} \leq u_{c8}^1$ | (b.169) | |
| | | $y_1^8 x_{12} + y_2^8 x_{22} + y_3^8 x_{32} + \dots + y_m^8 x_{m2} \leq u_{c8}^2$ | (b.170) | |
| | | \vdots | \vdots | |
| | | $y_1^8 x_{124} + y_2^8 x_{224} + y_3^8 x_{324} + \dots + y_m^8 x_{m24} \leq u_{c8}^{24}$ | (b.216) | |
| $\sum_{t \in T} x_m^t = 1, m \in Q$ $x_m^t \in \{0, 1\}, m \in Q$ $t \in [1, 2, \dots, 24]$ | A_{eq} | $x_{11} + x_{12} + x_{13} + \dots + x_{124} = 1$ | (d.1) | |
| | | $x_{21} + x_{22} + x_{23} + \dots + x_{224} = 1$ | (d.2) | |
| | | \vdots | \vdots | |
| | | $x_{m1} + x_{m2} + x_{m3} + \dots + x_{m24} = 1$ | (d.m) | |

4 Testing and results

The purpose of the test is to observe the impact of changing weights of evaluation objectives on performance. Meanwhile it aims to investigate following questions proposed above. 1) Whether there is an opportunity to reduce implementation difficulty while not to increase too much displacement. 2) Is it possible that movements with high priority will have a small displacement (HPSD) by our algorithm? 3) Can priority be considered as the cost of displacing the unit time to ensure that the high priority movement has a low probability of being displaced (HPLA)? 4) Considering priority as the cost of displacing the unit time, what are the differences in performance indicators when priority is batched into a computer program in priority order or in chronological order (morning-to-night final slot-table presentation) ?

Is it possible that movements with high priority will have a small displacement (HPSD) by our algorithm? Can priority be considered as the cost of displacing the unit time to ensure that the high priority movement has a

low probability of being displaced (HPLP)? Considering priority as the cost of displacing the unit time, what are the differences in performance indicators when priority is batched into a computer program in priority order or in chronological order (morning-to-night final slot-table presentation)?

4.1 Performance compare with different weight factors of evaluation objectives

Weights setting and results with different weights of objective functions are showed in Table 4.

Table 4. Performance compare with different weight factors of evaluation objectives

| Tests | Weights of disp,diff,prio | Total diff | Average diff | Total disp | Average disp | Capacity | Min (disp) | Max (disp) | Order | Figure |
|-------|---------------------------|------------|--------------|------------|--------------|----------|------------|------------|--------|--------|
| 1 | 1,0,0 | 8.2E+06 | 5752.40 | 27200 | 19.18 | 88,23,7 | -340 | 385 | SD | 1-2 |
| 2 | 0,1,0 | 4.5E+06 | 3177.89 | 29290 | 20.66 | 88,23,7 | -430 | 310 | SD | 3-4 |
| 3 | 0,0,1 | 6.4E+06 | 4480.08 | 29220 | 20.61 | 88,23,7 | -485 | 405 | SD | 5-6 |
| 4 | 0,0,9,0,1 | 5.1E+06 | 3582.03 | 30130 | 21.25 | 88,23,7 | -565 | 360 | SD | N |
| 5 | 0,1,1 | 6.4E+06 | 4480.08 | 27015 | 19.05 | 88,23,7 | -485 | 405 | SD | N |
| 6 | 0,0,5,0,5 | 6.6E+06 | 4650.95 | 29045 | 20.48 | 88,23,7 | -485 | 340 | SD | N |
| 7 | 0,1,1 | 9.0E+06 | 6367.90 | 44330 | 31.26 | 78,20,7 | -520 | 460 | SD | N |
| 8 | 0,0,5,0,5 | 7.2E+06 | 5064.14 | 28305 | 19.96 | 88,23,7 | -585 | 375 | not SD | 7-9 |
| 9 | 1,0,1,0,9 | 5.2E+06 | 3670.50 | 29385 | 21.00 | 88,23,7 | -545 | 350 | SD | N |
| 10 | 1,0,0 | 8.3E+06 | 5855.00 | 28170 | 20.00 | 88,23,8 | -270 | 419 | not SD | N |
| 11 | 0.8, 0.01, 0.09 | 5.0E+06 | 3544.65 | 28440 | 20.06 | 88,23,8 | -510 | 410 | SD | N |
| 12 | 100, 0.1, 0 | 5.3E+06 | 3707.60 | 28060 | 19.79 | 88,23,8 | -545 | 365 | SD | N |

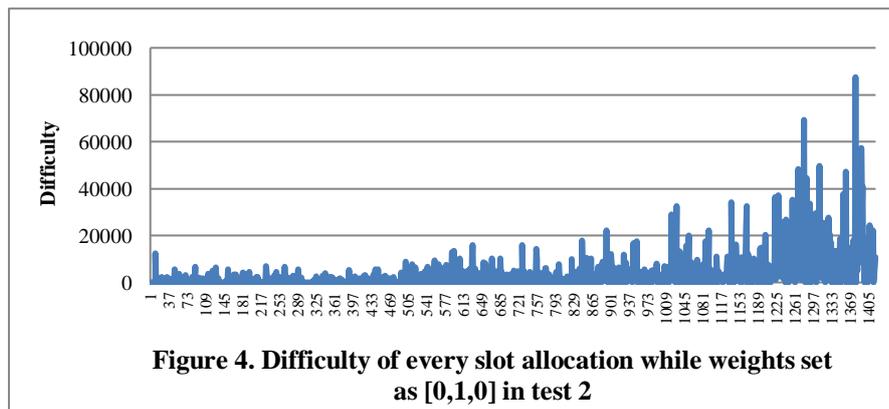
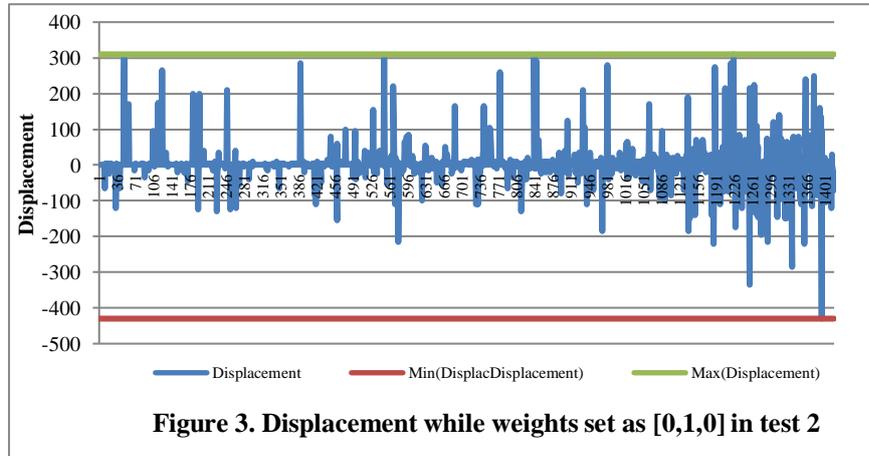
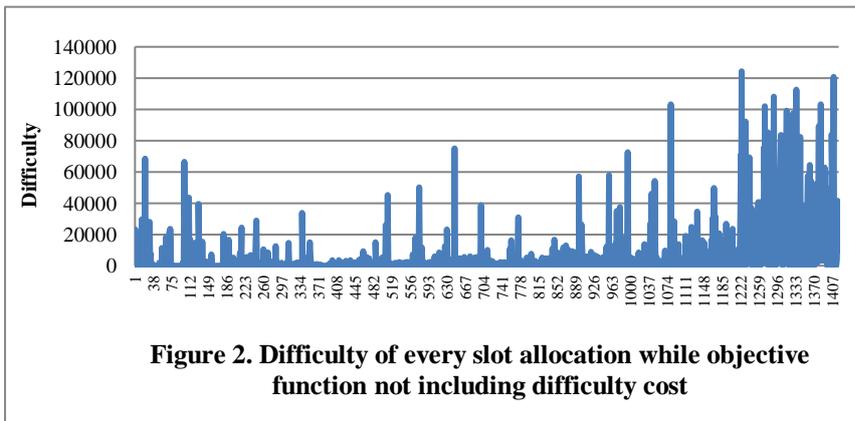
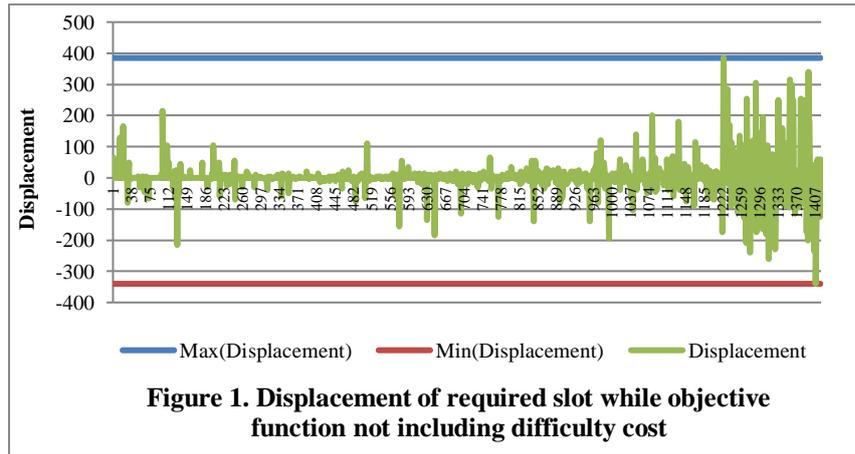
Note: disp=displacement, diff=difficulty, prio=priority, SD=sorting descent, N=not show, key information

➤ For question 1

According to the results of table 4, we found that there is a contradictory relationship between displacement and difficulty, thus the first question still cannot be answered. Thus the first question still cannot be answered, and will be further to investigate at next section.

➤ For question 2

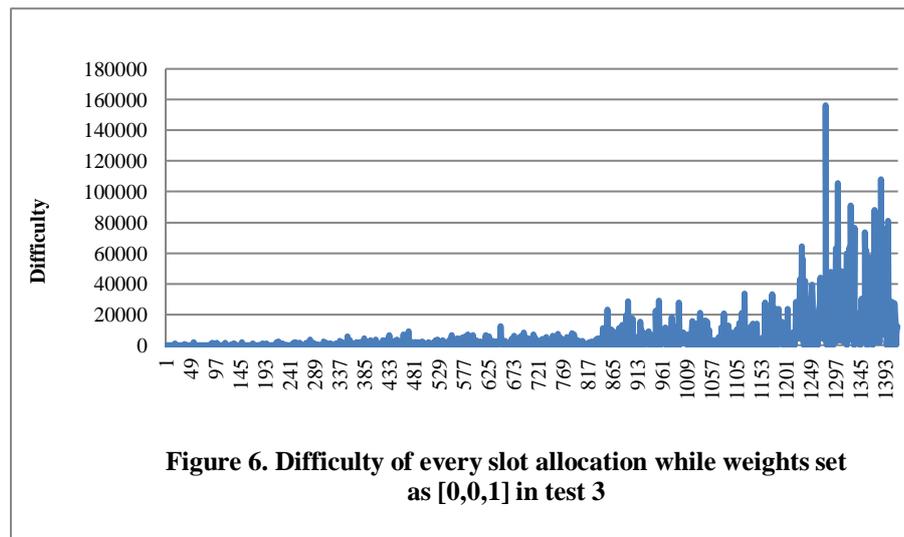
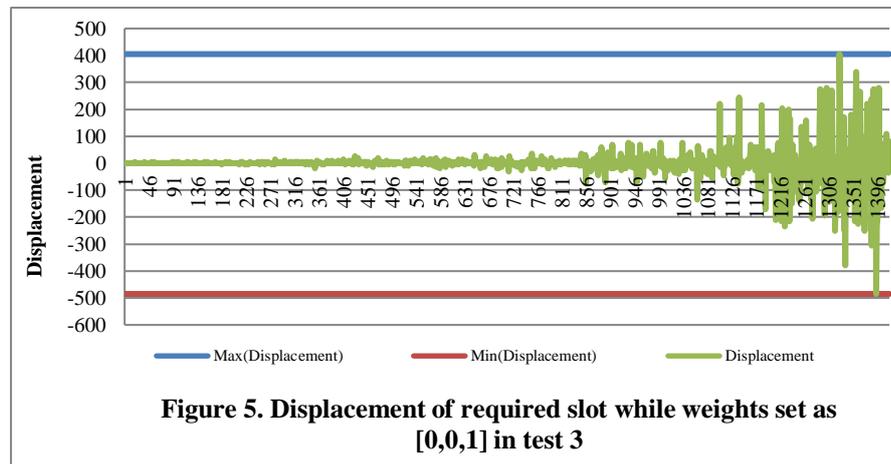
Displacement and difficulty of slot allocation and weights are set as [1,0,0] as showed in figure 1-2. In this case, the slot with small priority (late entering procedure) has a greater probability of being displaced, and the amount of slot displacement and its displaced probability will increase significantly with priority decreasing. While weights are set as [0,1,0] in test 2, displacement and difficulty of each slot allocation are showed in figure 3-4. In this case, as the priority decreasing, the displaced probability of a movement has the same trend as in test 1, but the difference is that even if the priority is high, there is a certain probability of being displaced greater. In test 1, the average slot displacement is relatively small, but the average difficulty of the slot displacement is greater than that in test 2, which means only using displacement or difficulty as the objective cannot guarantee HPSD, but both of them can guarantee HPLA, although using difficulty as the objective will lead to some movement being displaced greater.



➤ For question 3

Comparing test 3 and other tests, we found when priority is the solely evaluation objective, both of difficulty and amount of displacement will increase gradually with priority decreasing (figure 5-6). This means using priority as objective can guarantee HPSD& HPLA very well, although average displacement or average difficulty is not the smallest.

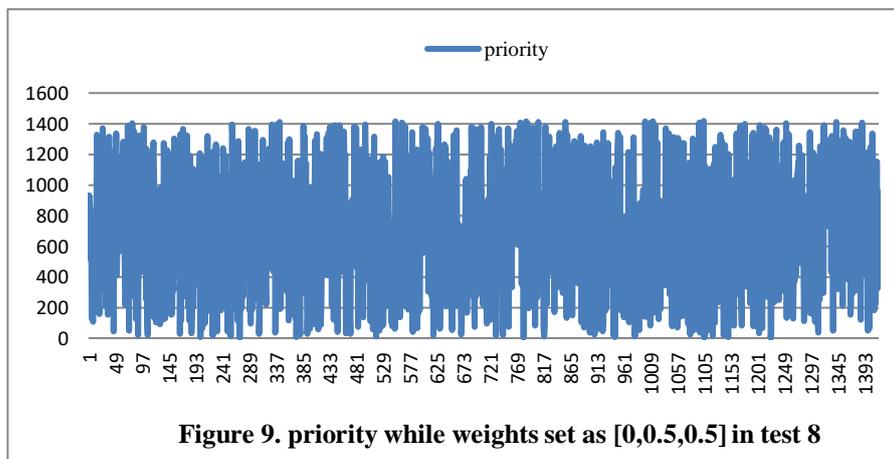
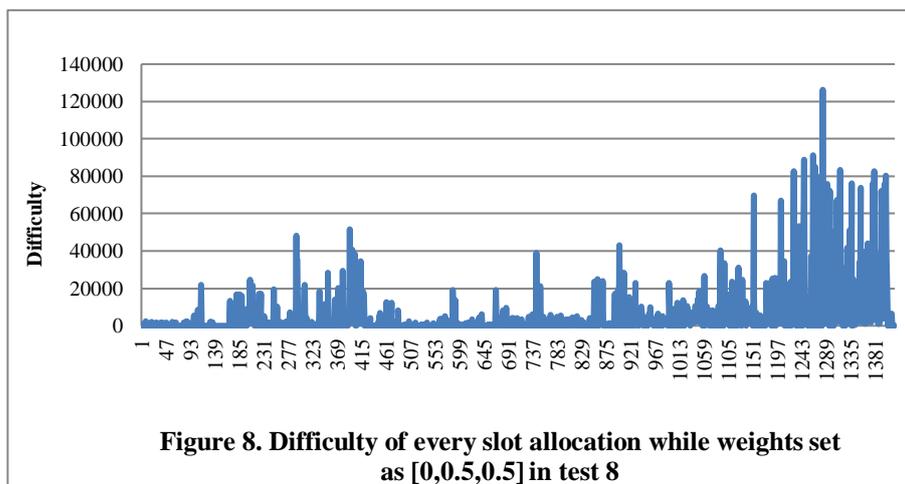
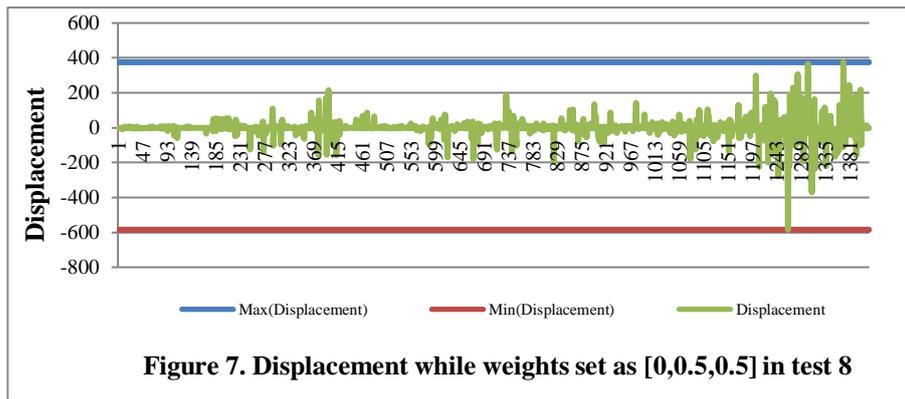
So, from investigation of question 2&3, we found that priority, displacement of difficulty should be included in slot model.



➤ For question 4

Comparing test 8 and test 6, we found that when other conditions remain unchanged, if application is sent to the process without application sorting, then the average difficulty will increase, but the average displacement remains almost unchanged (test 8). This means that the effect of sorting or not on difficulty is greater than that on displacement.

Based on test 8 and figure 7-9, we found that if movements are not listed according to the descent order of priority, total difficulty is the greatest in all tests with same capacity setting [88,23,7], but movements listed later in inputting data table still have more probability of being displaced and being displaced greater than a movement ahead of the list. It is very obvious that when a movement listed at the end of the roster, the remaining capacity becomes less and less, while the probability of being displaced becomes larger and larger.



➤ Another found

In test 4-6, 9, 11-12, when difficulty and priority exist simultaneously in evaluation objectives, the changes of the weight of these two objectives mainly affect the change of difficulty while the average displacement amount remains almost unchanged. This maybe implies that priority has not too much effect on displacement, but has effect on difficulty under the condition movements are sorted according to priority.

In test 7, due to the fact that the limited capacity (limited capacity equal to minimum in $[78 \ 20 \times 4 \ 7 \times 12]$, which is 78) is smaller than that (minimum in $[88 \ 23 \times 4 \ 7 \times 12]$ is 84) in other test, the average displacement amount and the displacement difficulty are significantly increased. This comparing result implies that capacity is the key factor affecting the displacement.

4.2 Analysis of the correlation between average displacement and average difficulty

The following formula of Pearson correlation coefficient is used to calculate the correlation coefficients of average displacement and average difficulty with the same capacity setting [88,23,7].

$$r = \frac{\sum XY - \sum X \sum Y}{\sqrt{(\sum X^2 - \frac{(\sum X)^2}{N})(\sum Y^2 - \frac{(\sum Y)^2}{N})}} \quad (11)$$

Three columns in table 4 (excluding test 7, because the capacity of this test is different from others) are extracted to table 5, and all rows are ranked according to the average displacement in ascending order. The first column is index of test. After calculating with formula 11 and table 5, the results show that there is a negative correlation between mean displacement and average difficulty with correlation coefficient $r=-0.54$. That is, one decreases while another one increases. If the average difficulty is sensitive to the average displacement, there is an opportunity to increase the average displacement a little to achieve a significant reduction in the average difficulty. Therefore, we need to test the sensitivity of the average difficulty to the average displacement.

Table 5. data sorting according to average displacement

| The first set of tests | Average disp | Average diff |
|------------------------|--------------|--------------|
| | X | Y |
| 5 | 19.05 | 4480.08 |
| 1 | 19.18 | 5752.40 |
| 12 | 19.79 | 3707.60 |
| 8 | 19.96 | 5064.14 |
| 10 | 20.00 | 5855.00 |
| 11 | 20.06 | 3544.65 |
| 6 | 20.48 | 4650.95 |
| 3 | 20.61 | 4480.08 |
| 2 | 20.66 | 3177.89 |
| 9 | 21.00 | 3670.50 |
| 4 | 21.25 | 3582.03 |

The following formula is used to test the sensitivity of average difficulty to average displacement.

$$\delta_i^X = X_i - X_{i-1}, i=2 \dots N; \quad (12)$$

$$\delta_i^Y = Y_i - Y_{i-1}, i=2 \dots N; \quad (13)$$

$$\sigma = \frac{(\sum_{i=2}^N \delta_i^X)/N}{(\sum_{i=2}^N \delta_i^Y)/N} = \frac{\sum_{i=2}^N \delta_i^X}{\sum_{i=2}^N \delta_i^Y}, i=2 \dots N; \quad (14)$$

Using the data in Table 5, we obtained the following sensitivity coefficients, $\sigma=-408.20$. This shows that the average difficulty is very sensitive to the average displacement. The average displacement only needs to increase a small amount, which can greatly reduce the implementation difficulty.

5 Discussion and conclusion

The test results show that even if we strictly implement the new slot regulation, applying different weights of performance objectives and even different processing technology, slot displacements and implementation difficulties are also different.

The average difficulty is very sensitive to the average displacement. This means the average displacement only needs to increase by a small amount, which can greatly reduce the implementation difficulty. It is worthwhile to increase some amount of slot displacement appropriately in return for a significant reduction in average execution difficulty. Especially when the number of seats on some regional flights is small, the air elapse time can

be displaced to a large margin, or when the linked airports are uncoordinated, it is reasonable to make more displacement for these flights.

Priority cost is not suitable for giving too much weight, because too much weight will lead to increased difficulty and adjustment. Furthermore, “lower displacement for movements with higher priority” has been guaranteed by the high priority listed ahead of data table. Therefore, airlines should pay more attention to “the position in the list” not priority itself. When applications are not listed according to priority, great differences in priorities make displacement become more difficult. For the management department of slot allocation, it is an effective way to ensure “lower displacement for movements with higher priority” by inputting programs in priority order.

The difficulty index proposed in this study is very useful in identifying which slot allocation scheme is better as shown and discussed in Section 4.1.

In summary, evaluation objectives including displacements, implementation difficulties and priority cost are suitable, in addition that the weights of the latter two objectives should be set smaller, and the weights of difficulty should be greater than the weights of priority cost. However, the values of specific weights still need to be evaluated by multi-stakeholders and specific market research.

The main limitation of the study is that priority is produced by simulation, because of limitation of available data. This implies that the relationship of implementation difficulty and priority needs to be discussed in further research. We believe that with the improvement of technical methods and awareness of slot allocation, it is promising to make slot allocation more scientific and reasonable by investigating new slot performance evaluation objectives and other constraints in future research.

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