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Article

Closing the Door on Quantum Nonlocality

Marian Kupczynski*

Département de l'Informatique, UQO, Case postale 1250, succursale Hull, Gatineau. QC, Canada J8X 3X7
marian.kupczynski@uqo.ca
* Correspondence: marian.kupczynski@uqo.ca; Tel.: +1-819-777-4493

Abstract: Bell type inequalities are proven using oversimplified probabilistic models and/or counterfactual definiteness (CFD). If setting-dependent variables describing measuring instruments are correctly introduced none of these inequalities may be proven. In spite of this a belief in a mysterious quantum nonlocality is not fading. Computer simulations of Bell tests allow studying different scenarios how the experimental data might have been created. They allow also to generate outcomes of various counterfactual experiments such as repeated or simultaneous measurements performed in different settings on the same '' photon-pair" etc. They allow reinforcing or relaxing CFD- compliance and /or to study the impact of various "photon identification procedures" mimicking those used in real experiments. Using a specific setting- dependent identification procedure data samples consistent with quantum predictions may be generated. It reflects an active role of instruments during the measurement process. Each setting dependent data samples are consistent with specific setting –dependent probabilistic models which may not be deduced using non-contextual local realistic or stochastic hidden variables. In this paper we discuss the results of these simulations. Since the data samples are generated in a locally causal way, these simulations provide additional strong arguments for closing the door on quantum nonlocality

Keywords: Bell-inequalities; quantum nonlocality; computer simulations of Bell tests; local causality; contextuality loophole; photon identification loophole.

1. Introduction

In classical physics we are used to think *that physical observables characterizing a state of a physical system have definite values even if they are not measured*. This assumption is called sometimes counterfactual definiteness (CFD).

When we look at a table we may define its different attributes: its size, its dimensions, its weight, type of material it is made of etc. It is natural to think that definite values of these attributes do exist, if we do not measure them and that by making more and more precise measurements we may estimate these definite values with an arbitrary precision. Even in classical physics many of these attributes are relative to the experimental contexts such a temperature, a place on the Earth, an ambient light etc. Nevertheless we may still believe that the attributes have definite values in each particular fixed environmental context.

According to CFD we may describe ensembles of physical systems using joint probability distributions of several physical variables even, if we may not measure them. CFD was applied with success in statistical mechanics but failed to explain the motion of electrons in atoms and in many other quantum phenomena.

According to quantum mechanics (QM), as Peres told, unperformed experiments have no results [1]. In QM outcomes of measurements are produced and recorded after the interaction of measuring instruments with physical systems in well-defined experimental contexts [2, 3].

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The measurements of incompatible quantum observables require mutually exclusive experimental set-ups. The joint probability distribution, of the outcomes of these measurements, does not exist and probabilistic models using such counterfactual distributions fail to describe the outcomes of these measurements.

The measurements of incompatible observables provide only complementary contextual information about studied physical systems. One may say that the classical filters are selectors of pre-existing properties, in contrast to quantum filters which are creators of contextual properties. In mathematical language the lattices of classical and quantum filters are incompatible [4-7].

QM predicts strong correlations between outcomes of spin polarization measurements in an idealized EPRB experiment, performed on "photon pairs" prepared in a singlet state [8, 9]. Several spin polarization correlations experiments (SPCE) have been performed in order to verify and to confirm these predictions [10-15].

In twin-photon beam experiments, Alice and Bob may choose two settings for their respective polarization beam splitters (PBS) and study correlations between clicks detected on their detectors. Since at a given time only one pair of settings may be used, thus the results form 4 samples coming from 4, mutually exclusive, random experiments.

In 1964 John Stewart Bell constructed local realistic hidden variable model (LRHVM) in which the clicks on detectors are completely determined by some hidden variables describing each-pair of photons prepared in the spin singlet state. LRHVM is CFD-compliant because the outcomes of all spin measurements are predetermined before the actual measurements are done. Using a unique parameter space and a joint probability distribution, to describe 4 mutually exclusive random experiments, he proved his famous inequalities and showed that according to QM they should be violated for some experimental settings [16, 17].

In LRHVM one does not assume that various spin projections may be measured simultaneously, in all directions, on a given "photon pair". One only assumes that there exists a probability distribution of some hidden variables λ from which all probabilistic predictions for feasible pairs of experiments may be deduced.

Local realistic hidden variable models are not Kolmogorov probabilistic models but they are isomorphic to particular Kolmogorov models in which joint probability distributions of all possible values of spin projections are well defined [18-21].

Similar inequalities were derived using stochastic hidden variable model (SHVM) in which the experimental outcomes are produced in independent random experiments run at distant locations [17, 22].

Bell did not understand the restricted applicability of his probabilistic model and believed that the violation of Bell-type inequalities would prove, that QM violates Einsteinian locality. This opinion is shared by many of his followers.

Idealized EPRB-experiment cannot be realized in the laboratory since we cannot follow "entangled photon pairs" from the moment when they are produced to the moment when they arrive to the detectors. .

We may only record clicks on distant detectors and time of their registration. To compare experimental outcomes with quantum predictions one has to identify which clicks correspond to "twin-photon pairs" we are interested in. These experimental uncertainties are called: time-coincidence, detection, fair-sampling or photon identification loophole [23-29].

The final post-selected data samples in real experiments strongly depend on the photon identification procedures used. The setting dependent post-selection is also a source of the apparent violation of Einsteinian no-signaling reported in the literature [30].

Various Bell- type inequalities are violated by quantum predictions and by experimental data [10-15]. In spite of several claims that all the loopholes have been closed, in recent experiments, we share the opinion that it is impossible to perform a completely loophole- free Bell-experiment [28, 29].

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At the same time we have no doubt that the violation of various Bell-type inequalities has been confirmed and in fact, no more Bell -experiments are needed. The violation of inequalities is not surprising,

It was pointed out by several authors [7, 18-21, 28-73], the list of references is by no means complete, that all these inequalities are proven using CFD and/or oversimplified probabilistic models which are inconsistent with the experimental protocols used in SPCE. A detailed discussion of the intimate relation of experimental protocols with probabilistic models may be found for example in [7, 30, 63].

LRHVM does not include supplementary variables describing measuring instruments. As Theo Nieuwenhuizen said, the *contextuality loophole* is fatal for the derivation of Bell inequalities [65-67].

Since several years Hans de Raedt , Kristel Michielsen and collaborators [74-81] have simulated event-by- event, in a locally causal way, several quantum experiments including EPRB .

In this paper we focus our discussion on recent papers [29, 80, 81]. We explain the probabilistic meaning of their algorithms and their results in a simple and a concise way.

The authors, using algorithms consistent with LRHV or with SHVM, simulate outcomes of real experiments. Estimated correlations, using the raw data, are not consistent with quantum predictions for EPRB. The agreement with QM is obtained, if a specific setting- dependent photon identification procedures, mimicking those used in real experiments, are used to extract final experimental samples.

In contrast to real experiments, the authors may also generate computer outputs for counterfactual experiments such as: instantaneous measurements performed, in all the different settings, for successive correlated "photon pairs. The authors prove, that the correlations between distant clicks, estimated using the raw data, from these counterfactual computer experiments, satisfy exactly all Bell-type inequalities for any finite sample.

It does not matter whether the outcomes are predetermined or not. What matters it is the existence of some joint probability distribution of counterfactual outcomes (each equal ±1).

This provide an explicit proof that Bell -type inequalities hold, only if the correlations may be deduced from the data drawn from a statistical population described by a joint probability distribution of different possible outcomes. Such joint probability distribution does not exist for Bell-experiments. Many years ago George Boole derived similar inequalities thus in fact Bell inequalities should be perhaps called Boole-Bell inequalities [46, 50, 70, 71].

Since computer simulations are explicitly causally local they close the door on all the speculations about mysterious quantum nonlocality, which is believed to be responsible for the correlations between distant outcomes studied in SPCE.

The paper is organized as follows.

In section 2 we discuss the results of the papers [80, 81,] in which outcomes are generated using an algorithm consistent with LRHV and "photon pairs" are identified using setting-dependent registration times.

In section 3 we discuss the results of the paper [29], in which outcomes are generated using an algorithms consistent with SHVM and "photons" are locally identified using voltage traces, compared with some detection threshold.

In section 4 we discuss the effect of post-selections on the properties of extracted samples.

- In section 5 we explain what we mean by saying that QT is a contextual theory and we clarify the definition of the *contextuality loophole*.

In section 6 we discuss how computer simulations may contribute to testing whether quantum theory is predictably complete

Section 7 contains several conclusions.

2. Computer simulation experiments using registration time delays.

As we mentioned in the introduction according to CFD "photon pair" in SPCE is described before a measurement by predetermined values of spin projections in all directions. In CFD-

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compliant LRHVM, joint probability distribution on a unique probability space is used to deduce the correlations between spin projections for different pairs of settings used in SPCE.

In their computer experiments [80, 81] Hans de Raedt, Kristel Michielsen and Karl Hess generate, for each member of a "photon pair", measurement outcomes and time delays of their registration.

At first, time delays are not taken into consideration. All outputted outcomes are used to estimate the correlations between clicks in "distant laboratories" and to verify the validity of CHSH inequality [82, 83].

Subsequently, using outputted time delays and their differences, correlated "photon pairs" are identified, as in real experiments [11], and post-selected data samples are consistent with the predictions of QM for EPRB.

The authors use the following CFD-compliant algorithm. A "photon pair" entering observation stations is represented by (φ, $\varphi + \pi/2$ r1, r2) . A polarizer is described by (*a*, T), where T is a fixed parameter related to a time unit and *a* is an angle of a chosen setting.

Two lines below explain how an outcome x and a time delay t* are calculated, for each "photon" passing by a polarizer:

$$\text{1: compute } c = \cos[2(a - \varphi)];\ s = \sin[2(a - \varphi)]; \quad (1)$$

$$\text{2: set } x = \text{sign}(c);\ t^* = rTs^2 \quad (2)$$

where φ is randomly drawn from $[0,2\pi]$ and *r* from [0,1]. Please note that outcomes x do not depend on the parameters r, thus these parameters could be assigned not to the incoming "photon pairs" but to the observation stations. Such interpretation is adopted in the paper [29] which we discuss in the next section.

Two different protocols are used to generate data samples. We call them: Protocol 1 (implementable) and Protocol 2 (counterfactual).

- Protocol 1 is consistent with a realisable protocol in SPCE. Outcomes, for any pair of settings, are generated, at each time, for a different "photon pair". Thus: 4 pseudo- random time series of data for 4 different pairs of settings are generated:

$$(x_1(t), x_2(t), t_1^*(t), t_2^*(t)) = G(\varphi_1(t), \varphi_2(t), r_1(t), r_2(t), a_1(t), a_2(t)) \quad (3)$$

where t=1,…4N and $a_1(t)\ a_2(t) = a_1\ a_2$ for t=1…N; $a_1(t)\ a_2(t) = a_1\ a'_2$ for t=N+1….2N ; $a_1(t)\ a_2(t) = a'_1\ a_2$ for t=2N+1…3N; $a_1(t)\ a_2(t) = a'_1\ a'_2$ for t=3N+1….4N .

- Protocol 2 is impossible to implement in SPCE and is forbidden by QM . For each "photon pair" predetermined outcomes and time delays, for all available settings, are outputted. A generated pseudo-random time series is:

$$(x_1(t), x'_1(t), x_2(t), x'_2(t), t_1^*(t), t_1'^*(t), t_2^*(t), t_2'^*(t)) = G_1(\varphi_1(t), \varphi_2(t), r_1(t), r_2(t), a_1, a'_1, a_2, a'_2) \quad (4)$$

where t=1,…4N.

The authors use a different definition of CFD [29.80, 81], According to their definition Protocol 1 is CFD non-compliant and Protocol 2 is CFD -compliant. According to our definition we call a protocol CFD –compliant, if and only if, it generates outcomes which are predetermined, by the parameters describing a "photon-pair".

Richard Gill studies in [84] an impact of CFD-compliance on samples which, are created in idealized SPCE experiment, with random choices of settings. He defines a 4 x 4N counterfactual spreadsheet. Each line of this spreadsheet describes each incoming "photon pair" by predetermined values ±1 of spin projections for 4 settings (a, a, b, b') used in SPCE. If no constraints are imposed one can have only 16 different lines in the spreadsheet which are permuted randomly depending on a sequence of "photon pairs" arriving to detectors.

In SPCE, pairs of settings are chosen randomly and only the results for one pair of settings can be observed for each incoming pair. Gill constructs experimental samples by choosing, from each line of the spreadsheet, two outcomes corresponding to randomly chosen settings. Finite samples, created following this protocol, may not violate CHSH inequality as significantly as it is predicted by QM and Gill makes a conjecture:

$$\Pr\left(\langle AB\rangle_{obs} + \langle AB'\rangle_{obs} + \langle A'B\rangle_{obs} - \langle A'B'\rangle_{obs} \geq 2\right) \leq \frac{1}{2} \quad (5)$$

The samples containing the outcomes x_1,x'_1, x_2 and x'_2 created using (1-4) have similar properties to the samples studied by Gill . Namely:

- Protocol 1 chooses, for each "photon pair "only two entries from a corresponding line of the spreadsheet: one for Alice and one for Bob. It is obvious from (3) that the first N pairs of entries are chosen from the same pair of columns since the settings are changed systematically, only at t=N+1, 2N+1, 3N+1, 4N+1. In spite of the fact that pairs of settings are not chosen randomly generated samples have the same properties as the samples in [84].

- Protocol 2 chooses, for each "photon pair", a complete line of the counterfactual spreadsheet. These lines form a sample drawn from some CFD -compliant joint probability distribution of 4 random variables. It is easy to see [62, 84] that in this case |S|=2 for any finite sample thus CHSH inequality is never violated.

For Protocol 1: results of computer simulations, before a post- selection, are consistent with Gill's conjecture (54 samples out of 100 violate |S| ≤2). For Protocol 2 : CHSH inequality is never violated. It confirms also the result of simulations of Sascha Vongher [85]. Vongher demonstrated that, if a fate of a "photon" is predetermined before the measurement, one may not , under the assumption of a fair sampling, violate Bell and CSHC inequalities, as significantly, as it is predicted by QM [62,85].

All computer simulations confirm that LRHVM is not consistent with QM.

In order to identify which pairs of computer outputs correspond to "photon pairs" the authors post-select their final data samples by comparing the differences between time delays with suitable time windows W: $|t_2^*(t) - t_1^*(t)| \leq W$.

Correlations, estimated using these post-selected setting- dependent samples, agree remarkably well with the correlations predicted by QM. Moreover time -window dependence of these estimates reproduces remarkably well such dependence observed in real experiment [11]

3. Computer simulation experiments using detection thresholds

In [29], the authors construct a faithful model of laboratory experiments of Giustina et al. [14] and Schalm et.al. [15], performed to check Eberhard [86] and Clauser- Horn inequalities [22].

Clauser –Horne inequalities are derived using SHVM. In contrast with LRHVM outcomes are not predetermined by the variables describing the inputted "photon pair". In SHVM:

$$P(x_1, x_2 \mid a_1, a_2) = \sum_{\lambda \in \Lambda} P(\lambda_1, \lambda_2) P(x_1 \mid a_1, \lambda_2) P(x_2 \mid a_2, \lambda_2) \quad (6)$$

where (λ_1, λ_2) describes "photon pairs", (a_1, a_2) , are chosen settings , $P(x_1|a_1, \lambda_1)$ and $P(x_2|a_2, \lambda_2)$ are probabilities describing stochastically independent measurements, performed in distant laboratories by Alice and Bob. If (λ_1, λ_2) are continuous variable, the sum in (6) is replaced by an integral More detailed discussion of SHVM may be found in [7,17,22].

In [29] a following (SHVM-compliant) algorithm is used:

For each k = 1, . . . , N a uniform random generator generates two floating-point numbers $0 \leq \phi_{1,k} \leq 2\pi$ and $\phi_{2,k} = \phi_{1,k} + \pi/2$, which are input to the stations with setting (a_1, a_1') and a_2, a_2) respectively. . The k-th event simulates the emission of a photon pair with maximally correlated, orthogonal polarizations for all the essential features of the laboratory experiments,

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Upon receiving the input, an observation station generates two pseudo- random numbers $(r, \hat{r})$ and computes :

$$c = cos\left[2(a - \phi)\right], \ s = sin\left[2(a - \phi)\right] \qquad (7)$$

and sets:

$$x = sign(1 + c - 2 * r), \ v = \hat{r}|s|^d\left(Vmax - Vmin\right) - Vmax \qquad (8)$$

where d is an adjustable parameter and positive numbers Vmin and Vmax define the range of the voltage traces , produced on the detectors by the incoming signals. Voltage signal v is identified as a "single photon" event, if and only if $-Vmax \leq v \leq V \leq -V\,min$, where V is a chosen detector threshold.

The choice of the specific functional forms of x and v in (7, 8) is inspired by a similar model which employs time-coincidence to identify pairs and exactly reproduces the single particle averages and two-particle correlations of the singlet state, if the number of events becomes very large [75, 79].

The algorithm (7), for each value of φ and uniformly distributed values of *r*, generates a sequence of randomly distributed x, such that the empirical frequency distribution of x=1 and x=-1 agrees with Malus' law:

$$P(X=1\,|\,a,\varphi) = \cos^2(a-\varphi) \ and \ P(X=-1\,|\,a,\varphi) = \sin^2(a-\varphi) \qquad (9)$$

where X is a random variable taking values x.

Voltage traces are only used to identify correlated "photon pairs", so we discuss, for the moment, the properties raw data samples of outcomes.

Using the algorithm (7, 8) the authors perform two computer experiments following two different protocols. Now the outcomes x are not predetermined and we call them: Protocol 1 and Protocol 2.

- Protocol 1 (implementable) is consistent with the experimental protocol used in SPCE because the outcomes, for any pair of settings , are generated each time for a different "photon pair". Namely 4 pseudo- random time series of data for 4 different pairs of settings may be generated:

  $$(x_1(t), x_2(t) = F_1(\varphi_1(t), \varphi_2(t), r_1(t), r_2(t), a_1(t), a_2(t)) \qquad (10)$$

  where t=1,…4N and $a_1(t)\ a_2(t)= a_1\ a_2$ for t=1…N; $a_1(t)\ a_2(t)= a_1\ a'_2$ for t=N+1….2N ; $a_1(t)a_2(t)= a'_1\ a_2$ for t=2N+1…3N; $a_1(t)\ a_2(t)= a'_1\ a'_2$ for t=3N+1….4N . The fact that the settings are not randomly chosen has no impact on the properties of the generated samples.

- Protocol 2 (counterfactual) is impossible to realize in SPCE and in QM , because it calculates and outputs at each step 4 pseudo-random values (simulating the outcomes of simultaneous joint measurement of incompatible observables performed on each "photon pair"). The generated pseudo-random time series of outputs for the 4 simultaneous measurement settings may be defined as:

  $$(x_1(t), x'_1(t), x_2(t), x'_2(t)) = F_2(\varphi_1(t), \varphi_2(t), r_1(t), r_2(t), r'_1(t), r'_2(t)\, a_1, a'_1, a_2, a'_2) \qquad (11)$$

  where t=1,…4N

Samples produced using the protocol 1 are consistent with a specific SHVM model (6):

$$P(x_1, x_2\,|\,a_1, a_2) = \int_0^{2\pi} d\varphi_1 \int_0^{2\pi} d\varphi_2 P(\varphi_1, \varphi_2) P(x_1\,|\,a_1, \varphi_1) P(x_2\,|\,a_2, \varphi_2) \qquad (12)$$

where the probability density $P(\varphi_1, \varphi_2) = \delta(\varphi_2 - \varphi_1 - \pi/2)/(2\pi)^2$ and the probabilities $P(x_1\,|\,a_1, \varphi_1) P(x_2\,|\,a_2, \varphi_2)$ are calculated using the Malus law (9).

Using (10) we find easily:

$$E(X_1) = E(X_2) = 0 \text{ and } E(X_1 X_2) = \frac{1}{2}\cos 2(a_1 - a_2) \qquad (13)$$

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where X_1 and X_2 are the random variables taking the values $x_1=\pm1$ and $x_2=\pm1$ respectively.

Single expectation values vanish as predicted by QM. QM predicts for EPRB $E(X_1X_2)=\cos 2(a_1-a_2)$. Expectation values $E(X_1X_2)$, displayed in (3,) contain a factor ½ thus they do not violate SHCH inequality. The agreement with quantum predictions is obtained only after the "photon identification procedure" which selects, from the raw data, final data samples.

In contrast with the protocol 1, any finite sample of quadruples $(x_1(t), x'_1(t), x_2(t), x'_2(t))$ generated using the protocol 2, obeys trivially all Bell and CHSH inequalities, similarly as in computer simulations discussed in the preceding section.

Again the data may be displayed using 4x 4N counterfactual spreadsheet and it does not matter that now its entries are not predetermined.

It confirms that , Bell- inequalities are the necessary and sufficient conditions for the existence of a joint probability distribution for 3 dichotomous random variables X_i taking the values $x_i=\pm1$ which can be measured pairwise, but not all simultaneously. As we mentioned in the introduction the similar inequalities were already derived by Georges Boole [87] and generalized by Vorob'ev [88].

Therefore the violation of Bell-type inequalities is void of any physical meaning and these inequalities are violated in several experiments: in successive spin polarization measurements [88], in social sciences [90-92] and even in a classical mechanics [59]. Claims that, they imply mysterious quantum nonlocality or the super-determinism are completely unfounded [63].

As we see, the raw data of computer simulations are not consistent with some quantum predictions. To produce data samples consistent with QM the authors have to apply setting-dependent "photon- pair identification procedure", mimicking the procedures used in real experiments In the next section we discuss the influence of a post-selection on the properties of the selected samples.

4. Setting–dependent post-selection.

It is well known, that in order to obtain reliable information about some statistical population, one has to study *simple random* samples drawn from this population.

In computer simulations compliant with LRHVM or with SHVM *simple pseudo- random* samples are generated. These samples are not consistent quantum probabilistic predictions and specific setting-dependent post selection of data items is needed.

Similarly raw data from real experiments may not be compared with theoretical predictions without applying "photon pair" identification procedures.

These procedures contain free parameters such as time windows, registration thresholds etc. The properties of the post-selected samples vary significantly in function of these parameters [11, 26, 30]

To illustrate a possible dramatic impact of a post-selection, let us consider two perfect simple random samples $S_1=\{x_1,\ldots..x_n\}$ and $S_2=\{y_1,\ldots.y_2\}$ where $x_i=\pm1$ and , $y_i=\pm1$. Using a simple pairing of outcomes we obtain a sample $S_3=\{(x_1, y_1),\ldots..(x_n, y_n)\}$, for which E(XY)=0.

- If we post- select data items, using a criterion keep only if $x_i + y_i = 2$, we extract completely correlated sub-sample of S_3, for which E (XY) =1
- If we post-select data items, using a criterion keep only if $x_i + y_i = -2$, we extract completely anti- correlated sub-sample of S_3, for which E (XY) = -1
- If we post-select randomly data items from S_3 we obtain uncorrelated sub-sample for which (E (XY) =0).

In spite of the fact that the post-selection in SPCE is necessary and justified, we share the opinion that a loophole –free test of Bell-type inequalities may not be performed [28, 51].

We demonstrated with Hans de Raedt [93] that sample inhomogeneity invalidates the standard significance tests, In spite of the fact , that in Bell-experiments [10-15] sample homogeneity could not or was not tested carefully enough , we do not doubt that Bell-type inequities have been violated in these experiments.

In view of our discussion presented in preceding sections the violation of Bell-type inequalities is by no means surprising and new tests are simply not needed.

It is well known that models which incorporate a setting-dependent post selection and/or exploit so called coincidence loophole may produce results [23-25] which violate CHSH inequality ($|S| \leq 2$) .

It is sometimes claimed that setting–dependence of experimental samples, may be only explained by some mysterious influences between distant experimental set-ups , super- determinism or conspiracy of detectors whose efficiency changes in order to comply with QM

There is much simpler explanation: the setting- dependence of experimental samples, in real and in computer experiments, reflects an active role played by the measuring instruments. The measurement outcomes are not predetermined and depend on the experimental settings.

Quantum observables are contextual and for each settings used in SPCE the data may only be described by a specific setting-dependent probabilistic model.

Probabilistic proofs of Bell-type inequalities suffer from the *contextuality loophole* because they try to describe various incompatible experiments using setting independent hidden variables [66,67]

In the next section we explain in more detail, what do we mean by the *contextuality loophole.* We demonstrate also how the setting –dependent photon identification procedure produce data samples consistent with specific probabilistic models which do not suffer from this loophole

5. The meaning of the *contextuality loophole.*

Contextuality has a different meaning for different authors [7, 44, 49, 54, 57, 66, 67, 94-98]. For some authors a theory is non-contextual, if values of physical observables do not depend on a <u>specific</u> experimental protocol used to measure these observables. For example a length of a table does not depend how this length is measured. Similarly QT does not say how a linear momentum of an electron and its energy are measured. <u>Quantum state vector represents all equivalent preparation procedures and a self-adjoint operator represents all equivalent measurement procedures of a corresponding physical variable</u> [98]. Therefore one can find statements in the literature that QM, as well as, the classical physics is non-contextual theory [96].

Physical systems may have different properties. Some of them are attributive what means that they do not change in the interaction with measuring instruments and they do not depend on the environment in which they are measured. Attributive properties are for example: the rest mass and the electric charge of an electron, Physical systems may be also described by contextual properties which are only revealed in particular experimental and/or environmental contexts for example: color, weight, magnetization, spin projection etc.

Interesting contextual property is a <u>probability</u> (understood not as a subjective belief of a human agent). A probability is neither a property of a coin nor a property of a flipping device. It is only a property of a whole random experiment [7, 21, 42, 49, 57].

In quantum theory, as Bohr insisted, we deal with: "the impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear" [2].

In particular QT gives probabilistic predictions on a statistical scatter of outcomes obtained in repeated "measurements" of some physical observable on "identically prepared physical systems". Since the probability is only a property of a random experiment, performed in a particular <u>experimental context</u> , thus QM provides a contextual description of the physical reality.

Similarly Kolmogorov probabilistic models provide a contextual description of random experiments. Namely each random experiment (in which there is a <u>statistical stabilization</u>) is described by its Kolmogorov model defined on a dedicated probability space [7, 42, 52].

Probabilistic models, as well as, QM do not enter into details how individual outcomes are obtained, but provide only predictions for the experiments as a whole. <u>If the context of an experiment changes</u> (for example we have two slits open instead of one) then <u>Kolmogorov and quantum</u>

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probabilistic models change. This is why we say : QT is a contextual theory. Only rarely different random experiments may be described using marginal probabilities deduced from some joint probability distribution [7, 48, 52, 63] and the joint probability distributions of incompatible quantum observables do not exist.

We say, following Theo Nieuwenhuizen [66, 67], that the proofs of Bell –type inequalities suffer from the fatal *contextuality loophole*. We also say that a probabilistic hidden variable model suffers from the *contextuality loophole,* if it uses the same probability space to describe different incompatible random experiments. To close the *contextuality loophole* a model has to incorporate supplementary variables describing measuring devices [65-67]. If setting dependent parameters are correctly incorporated in a hidden variable probabilistic model then Bell-type inequalities may not be proven.

For example, if $x_a(t))=f_a(\lambda_1(t), \lambda_a(t),a)$ and $x_b(t))=f_b(\lambda_2(t), \lambda_b(t),b)$ (where $(\lambda_1(t), \lambda_2(t))$ and $(\lambda_a(t), \lambda_b(t))$ describe respectively an "EPR-pair" and "microstates" of measuring instruments, in the setting (a, b), then Gill's counterfactual spreadsheet does not exist and the only constraint we have is $|S| \leq 4$. A detailed discussion of a model, able to reproduce any correlations in SPCE, may be found in [63]. The same constraint was derived by Andrei Khrennikov in his generalization of Kolmogorov model for SPCE experiments [99, 100].

Therefore any hidden variable model wanting to reproduce quantum predictions must include explicit dependence on the settings. In [29, 80, 81] "photon identification procedures" allow to generate samples, which may be described by a contextual setting-dependent probabilistic models, able to reproduced exactly quantum predictions for idealized EPRB experiments. For example we rewrite below Eq, 19 from [76] in explicitly contextual form:

$$P(x_1,x_2\,|\,\alpha_1,\beta_1,\mathrm{W}) = \int_0^{2\pi}\int_0^{2\pi} P(x_1\,|\,\alpha,\xi_1)\,\mathrm{P}(x_2\,|\,\beta,\xi_2)\,\mathrm{P}(\xi_1,\xi_2\,|\,\alpha,\beta,\mathrm{W})\,\mathrm{d}\,\xi_1\,\mathrm{d}\,\xi_2 \tag{14}$$

The setting- dependence in these models is a result of locally causal data generation and any speculations about a spooky action at the distance are unfounded

Contextual probabilistic models of spin polarization correlation experiments may also be defined in a more direct and intuitive way see for example [62, 63].

To describe in detail time-series of data, in different6 domains of science, we usually have to use stochastic processes. It is reasonable to ask, whether quantum probabilities may explain all fine structure in those time-series [53,101,102].

Computer simulations, entering into details how individual data items are created, allow to model real experiments, to incorporate time delays and time-windows , what is impossible in QM.

Successful computer simulations of several quantum experiments demonstrate how quantum probabilities might emerge a more detailed description of quantum phenomena. Testing the *predictable completeness* of QM does not depend on the existence of such detailed description. This is the topic of the next section.

6. Computer simulations and predictable completeness of QT

Bohr claimed, that any subquantum analysis of quantum phenomena is impossible and that QM provides a complete description of individual physical systems [99]. Einstein believed that quantum probabilities are emergent and that a complete theory should give more details how individual outcomes are created [103, 104]. Bohmian- mechanics, stochastic electrodynamics, hidden variable models and various computer simulations are attempts to realize this program in particular cases.

Event-by-event simulations of quantum experiments are trying to get more intuitive understanding of quantum phenomena, without evoking *quantum magic*. As we have already mentioned above, several quantum phenomena have been simulated [74-81]. These simulations have no ambition of replacing QM. They generate time series of outcomes, similar to those created, in real experiments, and study the effects, which QM is unable to address.

Different experimental protocols, based on the same probabilistic model, provide more detailed information how data are created and may produce significantly different finite samples [93].

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In [11-15] one use time-windows in order to identify correlated detection events. The estimated correlations strongly depend on time-windows used as those in the real experiments.

The main ingredient of the computer model, discussed in the section 2, is time delays. The authors say that these time delays are due to the existence of dynamic many-body interactions of a photon with a measuring apparatus. In our opinion the equations (1) and (2) allow to imagine more detailed mechanism how data in real experiment might have been created.

Let us change slightly the assignment of the variables, used in (1, 2), and describe an "EPR-pair" by (φ, $\varphi + \pi/2$) and measuring devices by (a_1, r_1, T; a_2, r_2, T). Before entering PBS, the "magnetic moments" of each pair are pointing in opposite directions. During the interaction with a PBS they are "aligned" along the directions a_1 and a_2 respectively, and subsequently they are "sent" to corresponding detectors. It is plausible, that the time needed for this alignment increases and decreases in the function of (a- φ). The model would have been closer to this physical intuition with parameters r drawn randomly from [1-c, 1], where c is a small number.

Of course the dependence of time delays on (a- φ) in (2) was chosen to reproduce quantum predictions. Nevertheless the existence of conjectured time delays, not predicted by QM, may be tested in specific dedicated experiments with polarized beams. The possibility of the existence and implications of time delays in Bell experiments was discussed, for the first time in detail, by Saverio Pascazio [24].

We don't know whether such delays were studied experimentally. If such time delays were discovered, it would give additional argument in favor of the idea that QM is not predictably complete.

In several papers we advocated that in order to check the *predictable completeness* of QM one has to search for unexpected regularities in the experimental time-series of data [53,101,102].

7. Conclusions

In classical physics statistical ensembles of physical systems may often be described by probabilistic models using joint probability distributions of several physical variables, even if we do not measure all of them at the same time. It is assumed that a measurement of any particular property does not change other properties which are not measured.

Bell did not realize, that his hidden variable model describes only a counter-factual random experiment in which, for each `photon-pair, `spin-projections are simultaneously measured by Alice and Bob in their available experimental settings. If outcomes are coded ±1, the recorded data form quadruples (±1, ±1, ±1, ±1) and pair-wise correlations, estimated using these data, may never violate Bell inequalities.

Pair-wise correlations, for different pairs of setting in Bell-experiments, are estimated using the data coming from mutually exclusive random experiments and they do not need to satisfy the constraints resulting from the existence of the counterfactual joint probability distribution.

Apparently theoretical arguments based on probability spaces and the properties of random variables have not been sufficiently convincing. The outcomes of computer simulations are perhaps be easier to understand.

Computer simulations allow generating data sets for several counterfactual experiments such as repeated measurements on the same "photon-pair", joint or successive measurements performed on the same" photon-pair" in different settings etc.

Using counterfactual protocols, quadruples (±1, ±1, ±1, ±1) are generated for each "photon- pair". These quadruples may be displayed in a 4x4N spreadsheet. If pair-wise correlations are estimated using this spreadsheet <u>Bell and CHSH inequalities are satisfied for any finite sample</u>.

It does not matter whether the outputs ±1 are predetermined [81] or generated randomly [29]. What does matter is : that we use quadruples to estimate pair-wise correlations Using the language of mathematical statistics we say, that quadruples form a finite sample drawn from a statistical population described by some joint probability distribution of 4 random variables.

Joint probability distributions are only well defined, if we measure several random variables, at the same time, such as: weight and height of a student, numbers on two dices rolled together etc. [63]

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If we want to study the correlations between outcomes of distant experiments, we must define how the outcomes are paired. The properties of the generalised joint probability distributions (GJPD) describing these paired outcomes depend in how the paring was defined. Detailed discussion of GJPD's and of pairing protocols may be found, for example, in [7, 90, 91].

If only pairs of outcomes are outputted for each "photon pair", some finite samples may violate Bell-type inequalities, but not as significantly as in real experiments.

Final data samples are created using "photon-pair identification" procedures, mimicking the procedures used in real experiments.

Similarly, as in real experiments, post-selected samples depend strongly on time -delays, time windows and voltage thresholds. Each of these post-selected samples is consistent with some specific setting dependent probabilistic model. For some specific values of adjustable parameters such model reproduces exactly quantum predictions for idealised EPRB experiment.

Computer simulations provided a direct proof that Boole-Bell-type inequalities are the necessary and sufficient condition for the existence of GJPD from which the pair-wise correlations between random variables may be deduced. Such GJPD does not exist for the random variables measured in Bell–tests.

Computer simulations, using locally causal algorithms,, give more detailed description of how individual data items might have been created in real experiments. Such detailed description is out of the scope of QM and was believed to be impossible. Therefore we cannot close the door on the Bohr-Einstein quantum debate [63, 81], in spite of what was claimed by Aspect [105].

Bell-type inequalities have been violated in [11-15], but it does not matter, whether there are or not spooky influences between distant experimental set-ups. It does not matter, whether experimentalists have a "free will" to choose their setting or not. What only matters, it is the impossibility to deduce the correlations between distant clicks in SPCE, for different pairs of setting, using the same counterfactual GJPD.

Completely different argument in favour of locality and *contextuality* has been recently given by Kurt Jung [106]. He derived polarization correlations of "photon pairs in triplet and singlet configurations, using the fact that circularly polarized wave packets associated with entangled "photon pairs" are phase shifted at the source. "…The linear polarization component of a circularly polarized photon is not defined before the measurement has taken place. Nonetheless the polarization correlation of entangled photons measured at distant locations is well defined and agrees with the predictions of quantum mechanics…"

We conclude that various theoretical arguments, reviewed in this paper, and locally causal simulations of Bell experiments allow closing the door on quantum nonlocality.

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