

1 *Type of the Paper (Article)*

# 2 **Influence of the fibre distribution and orientation in** 3 **the fracture behaviour of polyolefin fibre reinforced** 4 **concrete**

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14 **Abstract:** Polyolefin fibre reinforced concrete (PFRC) has become an attractive alternative to steel  
15 for the reinforcement of concrete elements mainly due to its chemical stability and the residual  
16 strengths that can be reached with lower weights. The use of polyolefin fibres can meet the  
17 requirements in the standards, although the main constitutive relations are based on the experience  
18 with steel fibres. Therefore, the structural contributions of the fibres should be assessed by inverse  
19 analysis. In this study, the fibre dosage has been fixed at 6kg/m<sup>3</sup> and both self-compacting concrete  
20 and conventional concrete have been used to compare the influence of the positioning of the fibres.  
21 An idealized homogeneous distribution of the fibres with such fibres crossing from side to side of  
22 the specimen has been added to self-compacting concrete. The experimental results of three-point  
23 bending tests on notched specimens have been reproduced by using the cohesive crack approach.  
24 Hence, the constitutive relations were found. The significance of this research relies on the  
25 verification of the formulations found to build the constitutive relations. Moreover, with these  
26 results it is possible to establish the higher threshold of the performance of PFRC and the difficulties  
27 of limiting the first unloading branch typical of fracture tests of PFRC.

28 **Keywords:** fibre reinforced concrete; polyolefin fibres; fibre distribution; fracture behaviour;  
29 structural fibres

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## 31 **1. Introduction**

32 Reinforced concrete was the most relevant construction material employed both in architecture  
33 and civil engineering during the XX century. The widespread use of such material was widened in  
34 the second half of the century when fibres began to be added to common concrete formulations. Since  
35 that moment, the applications of fibre-reinforced concrete (FRC) have been drastically enlarged due  
36 to the development of a significant variety of fibres produced by combining materials, sizes and  
37 shapes [1-4]. The improvement of the concrete properties that the fibres induce has enabled a large  
38 amount of uses such as cracking control, fire spalling prevention, and multifunctional concretes [5-7]  
39 that enable applications such as guiding vehicles or heating pavements. In some of these uses, the  
40 metallic nature of steel fibres, which are the most common, might be an issue due to their potential  
41 corrodible and magnetic nature. In order to address such situations, certain types of polymeric fibres,  
42 which can be considered as structural ones, have recently been developed. Those fibres, essentially  
43 polyolefin-based macro fibres, provide structural capacities with lower dosages in terms of weight.  
44 In addition, polyolefin fibres (PF) are chemically stable and reduce the overall cost of the material.

45 Polyolefin fibre reinforced concrete (PFRC) has been the centre of relevant research in recent years,  
46 especially focussed on the characterization of the mechanical properties [8-13] and engineering  
47 applications [14, 15].

48 Fibres are added in the final stages of FRC production. Pouring methods, formwork geometry  
49 and rheological properties of concrete add variations in the positioning and orientation of the fibres,  
50 as examined in references [16, 17, 18]. Such parameters have been studied through employing a wide  
51 variety of means, for example, use of stereological tools [19], statistical ones [20] and even rheological  
52 analysis [21, 22]. Most of the studies that deal with this matter have counted the amount of fibres  
53 placed in a sawn surface or in the fracture surfaces generated in bending fracture tests [23, 24, 25]. In  
54 such cases, the orientation factor proposed by Krenchel in 1975 [26] enables the use of a parameter  
55 that offers a coupled value of the orientation and the distribution of the fibres. With the use of the  
56 orientation factor  $\theta$ , it has been possible to determine the improvements in mechanical properties of  
57 SFRC by using self-compacting concrete (SCC) [27] and the effects of several types of vibration on  
58 conventional concrete (VCC) [23]. Nevertheless, at the time of writing there is a gap in experimental  
59 data and analytical models for predicting the upper threshold of the improvement that a certain  
60 amount of polyolefin fibres, if ideally oriented and distributed, can provide to a certain type of  
61 concrete.

62 The main improvement that fibres provide to concrete appears after concrete cracking has taken  
63 place. Consequently, such post-cracking behaviour has become the reference property, with several  
64 tests and recommendations being established [28-34] in order to enable a trustworthy comparison  
65 among the possible combinations of concrete and fibres. If the results comply with certain  
66 requirements established in the aforementioned recommendations, the contribution of the fibres  
67 added to the composite material might be taken into account. Therefore, it would be possible to  
68 reduce the amount of traditional steel rebars in reinforced concrete. Although this procedure is  
69 currently used, the development of predictive models that may allow the prediction of the number  
70 and positioning of the fibres placed in the critical sections of a structural piece is welcomed [35, 36,  
71 27, 37, 38, 39]. Such models might be a valuable tool for structural designers; they would also entail  
72 merging the laboratory test results and the everyday structural calculations.

73 Following this rationale, it would be even more significant to reproduce the fracture behaviour  
74 of FRC, and especially PFRC due to the novelty of the material, by means of numerical simulations.  
75 This study adopts the cohesive crack approach. Since the fracture behaviour of PFRC shows  
76 hardening tri-linear curves have been used. Such a type of curves has been used for SFRC in cases of  
77 intermediate branches with soft-unloading or even flat stages [40, 41]. However, the need for a branch  
78 with recharging strength has recently entailed an accurate reproduction of the mechanical behaviour  
79 of PFRC [42, 43]. Such a contribution was able to predict the mechanical response of a PFRC with a  
80 determined amount of fibres by changing the parameters that define a tri-linear softening function.  
81 The variations of such parameters were performed based on several assumptions and performing an  
82 inverse analysis. By combining the parameters obtained in the iterative analysis with a numerical  
83 regression, the functions that define the parameters of the softening functions were defined.  
84 However, such assumptions and the functions that were deduced have not been corroborated yet.

85 In order to address such shortcomings in the field of PFRC, an experimental study has been  
86 performed with self-compacting concrete and conventional vibrated concrete with a fibre dosage of  
87  $6\text{kg/m}^3$  of 60mm-long polyolefin fibres randomly distributed and named SCC6-60 and VCC6-60  
88 respectively. Additionally, self-compacting concrete (SCC) with the same amount of fibres ideally  
89 positioned with a homogeneous distribution in the cross section of the concrete elements was  
90 manufactured and termed SCC6-430. The reinforcement capacity of the fibres added has been  
91 assessed by comparing the behaviour of the material when subjected to three-point bending fracture  
92 tests for the three compositions manufactured. In addition, the influence of the orientation factor has  
93 been determined by counting the fibres placed in the fracture surfaces generated. Moreover, the test  
94 results, including those obtained in the specimens with an ideal homogeneous distribution of fibres,  
95 have been reproduced by using the constitutive fracture model shown in [42]. The changes added to  
96 the parameters that define the softening function have been compared for the formulations and the

97 assumptions performed and the functions that define such variations analysed. The significance of  
 98 this research lies in the definition of the threshold of the mechanical reinforcement of PFRC when the  
 99 fibres are ideally distributed. In addition, the importance of the distribution of the fibres would be  
 100 found when comparing the post-peak behaviour of SCC6-60 and the SCC6-60 with the behaviour of  
 101 SCC6-430. Moreover, the assumptions made in references [8, 17] that established a relation between  
 102 the maximum post-peak load ( $L_{REM}$ ) and the number of fibres in the fracture surface generated are  
 103 validated. In addition, the relation between the minimum post-cracking load and the amount of fibres  
 104 in the lower third of the fracture surface generate is also validated. Besides, the numerical simulations  
 105 performed would validate the hypothesis assumed in the literature and enable a confident use of the  
 106 functions proposed. Thus, the accuracy of the numerical simulations and the inverse analysis can  
 107 improve future modelling procedures and supply reliable constitutive relations for PFRC related  
 108 with physical parameters.  
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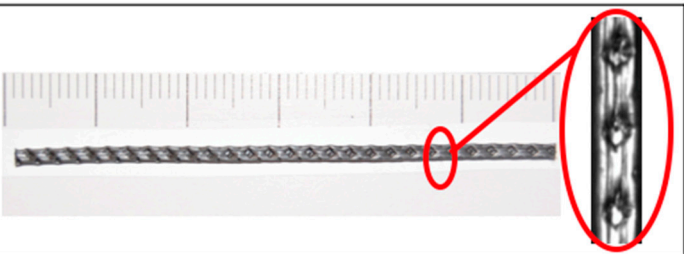
## 110 2. Concrete production

111 Both vibrated concrete (VC) and SCC were manufactured with siliceous aggregates composed  
 112 of two types of gravel of 4-8 mm and 4-12 mm and sand of 0-2 mm. The maximum aggregate size  
 113 was 12.7 mm. A Portland cement type EN 197-1 CEM I 52.5 R-SR 5 [44] was used in this study. A  
 114 mineral admixture of limestone powder as micro-aggregate was employed with specific gravity and  
 115 Blaine surface of 2700 kg/m<sup>3</sup> and 400-450 m<sup>2</sup>/kg, respectively. The calcium carbonate content of the  
 116 limestone powder was higher than 98%, with less than 0.05% being retained in the 45 µm sieve. In  
 117 the SCC and in the VC alike, an admixture named Sika Viscocrete 5720 (a polycarboxylate-based  
 118 superplasticizer with a solid content of 36% and 1090 kg/m<sup>3</sup> density) was employed. The formulations  
 119 of VC and SCC can be seen in Table 1.

120 **Table 1.** Concrete formulation per m<sup>3</sup>.

	Cement	Limestone powder	Water	Sand	Gravel	Grit	Superplasticizer (% cement weight)
SCC6-60 and SCC6-430	375	200	187.5	918	245	367	1.25
VCC	375	100	187.5	916	300	450	0.75

121 In this study, the polyolefin fibres employed boasted two different lengths. Firstly, 60mm-long  
 122 commercial fibres; and secondly, the same fibres but 600mm-long. Both fibres had the same  
 123 mechanical and chemical properties and only differed in their length. The outlook of the 60mm long  
 124 fibres where their rough surfaces appear can be seen in Figure 1. Likewise, the mechanical properties  
 125 supplied by the manufacturer can be seen in such an illustration.  
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Density (g/cm <sup>3</sup> )	0.910	
Length (mm)	60	
Eq. diameter. (mm)	0.903	
Tensile strength (MPa)	> 500	
Modulus of elasticity (GPa)	> 9	

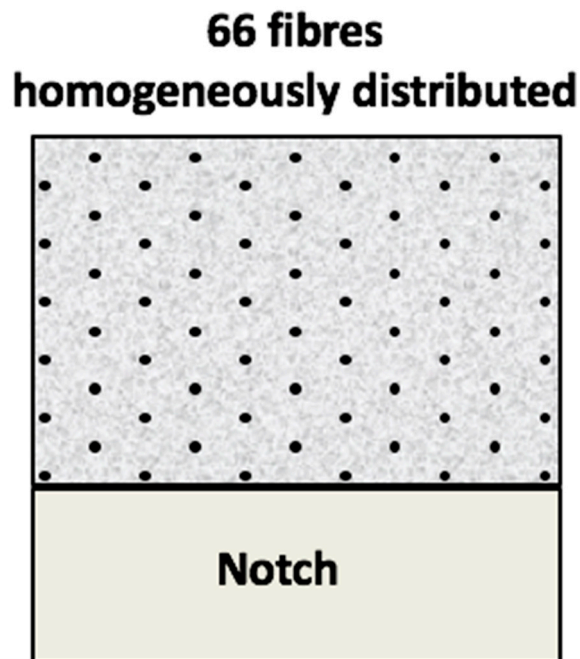
127 **Figure 1.** Commercial polyolefin fibre. Scale in mm. Mechanical properties supplied by the  
 128 manufacturer.  
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130 With the aforementioned materials, three types of concrete were prepared. The first two were a  
 131 vibrated conventional concrete and the second one a self-compacting concrete. In these two concretes,  
 132 6kg/m<sup>3</sup> of the 60mm-long polyolefin commercial fibres previously described were used. In addition

133 to VCC6-60 and SCC6-60, other specimens were manufactured with the same volumetric fraction of  
134 fibres but choosing the position and orientation of the fibres. This batch was named SCC6-430. In  
135 SCC6-60 a standard mixing procedure was used. The fibres were added in three stages and  
136 consequently were mixed with the rest of components, leading to a random positioning of the fibres  
137 within the bulk. Once the VCC6 concrete was prepared, the material was poured in  $430 \times 100 \times 100 \text{ mm}^3$   
138 moulds following the recommendation [30, 32]. In the case of the SCC6-60, the fresh material was  
139 poured from one of the sides of the mould to allow the fibres to align in the direction of the flow.

140 In the case of the SCC6-430, the fibres were positioned by choosing a homogeneous distribution  
141 and a perpendicular angle between the cross section and the fibres. The amount of fibres positioned  
142 in the specimens was selected by taking into account the volumetric fraction that entails a  $6 \text{ kg/m}^3$   
143 addition of fibres being homogeneously distributed in the ligament of the cross section.  
144 Consequently, as the volumetric fraction that corresponds to such addition is 0.66%, the amount of  
145 fibres in a  $100 \times 100 \text{ mm}^2$  section is 66. The disposition of the fibres in the middle section of the specimens  
146 can be seen in Figure 2. No fibres were positioned in the bottom third of the specimen because such  
147 a location coincided with the position and depth of the notch. The self-compacting concrete was  
148 mixed without adding any fibre and poured in the moulds without moving or causing any damage  
149 to the long fibres positioned. The four-step process can be seen in Figure 3.

150 In addition to the prismatic specimens, nine cylindrical specimens were prepared with 150mm  
151 diameter and 300mm height for each batch. All the specimens were stored in a climatic chamber at  
152  $20 \pm 2^\circ \text{C}$  and above 95% of humidity until the time of testing.



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**Figure 2.** Cross section of the specimens performed with a homogeneous distribution of fibres.



**Figure 3.** (a) Positioning of the long fibres in the moulds; (b) fixing the mould sides to avoid any damage in the fibres while concrete in fresh state; (c) Pouring of self-compacting concrete; (d) final appearance of the specimens.

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### 159 3. Material characterisation

160 The formulations were tested in order to obtain their main mechanical properties, such as  
161 compressive strength, indirect tensile strength and modulus of elasticity. Such tests were performed  
162 according to the following recommendations: EN 12390-3:2009 (compressive strength), EN 12390-  
163 6:2009 (indirect tensile strength) and EN 12390-13 (modulus of elasticity). Table 2 shows the  
164 mechanical properties of all the formulations performed.

165 **Table 2.** Mechanical properties of the concrete formulations.

	VCC6-60	SCC6-60	SCC6-430
Compressive strength, $f_{ck}$ (MPa)	32.9	41.4	39.0
Modulus of elasticity, $E$ (GPa)	29.9	31.6	36.0
Tensile strength, $f_{ct}$ (MPa)	3.75	4.09	3.80
Fracture energy*, $G_F$ (N/m)	131	130	130

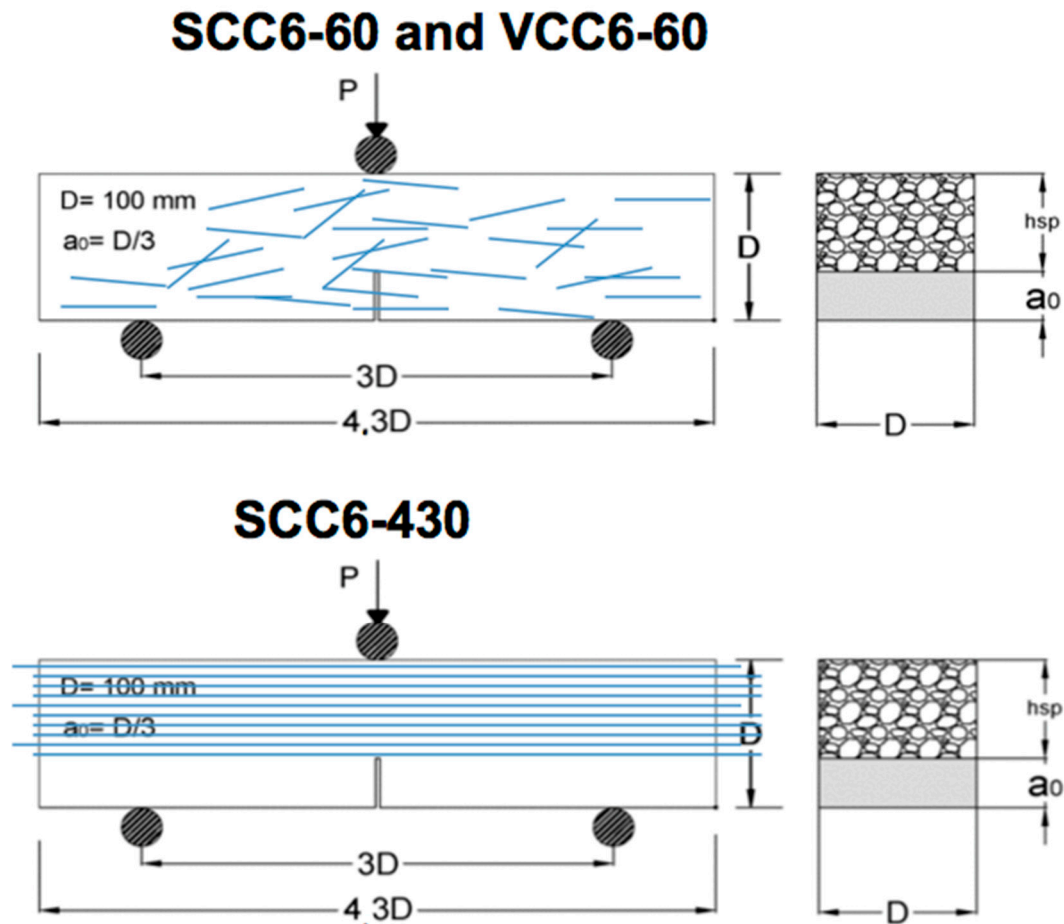
\*Values related with its correspondent plain concrete in reference [45]

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The mechanical properties shown in Table 2 show slight differences among the concrete types used. Although it can be clearly seen that the changes in the aggregate proportions and the presence of fibres caused variations in the mechanical properties, it could also be argued that such changes are of minor importance in the subject studied in this paper. That is to say, the principal effects of the

171 orientation of the fibres appear in the post-peak region when the contribution of the concrete bulk  
172 material to the mechanical response of the composite material can be considered negligible.

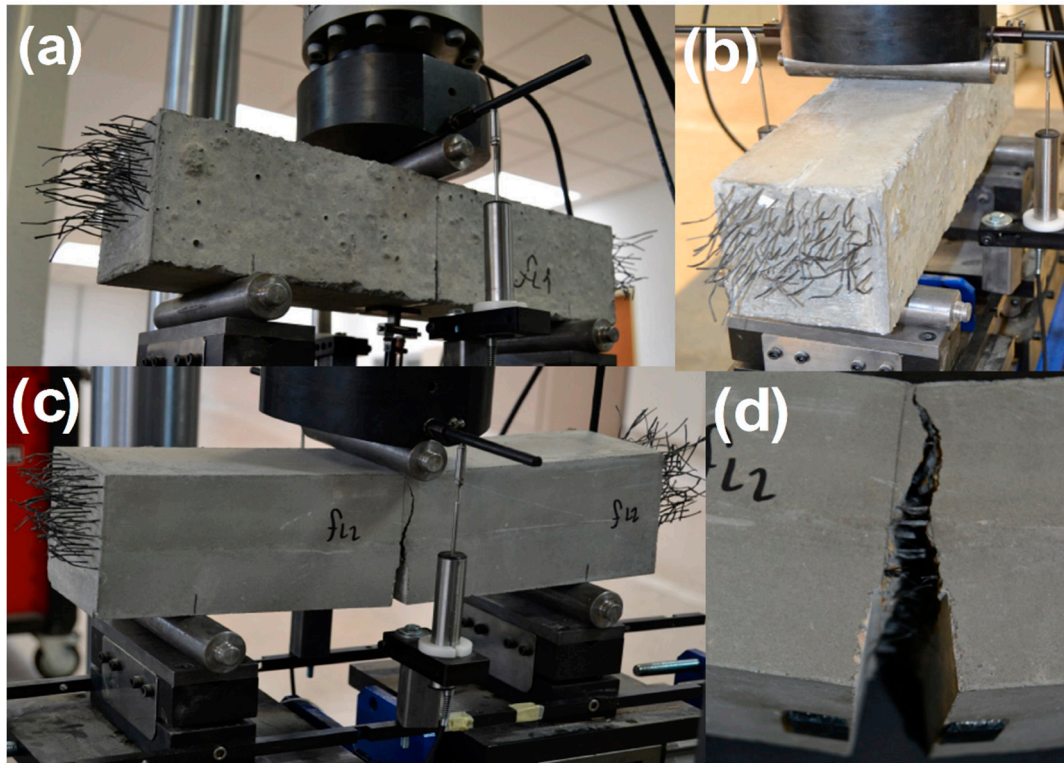
173 The fracture tests were carried out in accordance with RILEM TC-187 SOC [46] by using three  
174 prismatic specimens for each formulation. The setup of the three-point bending test, according to the  
175 standard noted above, was made in accordance with the depth of the beam. A span-to-depth ratio of  
176 3.0 and a notch in the centre of the span of 1/3 of the depth were chosen with the loading cylinder  
177 above the notch. The test geometry setup is shown in Figure 4.



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**Figure 4.** (a) Test geometry in the case of fibres randomly positioned; (b) test geometry in the case of fibres being aligned in the stress direction and homogeneously distributed.

181 The test was performed with crack-mouth opening displacement control (CMOD) by using a  
182 clip-on gauge device. Two more extensometers (linear variable differential transformer (LVDT)  
183 devices) were also placed on each side of the specimen to measure the deflection. During the test,  
184 time, load, deflection, CMOD and also the machine actuator position were recorded. Such a setup  
185 and procedure was used for testing the specimens of all formulations. An image of the testing  
186 procedure applied to a SCC6-430 specimen can be seen in Figure 5. The notch was machined with a  
187 water-cooled low-speed diamond cutting disc. The specimen positioning was carefully made by  
188 means of laser devices. The concrete beams rested on two rigid steel cylinders laying on two ground  
189 supports, which allowed free rotation out of the plane of the beam and guaranteed negligible friction  
190 rolling in the longitudinal direction of the beam. Thus, the results of the fracture tests on two beams  
191 showed a remarkably low degree of scatter. The latter is also supported by previous works in  
192 references [8, 42, 43].



**Figure 5.** (a) Specimen ready for the test; (b) test disposition in the testing machine; (c) crack fully developed while testing; (d) image of the fracture surface generated in the fracture test with the fibres bridging the crack sides.

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#### 197 4. Fracture tests results

198 Regardless of the fibre rupture, fibre bridging, fibre pull-out, fibre debonding or matrix cracking,  
199 these mechanisms prompted much higher deformations in all the specimens tested than in concrete  
200 and, consequently, the upper bound of the CMOD device was exceeded. In order to continue the test,  
201 when the upper bound of the referred device was reached the control of the fracture test was changed  
202 to the machine actuator position. With the latter it was possible to obtain deflection values up to 15  
203 mm. Nonetheless, all the tests were stopped without reaching the collapse of the specimens.

204 The curves obtained from the tests shown in Figure 6 could be divided in three distinct trends  
205 that can be easily identified. All can be defined by changes in the load-bearing capacity of the  
206 material. The first one begins at the start of the test and ends at the load at the limit of proportionality.  
207 Once the deflection at this point has been surpassed, an unloading branch can be perceived. The  
208 shape of such an unloading branch of the curves resembles the analogue part of the softening curve  
209 of conventional concrete. The unloading takes place until fibres are capable of absorbing the energy  
210 released in the fracture processes. The fibres added to the concrete were able to sustain higher loads  
211 and, consequently, the load-deflection curve showed a positive slope after the minimum post-peak  
212 load. Such a slope was constant from a certain deflection and afterwards, as certain damage  
213 mechanisms appeared, it decreased steadily until the maximum post-cracking load was reached.  
214 After reaching the maximum post-peak load, the load bearing capacity of the material decreases  
215 progressively due to the continuation of the damage in the material. Consequently, the load-  
216 deflection curve showed a stable reduction of the mechanical capacity of the material until the end of  
217 the test was reached.

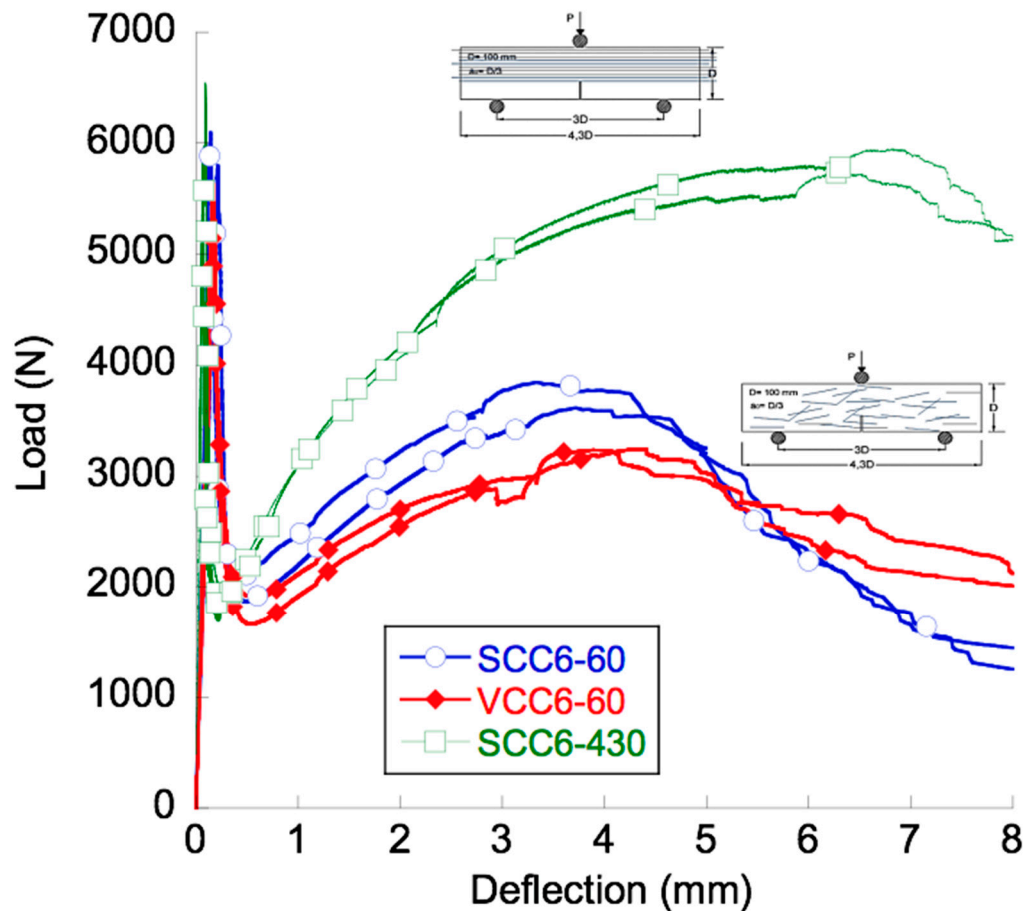


Figure 6. Three-point bending fracture test results.

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220 The aforementioned characteristics are common for the three formulations manufactured. The  
221 mechanical response of SCC6-60 was slightly better than the VCC6-60, though no major disparities  
222 were found. In addition, such differences had already been reported in previous studies [10] and, in  
223 the case of this contribution, will be analysed by means of the orientation factor in the following  
224 sections. Nevertheless, if the fracture curves of SCC6-430 are compared with their analogous VCC6-  
225 60 and SCC6-60 remarkable features should be outlined.

226 There were no changes in the peak load obtained in the fracture tests. This result is coherent  
227 because there were only slight changes in the concrete formulations used. In addition, such results  
228 show that the different pouring method employed in the SCC6-60 and SCC6-430 specimens did not  
229 have any influence. Another point that should be highlighted is that in all formulations the minimum  
230 post-peak load was similar and at around 30% of the peak load.

231 The slope of the reloading part of the curves in the case of the SCC6-430 specimens is noticeably  
232 greater. While in the case of the VCC6-60 and SCC6-60 specimens the maximum post-peak load is  
233 around 50%, when the curves of SCC6-430 are analysed it can be seen that such a point reaches almost  
234 the value of the peak load.

235 In addition, the deflection at the maximum loading capacity changes from 3.5mm to 6.0mm. It  
236 has to be emphasised that this observation was already perceived to a lesser extent in previously  
237 published literature [17, 16]. When reaching this deflection, different damaging mechanisms have  
238 appeared with dissimilar importance. In the case of the concrete formulations performed with short  
239 fibres, the following damage mechanisms appear: matrix cracking, fibre-matrix debonding, fibre  
240 pull-out and fibre rupture. However, the changes of the slope that perceived when comparing the  
241 SCC6-60 and VCC6-60 curves with the SCC6-430 curves can be attributed to the damage mechanisms  
242 that cannot appear when reinforcing with long fibres: fibre pull-out. Another point worth mentioning  
243 is the lack of scattering in the case of the SCC6-430 specimens when compared with the specimens



244 performed with short fibres. This observation would clearly explain that the inherent scattering that  
 245 appears in the fracture tests of FRC is caused by the differences in the amount of fibres and their  
 246 positioning in the fracture surface.

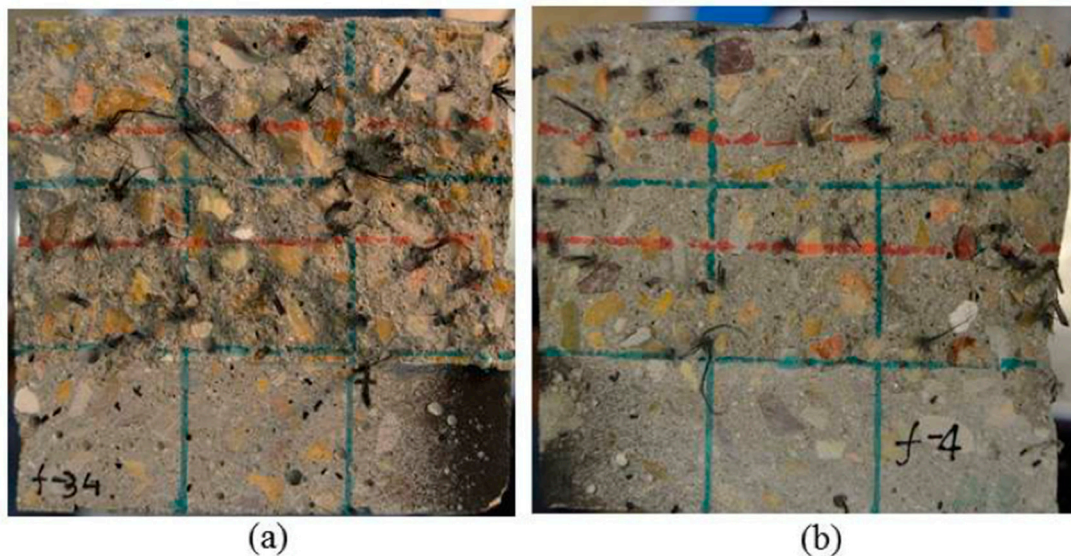
247 Analysing the final unloading part of the tests, it can be seen that the unloading process seems  
 248 to be more gradual in the case of the SCC6-430 specimens. However, a deeper study would be  
 249 required to obtain sound conclusions. In any case, as in previous studies by the same authors in  
 250 references [8, 10], the characteristic points of the experimental mean curves have been extracted in  
 251 Table 3 in order to ease their discussion.

252 **Table 3.** Minimum ( $L_{MIN}$ ) and maximum ( $L_{REM}$ ) post-cracking load and their corresponding crack  
 253 openings.

	$L_{MIN}$ (kN)	$w_{LMIM}$ (mm)	$L_{REM}$ (kN)	$w_{LREM}$ (mm)
SCC6-430	1.875	0.22	5.676	6.1
VCC6-60	1.788	0.54	3.246	4.1
SCC6-60	1.994	0.45	3.712	3.6

## 254 5. Fracture surface analysis

255 In order to quantify the importance of the orientation and distribution of the fibres, an analysis  
 256 of the fracture surfaces generated in the three-point bending tests was required. In the case of the  
 257 SCC6-430 specimens, the amount of fibres was predetermined in the manufacturing process to equal  
 258 the theoretical one, with it having a perfect distribution and positioning. However, a fibre-counting  
 259 exercise was required to assess the number of fibres in the fracture surfaces of the rest or formulations.  
 260 Figure 7 shows the appearance of two specimens of VCC6-60 and SCC6-60: the notch required to  
 261 perform the fracture test can be clearly seen.



262 **Figure 7.** Fracture surfaces generated: (a) VCC6-60;(b) SCC6-60.

264 The theoretical number of fibres placed in the fracture surface was obtained for each concrete by  
 265 using Eqs. (1) and (2), considering that the fibres were uniformly distributed and perpendicular to  
 266 the crack.

$$267 \quad V_f = \frac{W_f}{\rho \cdot V} \quad (1)$$

$$268 \quad th = \frac{A V_f}{A_f} \quad (2)$$

269 In (1) and (2)  $V_f$  is the fibre volumetric fraction,  $W_f$  the weight of the fibres for a reference volume  
 270 of 1 m<sup>3</sup>,  $\rho$  the fibre density and  $V$  the total volume. In addition,  $th$  is the theoretical number of fibres  
 271 placed in the fracture surface of a given specimen, with  $A$  being the cross section of the specimen and  
 272  $A_f$  the section of one fibre. The average total number of fibres obtained from the counting exercise  
 273 can be seen in Figure 8. Moreover, the relation between the fibres counted in a given cross-section ( $n$ )  
 274 and theoretical number of fibres ( $th$ ) are also shown in Figure 8. This relation  $\theta$  is the so-called in  
 275 previous research orientation factor [26] that assumes a homogeneous distribution of fibres in the  
 276 section and which may be seen in Eq. (3).

$$277 \quad \theta = \frac{n}{th} = n \frac{A_f}{V_f A} \quad (3)$$

278 In the case of the SCC6-430 specimens, the theoretical fibre content was homogeneously  
 279 distributed in all the specimen sections and consequently in such specimens the value of the  
 280 orientation factor is 1. The evaluation of the orientation factor was performed using the approach  
 281 aforementioned and the results are also shown in Figure 8, including the percentage of pull-out fibres.

Amount of fibres			Positioning					
SCC6-60			thirds	total fibres	SCC6-60	thirds		
11	10	6	26	78	13.6%	12.8%	7.2%	33.6%
6	4	6	15	th=99	7.7%	4.7%	7.2%	19.6%
9	2	7	18	$\theta=0.79$	11.5%	2.6%	8.5%	22.6%
19			19	pulled-out 34.1%	24.3%			24.3%
VCC6-60			thirds	total fibres	VCC6-60	thirds		
8	6	10	23	72	10.7%	7.9%	14.0%	32.6%
6	5	4	15	th=99	8.4%	6.5%	5.6%	20.5%
4	5	5	13	$\theta=0.72$	5.6%	6.5%	6.5%	18.6%
20			20	pulled-out 35.6%	28.4%			28.4%

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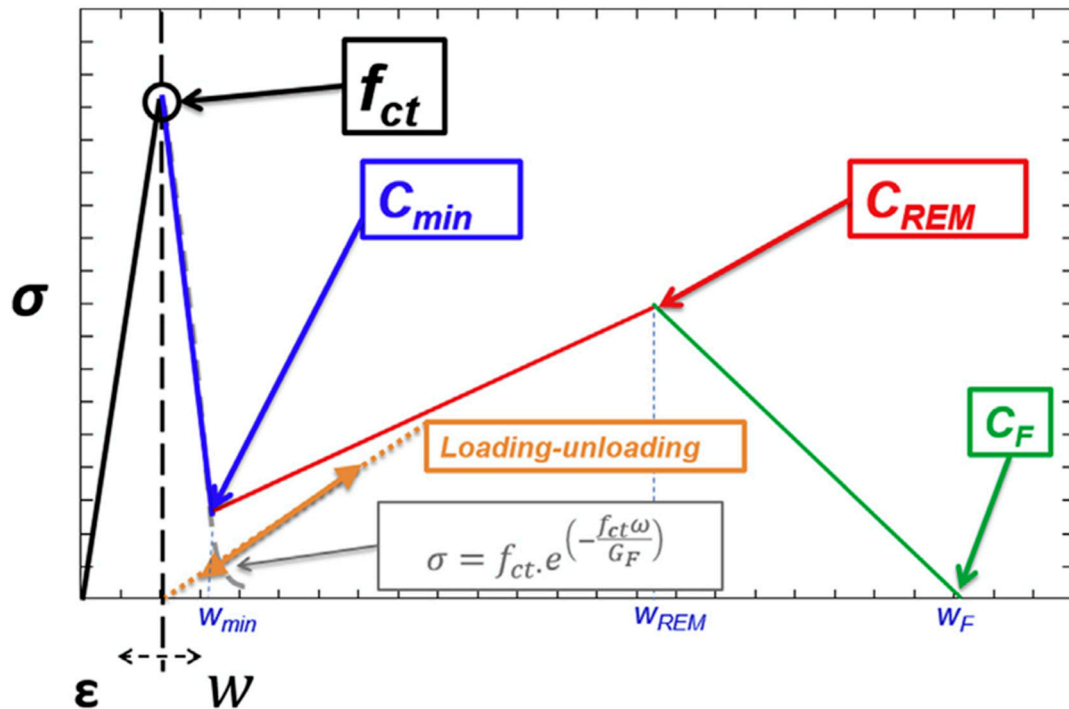
**Figure 8.** Counting exercise performed in SCC6-60 and VCC6-60 (including the percentage of pull-out fibres)

## 285 6. Numerical simulations

286 Similarly to the process followed in references [42, 43, 47], the fracture results obtained in the  
 287 experimental campaign were reproduced by means of numerical simulations. This process was  
 288 carried out by using the commercial software Abaqus through merging the features implemented in  
 289 such a code with a material user subroutine.

290 The fracture behaviour was reproduced by employing 2D numerical models meshed through  
 291 using three-node triangular elements with one Gauss point. The mechanical behaviour of the material  
 292 implemented in the UMAT was linear elastic without any damage when under compression.  
 293 Regarding the tensile behaviour, the material was linear elastic until the tensile stress was reached. If  
 294 the strain that corresponds to the tensile strength were surpassed, the mechanical response of the  
 295 material would be governed by a softening function implemented in a user material subroutine. As  
 296 in previous studies, it was shown that a tri-linear softening function was suitable for reproducing the  
 297 fracture behaviour of the material, with the same approach being used in the case of the specimens  
 298 studied in this paper. The shape and the characteristic points that define the softening function can  
 299 be seen in Figure 9.

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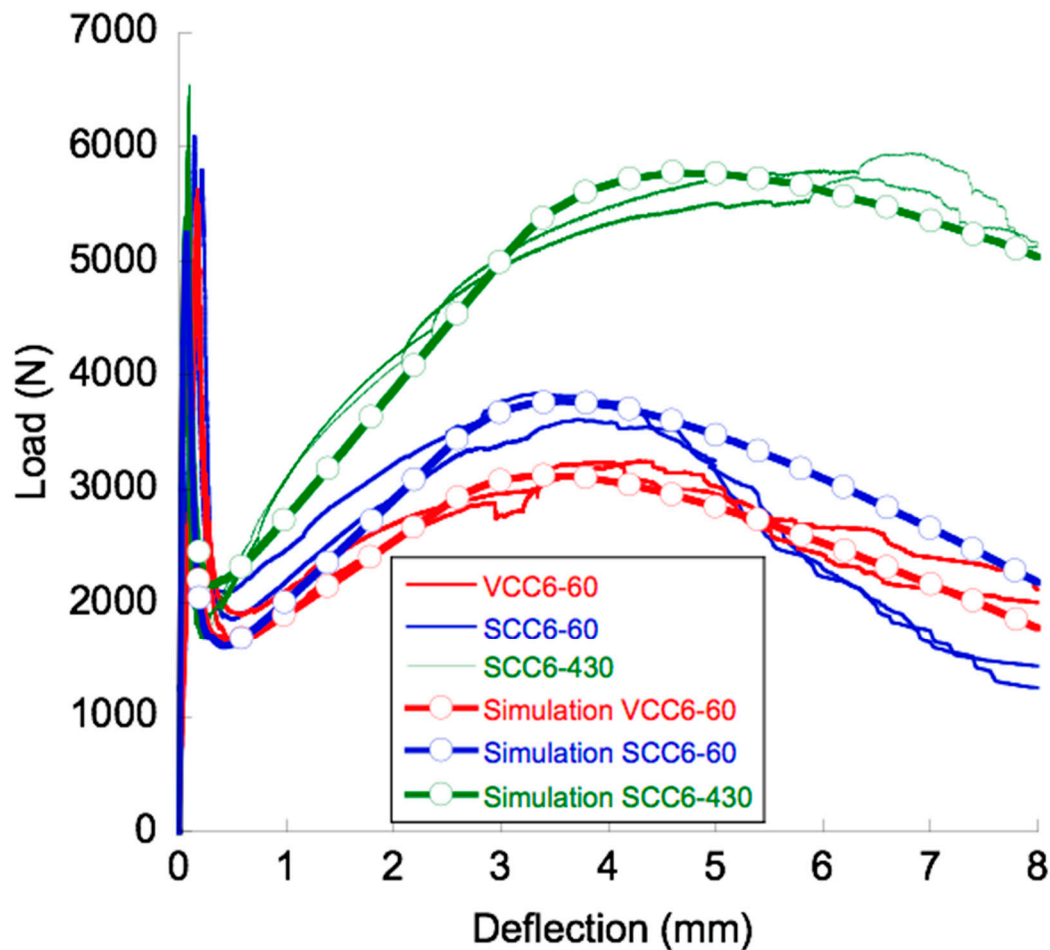
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**Figure 9.** Sketch of the shape and position of the turning points of the constitutive relations for PFRC.

303 The non-linear fracture process zone emerges in the elements placed on the crack. The behaviour  
304 of the fracturing elements depends on a constitutive relation that needs to be iteratively fit until  
305 finding the values of  $C_{MIN}$ ,  $C_{REM}$  and  $C_F$  that are able to reproduce the fracture behaviour of all the  
306 three formulations with a reasonable degree of accuracy.

307 The inverse analysis used has been explained in depth in previous papers [42]. The mechanical  
308 data of the material that was obtained in the mechanical test and used in the simulations can be seen  
309 in Table 2. For the specific fracture energy ( $G_F$ ) and Poisson coefficient of plain concrete, the values  
310 of 130 N/m and 0.2 were respectively adopted [42].

311 With the aforementioned data and by exerting the inverse analysis previously cited, it was  
312 possible to find an accurate reproduction of the experimental tests. In addition, the tri-linear softening  
313 functions were defined. In Figure 10 both numerical results and experimental results can be seen.



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**Figure 10** . Results of the numerical simulations and the experimental tests.

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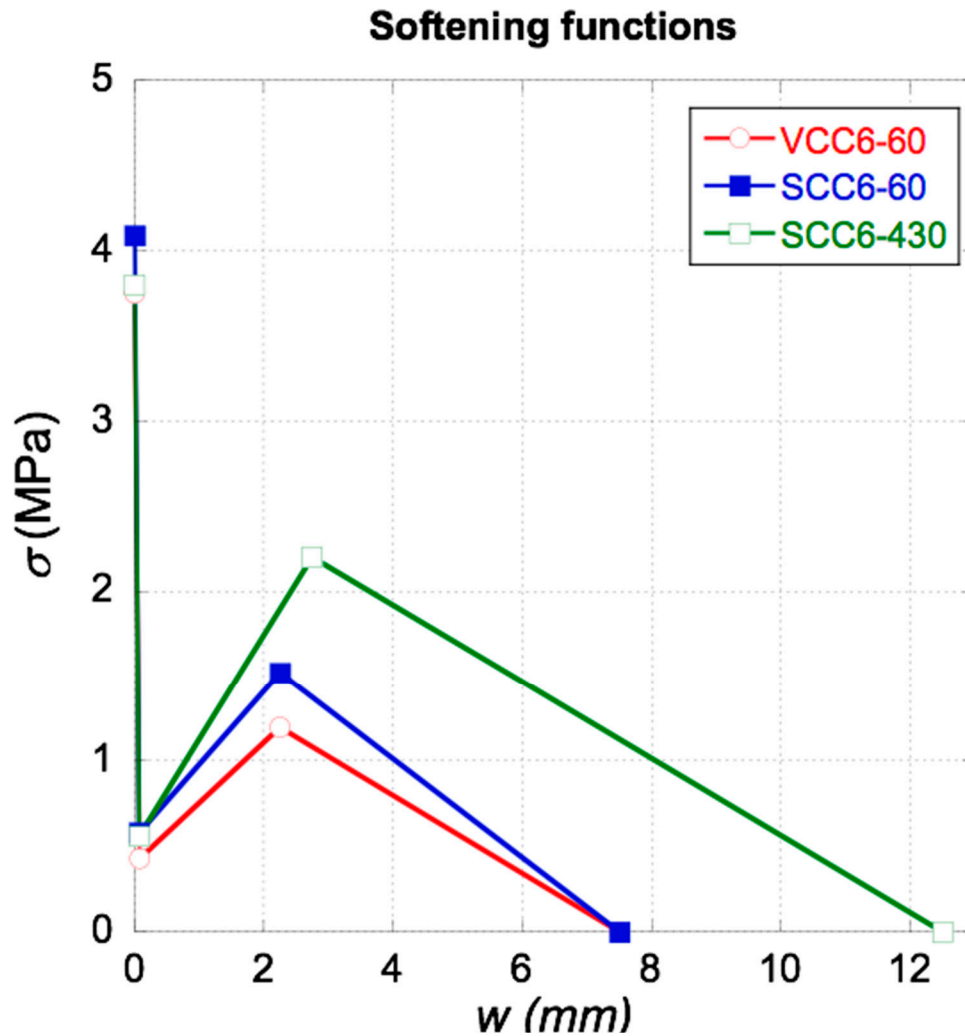
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The curves that appear in Figure 10 clearly reproduce, and with a significant degree of accuracy, the experimental results. It should be highlighted that the accuracy of the simulations in the cases of the SCC6-60 and VCC6-60 is notable before the post-peak maximum load has been reached. However, this similarity is reduced from this point onwards. The differences that appear between the experimental curves and the numerical ones are caused by the unpredictable nature of the appearance of the aforementioned damage mechanisms. This is clearly confirmed when the scattering of the experimental curves is more reduced, as in the case of the SCC6-430 specimens. It can be clearly seen that the curves obtained by the numerical simulations in this case fits the experimental results extremely well.

The reproduction of the mechanical behaviour of SCC6-60 and VCC6-60 implied changes in  $f_{ct}$ ,  $C_{MIN}$  and  $C_{REM}$ . Regarding those of  $C_{MIN}$  and  $C_{REM}$ , it can be seen in Table 4 that there were only slight changes. In the case of  $C_F$ , the value was stable for both formulations. In order to reproduce the tests of SCC6-430 specimens it was needed to change slightly the value of  $C_{MIN}$ . Nevertheless, in the case of  $C_{REM}$  not only the stress value had to be modified. The crack opening at which  $C_{REM}$  takes place changes noticeably in the specimens with the fibres homogeneously distributed and shifts from a value of 2.25 to 2.75. Similarly, the value of  $C_F$  has to be fixed at 12.5 in order to obtain an accurate reproduction of the experimental results.



334

335 **Figure 11.** Softening functions implemented336 **Table 4.** Turning points for the numerical simulation

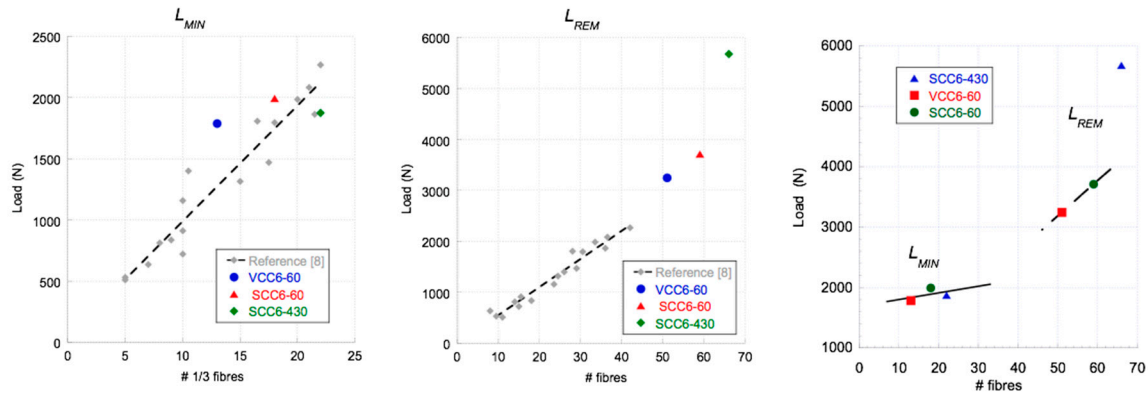
	$C_{MIN}$		$C_{REM}$		$C_F$	
	$w(mm)$	$\sigma(MPa)$	$w(mm)$	$\sigma(MPa)$	$w(mm)$	$\sigma(MPa)$
VCC6-60	0.08	0.43	2.25	1.20	7.5	0
SCC6-60	0.07	0.58	2.25	1.52	7.5	0
SCC6-430	0.07	0.56	2.75	2.20	12.5	0

337 **7. Discussion**

338 This section deals with the relevant aspects of the experimental results, connections between the  
 339 orientation and distribution of the fibres with such results and changes that were performed in the  
 340 material subroutine implementation in order to find an accurate reproduction of all the tests.

341 As could be seen in the correspondent section, the general shape of the fracture curves obtained  
 342 for the formulations were analogous. Both SCC6-60 and VCC6-60 were highly similar to each other.  
 343 Moreover, there were significant similarities among the fracture curves of SCC6-60, VCC6-60 and  
 344 SCC6-430. Although this can be seen in Figure 6, there are several aspects that should be underlined.  
 345 Firstly, in all formulations the minimum post-cracking loads were similar. This feature is of primary  
 346 importance because it clearly states that the bearing capacity for small deformations does not depend  
 347 on the positioning or orientation of the fibres because the orientation factor of the VCC6-60 specimens  
 348 on average is 0.72 and in the case of SCC6-60 is 0.78 which are noticeably smaller to the unity that the

349 SCC6-430 specimens boast. In addition, as can be seen in Figure 8 and Figure 12, although the  
 350 coefficient of variation is 0.25 between the amounts of fibres in the lower third of the fracture surfaces  
 351 generated, there are no remarkable changes of the value of  $L_{MIN}$  recorded. In addition, this tendency  
 352 fits the previous data available in literature. Therefore, as stated in reference [42] such a load value is  
 353 mainly influenced by the amount of fibres added and the rest of parameters that might be considered  
 354 to have only a limited impact.



355

356 **Figure 12.** Relation among  $L_{MIN}$  and  $L_{REM}$  with the number of fibres in the ligament.

357 Another point that should be highlighted is that the slope of the reloading part of the curves is  
 358 noticeably greater in the case of the SCC6-430 specimens. The slope of the SCC6-430 formulation in  
 359 this part of the experimental curve is 181% higher than in the case of the VCC6-60 formulation and  
 360 156% greater than in the case of the SCC6-60. However, there was no clear relation between such  
 361 increments and the difference in the presence of fibres in any of the combinations obtained by adding  
 362 the amount of fibres in the thirds of the ligament surface.

363 Regarding the maximum post peak value,  $L_{REM}$ , it should be underlined that the experimental  
 364 values obtained in the curves of the VCC6-60 and SCC6-60 specimens are above the values obtained  
 365 in literature. The linear tendency that relates the amount of fibres and  $L_{REM}$  seems to be valid.  
 366 Conversely, the value obtained in the case of the specimens manufactured with the 430mm-long  
 367 fibres is clearly above the values or even the predictions that might be applicable to all the  
 368 formulations performed with short fibres of the same characteristics. It could be argued that there  
 369 were a greater number of fibres in the ligament of the specimen and thus the load that the specimens  
 370 were able to sustain is also greater. Following this rationale, in the whole section there was an amount  
 371 of fibres 128% and 111% greater in the case of SCC6-430 when compared with VCC6-60 and SCC6-60  
 372 respectively. However, if the values of the  $L_{REM}$  obtained for SCC6-430 is multiplied by the coefficient  
 373 of orientation, the  $L_{REM}$  values of the VCC6-60 and SCC6-60 should have been 4472N and 4071N. Such  
 374 values are considerably higher than those obtained. Consequently,  $L_{REM}$  values achieved by the  
 375 formulations performed with 60mm-long fibres should show damage mechanisms which only  
 376 appear when short fibres are added. One of such mechanisms could be the pull-out of the fibres  
 377 (see reference [48]) that play a major role in the composite material behavior. Therefore, it is important  
 378 to highlight that not only the orientation factor reduces the load bearing capacity of the specimen in  
 379 VCC6-60 and SCC6-60 formulations.

380 In the case of the unloading branch of the curves, it can be seen that the unloading process is  
 381 steadier in the case of the SCC6-430 specimens. While in the cases of SCC6-60 and VCC6-60 the  
 382 decrement of load bearing capacity is noticeable from  $L_{REM}$  onwards, in the case of the VCC6-430  
 383 specimens it seems that this process is slower. It should be remembered that in the VCC6-430  
 384 specimens there is no possible pull-out of the fibres while this damage mechanism appears in a  
 385 certain amount of fibres present in the fracture surface of SCC6-60 and VCC6-60 specimens

386 Regarding the numerical analysis performed, it can be concluded that only by applying minor  
 387 changes in the tri-linear softening function found in the literature [42] it was possible to reproduce  
 388 both the experimental results of VCC6-60 and SCC6-60 specimens. However, several changes had to

389 be applied in order to simulate the behaviour of SCC6-430 specimens. While in the cases of the VCC6-  
 390 60 and SCC6-60 specimens the changes of the values of  $C_{REM}$  were limited to the strain value, in the  
 391 case of the SCC6-430 specimens it was necessary to change also the crack width where the  $C_{REM}$  takes  
 392 place. Moreover, the value at which  $C_F$  takes place had to be noteworthy modified. This is in  
 393 accordance with what was proposed in previous studies [11, 42, 43].

394 Concerning the functions derived in [11, 42], the inverse analysis performed in this contribution  
 395 enabled checking of the validity of the equations deduced. If expression (1) is considered, it can be  
 396 seen that the only parameter of the material is the volumetric fraction and, consequently, all the  
 397 predictions of the  $C_{MIN}$  value were unique for all formulations. In Table 6 the values obtained by  
 398 means of the combination of the inverse analysis and the numerical simulations are shown. As can  
 399 be seen in such a table, the predicted value through using equation (1) provides an accurate fit of  $C_{MIN}$   
 400 that corresponds to VCC6-60.

$$401 \quad \Phi = -3.6046 + 5.0625(1 - e^{6.55Vf}) \quad (4)$$

402 However, this is not the case of SCC6-60 or even SCC6-430. If the  $C_{MIN}$  value is considered as  
 403 correct, the volumetric fraction could be deduced. In the case of the two formulations, such a  
 404 volumetric fraction deduced corresponds to a PFRC formulation with 10 kg/m<sup>3</sup> addition of fibres.  
 405 This value could be explained if the homogeneous, and artificially prepared, disposition of the fibres  
 406 were taken into account. Nevertheless, such an argument cannot be applied to the SCC6-60 result  
 407 where an equal volumetric fraction of 1% is obtained. Such a remarkable variation is not only caused  
 408 by the number of fibres in the lower third of the ligament, but also by the positioning of the fibres  
 409 and the improvements obtained in the distribution of the fibres due to the flux of self-compacting  
 410 concrete [16]. In such a lower third, the average position of the fibres was not in the centre of gravity  
 411 of such an area but closer to the tip of the notch. Consequently the contribution of the fibres to  
 412 regaining the load-bearing capacity was greater than in a normal situation because in average their  
 413 strain was greater as it was the strain bore by them.

414 Considering the value of  $C_{REM}$  that can be obtained by means of expression (2), two conditions  
 415 should be pointed out. While in the case of VCC6-60 and SCC6-60 the amount of pulled-out fibres  
 416 and the distribution of them had to be found, with both formulations being close to 20% and 0.72 and  
 417 0.79 respectively, in the case of the SCC6-430 specimens the value of fibres pulled out is zero while  
 418 the value of  $\theta$  is 1. By applying these conditions to equation (2), the value of  $C_{REM}$  for all the  
 419 formulations can be seen in Table 5. In the case of SCC6-60, there is an accurate prediction of the  
 420 values used. However, in the case of the other two formulations there was a deviation of up to 18%.

$$421 \quad \sigma_{CREM} = (1 - \% \text{ pulled - out}) Vf \theta \sigma_u \quad (5)$$

422 **Table 5.** Comparison of the predicted values in references [11, 42] and those obtained in this study by  
 423 inverse analysis.

	$C_{MIN}$			$C_{REM}$		
	inverse analysis	predicted by [42]	$\Delta$	inverse analysis	predicted by [42]	$\Delta$
VCC6-60	0.43	0.43	0.00%	1.20	1.42	-18.33%
SCC6-60	0.58	0.43	25.85%	1.52	1.57	-3.29%
SCC6-430	0.56	0.43	23.21%	2.20	2.48	-12.73%

424 Another point that should be highlighted is that with longer fibres the deflection where  $C_{REM}$   
 425 occurs changes. Such an idea was suggested when comparing the fracture test results shown in  
 426 reference [16] but, due to the reduced difference in the fibre length, it was difficult to perceive.  
 427 Nevertheless, when using 430mm-long fibres it has been easy to confirm this phenomenon. Even  
 428 when there was a reflection in the softening function implemented. There was swift shift of 0.5mm  
 429 in the case of the crack opening of the  $C_{REM}$  and in the case of  $C_F$  there was a 5mm shift. Consequently,  
 430 it can be stated that the longer the fibres the greater is the value of  $C_F$  that should be implemented.  
 431

## 432 8. Conclusions

433 The failure mechanisms prompted when PFRC is subjected to a fracture test are various and  
434 complex. Such ones are fibre rupture, fibre bridging, fibre pull-out, fibre debonding or matrix  
435 cracking. Given that some of them appear at the same strain states, they compete which means that  
436 it is difficult to separate the influence of each of them. As it is hard to isolate the effect of each  
437 mechanism, the question of what would be the mechanical threshold of the reinforcement provided  
438 by the polyolefin fibres remained unsolved. This threshold is influenced by two intrinsic  
439 characteristic of the fibre reinforced concrete: the fibre distribution and the fibre orientation. In order  
440 to assess the optimum reinforcement that a certain amount of polyolefin fibres can provide, a series  
441 of tests was performed in specimens reinforced with 430mm-long polyolefin fibres perfectly  
442 distributed and oriented. The test results showed that if fibres are not pulled out, and they are  
443 homogeneously distributed, the mechanical capacity of the fibres added enables reaching stress  
444 values close to those obtained in the limit of proportionality which is also the peak load. It is also  
445 worth underlining that with such changes the performance of only 6kg/m<sup>3</sup> of fibres was even superior  
446 to a formulation with a 10kg/m<sup>3</sup> addition of short fibres.

447 The analysis of the fracture tests performed in the VCC6-60, SCC6-60 and SCC6-430 formulations  
448 showed that the  $L_{MIN}$  was essentially related with the amount of fibres present in the lower third of  
449 the ligament, with the uniform distribution of fibres within this third or even the length of the fibres  
450 not being of significant importance. However, when  $L_{REM}$  is considered, the behaviour of the material  
451 was greatly influenced by the fibre length due to the damage mechanisms that appear. As all the  
452 430mm-long fibres break during the test, the amount of load that such fibres are capable of sustaining  
453 and, therefore, the amount of energy that they absorb is noticeably greater than in a standard  
454 situation. Such differences were caused by the 20% of fibres which were pulled out from the concrete  
455 matrix and the changes in the fibres positioning that appeared in VCC6-60 and SCC6-60 formulations.

456 Regarding the numerical simulations, the fracture tests of the formulations were reproduced and  
457 provided a remarkable degree of accuracy. The applicability of such an approach to the SCC6-430  
458 formulation showed the robustness and versatility of the cohesive crack approach and the accuracy  
459 of the inverse analysis performed. In addition, the expressions proposed in [11, 42] have been applied  
460 to the tests conducted in this contribution with relative success. It should be noted that as the  
461 expressions had been obtained based only on a reduced amount of data the values of  $C_{MIN}$  and  $C_{REM}$   
462 proposed were not accurate. However, if the aforementioned argument is considered, the variations  
463 of around of 25% that were assessed might be considered within the typical scattering of FRC, with  
464 them being reasonably correct and their physical meaning being considered adequate.

465 Lastly, the results of both the numerical simulations and experimental results have provided  
466 valuable discussion and verification of the parameters and expressions proposed in previous research  
467 as regards the influence of the coefficient of orientation. The importance of the distribution and  
468 orientation of fibres has been clearly stated, as well as the need of models and formulations that help  
469 structural designers to take into consideration this type of predictive tools and numerical results.

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