Derivation of generalized Einstein's equations of gravitation based on a mechanical model of vacuum and a sink flow model of particles

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J. C. Maxwell, B. Riemann and H. Poincaré have proposed the idea that all microscopic particles are sink flows in a fluidic aether. Following this research program, a previous theory of gravitation based on a mechanical model of vacuum and a sink flow model of particles is generalized by methods of special relativistic continuum mechanics. In inertial reference frames, we construct a tensorial potential which satisfies the wave equation. Inspired by the equation of motion of a test particle, a definition of a metric tensor of a Riemannian spacetime is introduced. Applying Fock's theorem, generalized Einstein's equations in inertial systems are derived based on some assumptions. These equations reduce to Einstein's equations in case of weak field in harmonic reference frames. In some special non-inertial reference frames, generalized Einstein's equations are derived based on some assumptions. If the field is weak and the reference frame is quasi-inertial, these generalized Einstein's equations reduce to Einstein's equations. Thus, this theory may also explains all the experiments which support the theory of general relativity. There exists some differences between this theory and Einstein's theory of general relativity.

Keywords: Einstein's equations; gravitation; general relativity; sink; gravitational aether.

I. INTRODUCTION

The Einstein's equations of gravitational fields in the theory of general relativity can be written as [1, 2]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T^m_{\mu\nu}, \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor of a Riemannian spacetime, $R_{\mu\nu}$ is the Ricci tensor, $R \equiv g^{\mu\nu} R_{\mu\nu}$ is the scalar curvature, $g^{\mu\nu}$ is the contravariant metric tensor, $\kappa = 8\pi\gamma_N/c^4$, γ_N is Newton's gravitational constant, c is the speed of light in vacuum, $T^m_{\mu\nu}$ is the energy-momentum tensor of a matter system.

The Einstein's equations (1) is a fundamental assumption in the theory of general relativity [1, 2]. It is remarkable that Einstein's theory of general relativity, born in 1915, has held up under every experimental test, refers to, for instance, [3].

There is a long history of researches of derivations or interpretations of Einstein's theory of general relativity. For instance, C. Misner et al. introduce six derivations of the Einstein's equation Eq.(1) in their great book ([2], p417). S. Weinberg proposed two derivations ([1], p151).

However, these theories still face the following difficulties. (1) Attempts to reconcile the theory of general relativity and quantum mechanics have met some mathematical difficulties ([4],p101); (2) The cosmological constant problem is still a puzzle, refers to, for instance, [5]; (3) The existence of black hole is still controversy, refers to, for instance, [6]; (4) Theoretical interpretation of P. A. M. Dirac's dimensionless large number ([7], p73) is still open; (5) The existences and characters of dark matter and dark energy are still controversy, refers to, for instance, [8]; (6) The existence and characters of gravitational aether are still not clear, refers to, for instance, [9]; (7) Whether Newton's gravitational constant γ_N depends on time and space is still not clear [10]; (8) Whether the speed of light in vacuum depends on time or space is controversy, refers to, for instance, [11].

Furthermore, there exists some other problems related to the theories of gravity, for instance, the definition of inertial system, origin of inertial force, the velocity of the propagation of gravity [12], the velocity of individual photons [13, 14], unified field theory, etc.

The purpose of this manuscript is to propose a derivation of the Einstein's equation (1) in some special reference frames based on a mechanical model of vacuum and a sink flow model of particles [15].

II. INTRODUCTION OF A PREVIOUS THEORY OF GRAVITATION BASED ON A SINK FLOW MODEL OF PARTICLES BY METHODS OF CLASSICAL FLUID MECHANICS

The idea that all microscopic particles are sink flows in a fluidic substratum is not new. For instance, in order to compare fluid motions with electric fields, J. C. Maxwell introduced an analogy between source or sink flows and electric charges ([16], p. 243). B. Riemann speculates that:"*I make the hypothesis that space is filled with a substance which continually flows into ponderable atoms, and vanishes there from the world of phenomena, the corporeal world*"([17], p. 507). H. Poincaré also suggests that matters may be holes in fluidic aether ([18], p. 171). A. Einstein and L. Infeld said ([19], p. 256-257):"*Matter is where the concentration of energy is great, field where the concentration of energy is small. ⋯ What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong.*"

Following these researchers, we suppose that all the microscopic particles were made up of a kind of elemen-

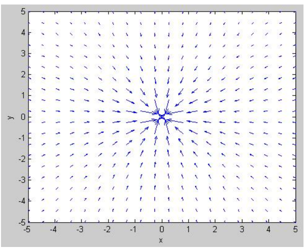

FIG. 1: an illustration of the velocity field of a sink flow.

tary sinks of a fluidic medium filling the space [15]. Thus, Newton's law of gravitation is derived by methods of hydrodynamics based on the fluid model of vacuum and the sink flow model of particles [15].

We briefly introduce this theory of gravitation [15]. Suppose that there exists a fluidic medium filling the interplanetary vacuum. For convenience, we may call this medium as the $\Omega(0)$ substratum, or gravitational aether, or tao [15]. Suppose that the following conditions are valid: (1) the $\Omega(0)$ substratum is an ideal fluid; (2) the ideal fluid is irrotational and barotropic; (3) the density of the $\Omega(0)$ substratum is homogeneous; (4) there are no external body forces exerted on the fluid; (5) the fluid is unbounded and the velocity of the fluid at the infinity is approaching to zero.

An illustration of the velocity field of a sink flow can be found in Figure 1. If a point source is moving with a velocity $\mathbf{v}_s$, then there is a force [15]

$$\mathbf{F}_Q = -\rho_0 Q(\mathbf{u} - \mathbf{v}_s) \tag{2}$$

is exerted on the source by the fluid, where ρ_0 is the density of the fluid, Q is the strength of the source, $\mathbf{u}$ is the velocity of the fluid at the location of the source induced by all means other than the source itself.

We suppose that all the elementary sinks were created simultaneously [15]. For convenience, we may call these elementary sinks as monads. The initial masses and the strengths of the monads are the same. Suppose that (1) $\mathbf{v}_i \ll \mathbf{u}_i, i = 1, 2$, where $\mathbf{v}_i$ is the velocity of the particle with mass m_i, $\mathbf{u}_i$ is the velocity of the $\Omega(0)$ substratum at the location of the particle with mass m_i induced by the other particle; (2) there are no other forces exerted on the particles, then the force $\mathbf{F}_{21}(t)$ exerted on the particle with mass $m_2(t)$ by the velocity field of $\Omega(0)$ substratum induced by the particle with mass $m_1(t)$ is [15]

$$\mathbf{F}_{21}(t) = -\gamma_N(t)\frac{m_1(t)m_2(t)}{r^2}\hat{\mathbf{r}}_{21}, \tag{3}$$

where $\hat{\mathbf{r}_{21}}$ denotes the unit vector directed outward along the line from the particle with mass $m_1(t)$ to the particle with mass $m_2(t)$, r is the distance between the two particles, $m_0(t)$ is the mass of monad at time t, $-q_0(q_0 > 0)$ is the strength of a monad, and

$$\gamma_N(t) = \frac{\rho_0 q_0^2}{4\pi m_0^2(t)}. \tag{4}$$

For continuously distributed matter, we have

$$\frac{\partial \rho_0}{\partial t} + \nabla\cdot(\rho_0\mathbf{u}) = -\rho_0\rho_s, \tag{5}$$

where $\mathbf{u}$ is the velocity of the $\Omega(0)$ substratum, $\nabla = \mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y + \mathbf{k}\partial/\partial z$ is the nabla operator introduced by Hamilton, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are basis vectors, $-\rho_s(\rho_s > 0)$ is the density of continuously distributed sinks, i.e.,

$$-\rho_s = \lim_{\triangle V\to 0}\frac{\triangle Q}{\triangle V}, \tag{6}$$

where $\triangle Q$ is the source strength of the continuously distributed matter in the volume $\triangle V$ of the $\Omega(0)$ substratum.

Since the $\Omega(0)$ substratum is homogeneous, i.e., $\partial\rho_0/\partial t = \partial\rho_0/\partial x = \partial\rho_0/\partial y = \partial\rho_0/\partial z = 0$, and irrotational, i.e., $\nabla\times\mathbf{u} = 0$, Eq. (5) can be written as [20]

$$\nabla^2\varphi = -\rho_s, \tag{7}$$

where φ is a velocity potential such that $\mathbf{u} = \nabla\varphi$, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplace operator.

We introduce the following definitions

$$\Phi = \frac{\rho_0 q_0}{m_0}\varphi, \quad \rho_m = \frac{m_0\rho_s}{q_0}, \tag{8}$$

where ρ_m denotes the mass density of continuously distributed particles.

Using Eq. (8) and Eq. (4), Eq. (7) can be written as

$$\nabla^2\Phi = -4\pi\gamma_N\rho_m. \tag{9}$$

III. A MECHANICAL MODEL OF VACUUM

According to our previous paper [21] we suppose that vacuum is filled with a kind of continuously distributed material which may be called $\Omega(1)$ substratum or electromagnetic aether. Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this mechanical model of vacuum and a source and sink flow model of electric charges [21]. We speculate that the electromagnetic aether may also generate gravity. Thus, we introduce the following assumption.

Assumption 1 *The particles that constitute the $\Omega(1)$ substratum, or the electromagnetic aether, are sinks in the $\Omega(0)$ substratum.*

Then, according to the previous theory of gravitation [15], these $\Omega(1)$ particles gravitate with each other and also attract with matters. Thus, vacuum is composed of at least two kinds of interacting substratums, i.e., the gravitational aether $\Omega(0)$ and the electromagnetic aether $\Omega(1)$.

From Eq. (2), we see that there exists a following universal damping force $\mathbf{F}_d = -\rho_0 q_0 m\mathbf{v}_p/m_0$ exerted on each particle by the $\Omega(0)$ substratum [15], where $\mathbf{v}_p$ is the velocity of the particle. Based on this universal damping force $\mathbf{F}_d$ and some assumptions, we derive a generalized Schrödinger equation for microscopic particles [22]. For convenience, we may call these theories [15, 21, 22] as the theory of vacuum mechanics.

IV. CONSTRUCTION OF A LAGRANGIAN FOR FREE FIELDS OF THE $\Omega(0)$ SUBSTRATUM BASED ON A TENSORIAL POTENTIAL IN THE GALILEAN COORDINATES

There exists some approaches ([23], page vii;[2], p. 424), which regards Einstein's general relativity as a special relativistic field theory in an unobservable flat spacetime, to derive the Einstein's equations (1). However, these theories can not provide a physical definition of the tensorial potential of gravitational fields, refers to, for instance, [2, 24, 25]. Thus, similar to the theory of general relativity, these theories may be regarded as phenomenological theories of gravitation.

Inspired by these special relativistic field theories of gravitation, we explore the possibility of establishing a similar theory based on the theory of vacuum mechanics [15, 21, 22]. Thus, first of all, we need to construct a Lagrangian for free fields of the $\Omega(0)$ substratum based on a tensorial potential in the Galilean coordinates. In this section, we will regard the $\Omega(0)$ substratum in the previous theory of gravitation [15] as a special relativistic fluid. Then, we will study the $\Omega(0)$ substratum by methods of special relativistic continuum mechanics [26].

In this article, we adopt the mathematical framework of the theory of special relativity [1]. However, the physical interpretation of the mathematics of the theory of special relativity may be different from Einstein's theory. It is known that Maxwell's equations are valid in the frames of reference that attached to the $\Omega(1)$ substratum [21]. We introduce a Cartesian coordinate system $\{o, x, y, z\}$ for a three-dimensional Euclidean space that attached to the $\Omega(1)$ substratum. Let $\{0, t\}$ be a one-dimensional time coordinate. We denote this reference frame as $S_{\Omega(1)}$.

Based on the Maxwell's equations, the law of propagation of an electromagnetic wave front in this reference frame $S_{\Omega(1)}$ can be derived and can be written as ([27],p. 13)

$$\frac{1}{c^2}\left(\frac{\partial\omega}{\partial t}\right)^2-\left(\frac{\partial\omega}{\partial x}\right)^2-\left(\frac{\partial\omega}{\partial y}\right)^2-\left(\frac{\partial\omega}{\partial z}\right)^2=0, \quad (10)$$

where $\omega(t, x, y, z)$ is an electromagnetic wave front, c is the velocity of light in the reference frame $S_{\Omega(1)}$.

An electromagnetic wave front is a characteristics. According to Fock's theorem of characteristics ([27], p. 432), we obtain the following metric tensor $\eta_{\alpha\beta} = \mathrm{diag}[c^2, -1, -1, -1]$ of a Minkowski spacetime for vacuum ([28], p. 57).

For convenience, we introduce the following Galilean coordinate system

$$x^0 \equiv ct, \quad x^1 \equiv x, \quad x^2 \equiv y, \quad x^3 \equiv z. \quad (11)$$

We will use Greek indices α, β, μ, ν, etc., denote the range $\{0, 1, 2, 3\}$ and use Latin indices i, j, k, etc., denote the range $\{1, 2, 3\}$. We will use Einstein's summation convention, that is, any repeated Greek superscript or subscript appearing in a term of an equation is to be summed from 0 to 3. We introduce the following definition of spacetime interval

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu, \quad (12)$$

where $\eta_{\mu\nu}$ is the metric tensor of the Minkowski spacetime defined by $\eta_{\mu\nu} = \mathrm{diag}[1, -1, -1, -1]$.

Suppose that the $\Omega(0)$ substratum is an incompressible viscous fluid. Then, there is no elastic deformations in the fluid and the internal stress states depend on the instantaneous velocity field. Thus, we can choose the reference frame $S_{\Omega(1)}$ as the co-moving coordinate system. The internal energy U is the sum of the internal elastic energy U_e and the dissipative energy U_d, i.e., $U = U_e + U_d$. Since there is no elastic deformations in the fluid, we have $U_e = 0$. We introduce the following definition of deviatoric tensor of strain rate $\dot{\gamma}^i_j$ ([29],p. 331)

$$\dot{\gamma}^i_j = \dot{S}^i_j - \dot{S}^k_k\delta^i_j, \quad (13)$$

where $\dot{S}^i_j$ is the tensor of strain rate, $\dot{S}^k_k$ is the rate of volume change, δ^i_j is the Kronecker delta.

Suppose that the rate of dissipative energy $\dot{U}_d$ is the Rayleigh type, then, we have ([29],p. 332)

$$\dot{U}_d = \mu_0\dot{\gamma}^i_j\dot{\gamma}^j_i, \quad (14)$$

where μ_0 is the coefficient of viscosity.

Since the $\Omega(0)$ substratum is incompressible, we have $\dot{S}^k_k = 0$. Thus, from Eqs. (14) and Eqs. (13), we have

$$\dot{U}_d = \mu_0\dot{S}^i_j\dot{S}^j_i. \quad (15)$$

In the low velocity limit, i.e., $u/c \ll 1$, where $u = |\mathbf{u}|$, the Lagrangian $L_{\Omega(0)}$ for free fields of the $\Omega(0)$ substratum can be written as ([29],p. 332)

$$L_{\Omega(0)} = \frac{1}{2}\rho_0 u^2 + \int_{t_0}^{t}\dot{U}_d(\dot{S}^i_j)dt, \quad (16)$$

where $u = |\mathbf{u}|$, t_0 is an initial time.

Suppose that the $\Omega(0)$ substratum is a Newtonian fluid and the stress tensor σ^i_j is symmetric, then we have ([30],p. 46)

$$\sigma^i_j = -p\delta^i_j + 2\mu_0\dot{S}^i_j, \quad (17)$$

where p is the pressure of the $\Omega(0)$ substratum.

Using Eqs. (17) and Eqs. (15), Eqs. (16) can be written as

$$L_{\Omega(0)} = \frac{1}{2}\rho_0 u^2 + \int_{t_0}^{t} (\sigma^i_j + p\delta^i_j)\frac{\dot{S}^j_i}{2}dt, \tag{18}$$

For a macroscopic observer, the relaxation time t_ε of the $\Omega(0)$ substratum is so small that the tensor of strain rate $\dot{S}^i_j$ may be regarded as a slow varying function of time, i.e., $\partial\dot{S}^i_j/\partial t \ll 1$. Thus, in a small time interval $[t_0, t]$, we have $\dot{S}^i_j \geq 0$, or, $\dot{S}^i_j \leq 0$. Then, it is possible to choose a value $\bar{\sigma}^i_j + \bar{p}\delta^i_j$ of $\sigma^i_j + p\delta^i_j$ in the time interval $[t_0, t]$ such that Eqs. (18) can be written as

$$L_{\Omega(0)} = \frac{1}{2}\rho_0 u^2 + (\bar{\sigma}^i_j + \bar{p}\delta^i_j)\int_{t_0}^{t} \frac{\dot{S}^j_i}{2}dt. \tag{19}$$

We introduce the following definition

$$\psi_{ij} \triangleq \int_{t_0}^{t} \frac{\dot{S}_{ij}}{2f_0}dt, \tag{20}$$

where f_0 is a parameter to be determined.

Using Eqs. (20), Eqs. (19) can be written as

$$L_{\Omega(0)} = \frac{1}{2}\rho_0 u^2 + f_0\psi^j_i(\bar{\sigma}^i_j + \bar{p}\delta^i_j). \tag{21}$$

Since the coefficient of viscosity μ_0 of the $\Omega(0)$ substratum may be very small, we introduce the following assumption.

Assumption 2 *In the low velocity limit, i.e., $u/c \ll 1$, where $u = |\boldsymbol{u}|$, $\boldsymbol{u}$ is the velocity of the $\Omega(0)$ substratum, we suppose that $\mu_0 \approx 0$ and we have the following conditions*

$$\psi_{ij} \approx 0, \quad \partial_\mu\psi_{ij} \approx 0, \quad \partial_\mu\partial_\nu\psi_{ij} \approx 0, \tag{22}$$

where

$$\partial_\mu \equiv \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right). \tag{23}$$

According to the Stokes-Helmholtz resolution theorem, refers to, for instance, [31], every sufficiently smooth vector field can be decomposed into irrotational and solenoidal parts. Thus, there exists a scalar function φ and a vector function $\mathbf{R}$ such that the velocity field $\mathbf{u}$ of the $\Omega(0)$ substratum can be represented by [31]

$$\mathbf{u} = \nabla\varphi + \nabla\times\mathbf{R}, \tag{24}$$

where $\nabla\times\varphi = 0$, $\nabla\cdot\mathbf{R} = 0$.

We introduce the following definition of a vector function $\vec{\xi}$

$$\frac{\partial\vec{\xi}}{\partial(ct)} = \nabla\times\mathbf{R}. \tag{25}$$

Putting Eq. (25) into Eq. (24), we have

$$\mathbf{u} = \nabla\varphi + \frac{\partial\vec{\xi}}{\partial(ct)}. \tag{26}$$

Based on Assumption 2 and using Eq. (8) and Eq. (26), Eq. (21) can be written as

$$\begin{aligned} L_{\Omega(0)} &= \frac{1}{2}\rho_0 u^2 = \frac{1}{2}\rho_0\left(\nabla\varphi + \frac{\partial\vec{\xi}}{\partial(ct)}\right)^2 \\ &= \frac{1}{2}\rho_0\left(\frac{m_0}{\rho_0 q_0}\nabla\Phi + \frac{\partial\vec{\xi}}{\partial(ct)}\right)^2. \end{aligned} \tag{27}$$

We introduce the following definitions

$$\psi_{00} = -a_{00}\Phi, \quad \psi_{0i} = \psi_{i0} = a_{0i}\xi_i, \tag{28}$$

$$\vec{\psi}_0 = \psi_{01}\mathbf{i} + \psi_{02}\mathbf{j} + \psi_{03}\mathbf{k}. \tag{29}$$

where $a_{00} > 0$ and $a_{0i} > 0$ are 4 parameters to be determined.

Eqs. (28) and Eqs. (20) have defined a rank 2 symmetric tensor $\psi_{\mu\nu}$. We require that for some special values of a_{00} and a_{0i}, Eq. (27) can be written as

$$\begin{aligned} L_{\Omega(0)} &= \left(\frac{m_0}{q_0\sqrt{2\rho_0}}\frac{1}{a_{00}}\nabla\psi_{00} - \sqrt{\frac{\rho_0}{2}}\frac{\partial(\psi_{0i}/a_{0i})}{\partial(ct)}\mathbf{e}^i\right)^2 \\ &\equiv \left(\nabla\psi_{00} - \frac{\partial\vec{\psi}_0}{\partial(ct)}\right)^2, \end{aligned} \tag{30}$$

where $\mathbf{e}^1 \equiv \mathbf{i}$, $\mathbf{e}^2 \equiv \mathbf{j}$, $\mathbf{e}^3 \equiv \mathbf{k}$.

Comparing the left- and right-hand parts of Eq. (30), we have

$$a_{00} = \sqrt{\frac{m_0^2}{2\rho_0 q_0^2}}, \quad a_{0i} = \sqrt{\frac{\rho_0}{2}}. \tag{31}$$

In order to construct the Lagrangian $L_{\Omega(0)}$ described in Eq. (30) based on the tensorial potential $\psi_{\mu\nu}$, we should consider all the possible products of derivatives of the tensor $\psi_{\mu\nu}$. If we require that the two tensor indices of $\psi_{\mu\nu}$ are different from each other and the two tensor indices of $\psi_{\mu\nu}$ are different from the derivative index, we have the following two possible products ([23], p. 43):

$$L_1 = \partial_\sigma\psi_{\mu\nu}\partial^\sigma\psi^{\mu\nu}, \quad L_2 = \partial_\sigma\psi_{\mu\nu}\partial^\mu\psi^{\sigma\nu}, \tag{32}$$

where $\psi^{\mu\nu} = \eta^{\mu\lambda}\eta^{\nu\sigma}\psi_{\lambda\sigma}$ is the corresponding contravariant tensor of $\psi_{\mu\nu}$.

If there are two indices of $\psi_{\mu\nu}$ which are equal, or one of the indices of $\psi_{\mu\nu}$ is the same as the derivative index, we may have the following three possible products ([23], p. 43):

$$L_3 = \partial_\nu\psi^{\mu\nu}\partial_\sigma\psi^\sigma{}_\mu, \quad L_4 = \partial^\mu\psi_{\mu\nu}\partial^\nu\psi, \tag{33}$$

$$L_5 = \partial_\lambda\psi\partial^\lambda\psi. \tag{34}$$

where ψ is the trace of $\psi_{\mu\nu}$, i.e., $\psi \equiv \psi^\lambda_\lambda = \eta_{\alpha\beta}\psi^{\alpha\beta}$,

$$\partial^\mu \equiv \eta^{\mu\nu}\partial_\nu = \left(\frac{\partial}{\partial x^0}, -\frac{\partial}{\partial x^1}, -\frac{\partial}{\partial x^2}, -\frac{\partial}{\partial x^3}\right). \tag{35}$$

L_3 may be omitted because it can be converted to L_2 by integration by parts ([23], p. 43).

such that the new tensorial potential $\bar{\psi}^{\mu\nu}$ does satisfy the Hilbert gauge condition Eqs. (88).

Using Eqs. (86), the field equations (90) can be written as

$$\partial_\lambda \partial^\lambda \phi^{\mu\nu} = -f_0(T^{\mu\nu} - T^{\mu\nu}_\omega). \tag{92}$$

The field equations (92) can also be written as

$$\eta^{\alpha\beta}\frac{\partial^2 \phi^{\mu\nu}}{\partial x^\alpha \partial x^\beta} = -f_0(T^{\mu\nu} - T^{\mu\nu}_\omega). \tag{93}$$

We noticed that the tensorial field equations (93) are similar to the wave equations of electromagnetic fields.

VII. CONSTRUCTION OF A TENSORIAL POTENTIAL IN INERTIAL REFERENCE FRAMES

The existence of the $\Omega(1)$ substratum allows us to introduce the following definition of inertial reference frames.

Definition 10 *If a coordinates system S is static or moving with a constant velocity relative to the reference frame $S_{\Omega(1)}$, then, we call such a coordinates system as an inertial reference frame.*

The field equations Eqs. (87) and Eqs. (90) are valid in the reference frame $S_{\Omega(1)}$. We will explore the possibility of constructing a tensorial potential in an arbitrary inertial system S'. In an inertial reference frame S, an arbitrary event is characterized by the four spacetime coordinates (t, x, y, z). In an inertial system S', this event is characterized by four other coordinates (t', x', y', z'). We assume that the origins of the Cartesian coordinates in the two inertial systems S and S' coincide at the time $t = t' = 0$. Then, the connections between these spacetime coordinates are given by a homogeneous linear transformation keeping the quantity $s^2 = c^2t^2 - x^2 - y^2 - z^2$ invariant, i.e., ([26], p. 90)

$$s^2 = c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2 = s'^2. \tag{94}$$

We introduce the following two coordinate systems

$$\begin{aligned} &x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z, \\ &x'^0 = ct', \quad x'^1 = x', \quad x'^2 = y', \quad x'^3 = z'. \end{aligned} \tag{95}$$

The homogeneous linear transformation keeping the quantity s^2 invariant, which is usually called the Lorentz transformation, can be written as ([33], p. 57; [26], p. 90)

$$x'^\mu = a^\mu{}_\nu x^\nu, \tag{96}$$

where $a^\mu{}_\nu$ are coefficients depend only on the angles between the spatial axes in the two inertial systems S and S' and on the relative velocity of S and S'.

Applying the standard methods in theory of special relativity [26], we have the following results.

Proposition 11 *Suppose that the field equations Eqs. (92) is valid in the the reference frame $S_{\Omega(1)}$. Then, in an arbitrary inertial system S', there exists a symmetric tensor $\phi'_{\mu\nu}$ satisfies the following wave equation*

$$\partial'_\lambda \partial'^\lambda \phi'^{\mu\nu} = -f_0(T'^{\mu\nu} - T'^{\mu\nu}_\omega), \tag{97}$$

where $T'^{\mu\nu}$ and $T'^{\mu\nu}_\omega$ are corresponding tensors of $T^{\mu\nu}$ and $T^{\mu\nu}_\omega$ in the arbitrary inertial reference frame S' respectively.

Proposition 12 *Suppose that the field equations Eqs. (87) is valid in the reference frame $S_{\Omega(1)}$. Then, in an arbitrary inertial system S', there exists a symmetric tensor $\phi'_{\mu\nu}$ satisfies the following field equation*

$$\begin{aligned} \partial'_\lambda \partial'^\lambda \phi'^{\mu\nu} &- \partial'_\lambda \partial'^\mu \phi'^{\nu\lambda} - \partial'_\lambda \partial'^\nu \phi'^{\mu\lambda} \\ &+ \eta^{\mu\nu} \partial'_\sigma \partial'_\lambda \phi'^{\sigma\lambda} = -f_0(T'^{\mu\nu} - T'^{\mu\nu}_\omega). \end{aligned} \tag{98}$$

VIII. THE EQUATIONS OF MOTION OF A POINT PARTICLE IN A GRAVITATIONAL FIELD AND INTRODUCTION OF AN EFFECTIVE RIEMANNIAN SPACETIME

In this section, we study the equations of motion of a free point particle in a gravitational field. The Lagrangian of a free point particle can be written as ([23], p. 57;[24])

$$L_0 = \frac{1}{2}m\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} = \frac{1}{2}mu^\mu u_\mu = \frac{1}{2}m\eta_{\mu\nu}u^\mu u^\nu, \tag{99}$$

where m is the rest mass of the point particle, $d\tau \equiv \frac{1}{c}\sqrt{dx^\mu dx_\mu}$ is the infinitesimal proper time interval, $u^\mu \equiv dx^\mu/d\tau$.

Suppose that $T^{\mu\nu}_{\Omega(1)} \approx 0$. Ignoring those higher terms $O[(f_0\psi_{\mu\nu})^2]$ in Eq. (51), the interaction term of the Lagrangian of a system of the $\Omega(0)$ substratum, the $\Omega(1)$ substratum and the point particle can be written in the following form ([23], p. 57;[24])

$$L_{\rm int} = f_0\psi_{\mu\nu}mu^\mu u^\nu. \tag{100}$$

Using Eq. (100) and Eq. (99), the total Lagrangian $L_{\rm p}$ of a system of the $\Omega(0)$ substratum, the $\Omega(1)$ substratum and the point particle can be written as ([23], p. 57)

$$L_{\rm p} = L_0 + L_{\rm int} = \frac{1}{2}mu^\mu u_\mu + f_0\psi_{\mu\nu}mu^\mu u^\nu. \tag{101}$$

The Euler-Lagrange equations for the total Lagrangian $L_{\rm p}$ can be written as ([33],p. 111)

$$\frac{d}{d\tau}\left[(\eta_{\mu\nu} + 2f_0\psi_{\mu\nu})\frac{dx^\nu}{d\tau}\right] - f_0\frac{\partial\psi_{\alpha\beta}}{\partial x^\mu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0. \tag{102}$$

We notice that the equations of motion (102) of a point particle in gravitational field are similar to the equations of a geodesic line (104) in a Riemannian spacetime. Thus, it is natural for us to introduce the following definition of a metric tensor $g_{\mu\nu}$ of a Riemannian spacetime ([23], p. 57)

$$g_{\mu\nu} = \eta_{\mu\nu} + 2f_0\psi_{\mu\nu}. \tag{103}$$

Then, the equations of motion Eqs. (102) can be written as ([23], p. 58)

$$\frac{d}{d\tau_g}\left(g_{\mu\nu}\frac{dx^\nu}{d\tau_g}\right) = \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^\mu}\frac{dx^\alpha}{d\tau_g}\frac{dx^\beta}{d\tau_g}, \tag{104}$$

where $d\tau_g$ is the infinitesimal proper time interval in the Riemannian spacetime with a metric tensor $g_{\mu\nu}$.

Eqs. (104) represent a geodesic line in a Riemannian spacetime with a metric tensor $g_{\mu\nu}$, which can also be written as ([34], p. 51)

$$\frac{d^2x^\mu}{d\tau_g^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{d\tau_g}\frac{dx^\sigma}{d\tau_g} = 0, \tag{105}$$

where

$$\Gamma^\nu_{\alpha\beta} \triangleq \frac{1}{2}g^{\mu\nu}\left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu}\right) \tag{106}$$

are the Christoffel symbols.

Thus, we find that the equations of motion (102) of a point particle in gravitational field represent a geodesic line described in Eqs. (105) in a Riemannian spacetime with a metric tensor $g_{\mu\nu}$.

According to Assumption 1, the particles that constitute the $\Omega(1)$ substratum are sinks in the $\Omega(0)$ substratum. Thus, the movements of the $\Omega(1)$ substratum in gravitational field will be different from the Maxwell's equations. We notice that the equations of motion of a point particle in gravitational field (104) are generalizations of the equations of motion of a point particle in vacuum free of gravitational field. The law of propagation of an electromagnetic wave front in vacuum free of gravitational field is Eqs. (10). Thus, the law of propagation of an electromagnetic wave front in gravitational field may be a kind of generalization of Eq. (10). Therefore, we introduce the following assumption.

Assumption 13 *To first order of $f_0\psi_{\mu\nu}$, the law of propagation of an electromagnetic wave front $\omega(x^0, x^1, x^2, x^3)$ in gravitational field is*

$$g_{\mu\nu}\frac{\partial\omega}{\partial x^\mu}\frac{\partial\omega}{\partial x^\nu} = 0, \tag{107}$$

where $\omega(x^0, x^1, x^2, x^3)$ is the electromagnetic wave front, $g_{\alpha\beta}$ is the metric tensor defined in Eqs. (103).

The measurements of spacetime intervals are carried out using light rays and point particles, which are only subject to inertial force and gravitation. Thus, according to Eqs. (104) and Eq. (107), the physically observable metric of spacetime, to first order of $f_0\psi_{\mu\nu}$, is $g_{\mu\nu}$. Thus, the initial flat background spacetime with metric $\eta_{\mu\nu}$ is no longer physically observable [24].

If we can further derive the Einstein's equations (1) using the definition (103) of a metric tensor $g_{\mu\nu}$ of a Riemannian spacetime, then, we may provide a geometrical interpretation of Einstein's theory of gravitation based on the theory of vacuum mechanics [15, 21, 22]. This is the task of the next section.

IX. GENERALIZED EINSTEIN EQUATIONS IN INERTIAL REFERENCE FRAMES

Definition 14 *The Einstein tensor $G_{\mu\nu}$ is defined by*

$$G_{\mu\nu} \triangleq R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R, \tag{108}$$

where $g_{\mu\nu}$ is a metric tensor of a Riemannian spacetime, $R_{\mu\nu}$ is the Ricci tensor, $R \triangleq g^{\mu\nu}R_{\mu\nu}$, $g^{\mu\nu}$ is the corresponding contravariant tensor of $g_{\mu\nu}$ such that $g_{\mu\lambda}g^{\lambda\nu} = \delta^\nu_\mu = g^\nu_\mu$ ([34], p. 40).

According to the geometrical interpretation of some theories of gravitation in flat spacetime [24], the physically observable metric $g_{\mu\nu}$ of spacetime in Eqs. (103) can be written as

$$g^{\mu\nu} = \eta^{\mu\nu} - 2f_0\psi^{\mu\nu} + O[(f_0\psi^{\mu\nu})^2]. \tag{109}$$

Following the clue showed in Eqs. (109) and noticing the methods of S. N. Gupta [35] and W. Thirring [24], we introduce the following definition of a metric tensor of a Riemannian spacetime.

Definition 15

$$\tilde{g}^{\mu\nu} \triangleq \sqrt{-g_0}g^{\mu\nu} \triangleq \eta^{\mu\nu} - 2f_0\phi^{\mu\nu}, \tag{110}$$

where $g_0 = \mathrm{Det}\ g_{\mu\nu}$.

We have the following expansion of the contravariant metric tensor $g^{\mu\nu}$ [35]

$$\begin{aligned} g^{\mu\nu} &= \eta^{\mu\nu} - 2f_0\phi^{\mu\nu} + f_0\eta^{\mu\nu}\eta_{\alpha\beta}\phi^{\alpha\beta} \\ &\quad -2f_0^2\eta_{\alpha\beta}\phi^{\alpha\beta}\phi^{\mu\nu} + f_0^2\eta^{\mu\nu}\eta_{\alpha\sigma}\eta_{\beta\lambda}\phi^{\alpha\beta}\phi^{\lambda\sigma} \\ &\quad +\frac{1}{2}f_0^2\eta^{\mu\nu}\eta_{\alpha\beta}\eta_{\lambda\sigma}\phi^{\alpha\beta}\phi^{\lambda\sigma} + O[(f_0\phi^{\mu\nu})^3]. \end{aligned} \tag{111}$$

Definition 16 *If $\phi^{\mu\nu}$ and their first and higher derivatives satisfy the following conditions*

$$|2f_0\phi^{\mu\nu}| \ll 1, \tag{112}$$

$$\left|\frac{\partial^n(2f_0\phi^{\mu\nu})}{\partial(x^\alpha)^n}\right| \ll 1, n = 1, 2, 3, \cdots \tag{113}$$

then we call this field $\phi^{\mu\nu}$ weak.

For weak fields, $\psi \approx \phi \approx 0$. Thus, $\phi^{\mu\nu} = \psi^{\mu\nu} - 1/2 \cdot \eta^{\mu\nu}\psi \approx \psi^{\mu\nu}$. From Eqs. (111), we see that the definition (110) is compatible with Eqs. (109).

the possibility to derive the Einstein's equations (1) in non-inertial reference frames.

When solving the Einstein's equations (1) for an isolated system of masses, V. Fock introduces harmonic reference frame and obtains an unambiguous solution ([27],p. 369). Furthermore, in the case of an isolated system of musses, he concludes that there exists a harmonic reference frame which is determined uniquely apart from a Lorentz transformation if suitable supplementary conditions are imposed ([27],p. 373). It is known that wave equations, for instance, Eqs. (97), keep the same form under Lorentz transformations [26]. Thus, we speculate that Fock's special harmonic reference frames may have provided us a clue for us to derive the Einstein's equations (1) in some special class of non-inertial reference frames.

We introduce an arbitrary coordinate system (x'^0, x'^1, x'^2, x'^3) and denote it as S_n. It is known that a particle in a non-inertial reference frame will experiences an inertial force. Unfortunately, we have no knowledge about the origin of inertial forces.

The equivalence between inertial mass and gravitational mass implies that to some degree gravitational forces behave in the same way as inertial forces that result from non-inertial reference frames ([33],p. 17). Thus, we introduce the following concepts for inertial forces similar to those concepts for gravitational interactions.

Definition 25 *Inertial potential $\psi^{\rm iner}_{\mu\nu}$ is an interaction potential between a matter system and vacuum resulting from the inertial force $\mathbf{F}_{iner}$ exerted on the matter system by vacuum in a non-inertial reference frame S_n.*

Definition 26 *Inertial force Lagrangian $L_{\rm iner}$ is an interaction Lagrangian between a matter system and vacuum resulting from the inertial force $\mathbf{F}_{iner}$ exerted on the matter system by vacuum in a non-inertial reference frame S_n.*

Now our task is to explore possible expressions of inertial potential $\psi^{\rm iner}_{\mu\nu}$ and inertial force Lagrangian $L_{\rm iner}$. Similar to Eqs. (159), the inertial acceleration $\mathbf{a}$ of a test point particle in the non-inertial reference frame S_n can be written as

$$a_i = -c^2\frac{\partial}{\partial x^i}\left(\frac{1-\eta'_{00}}{2}\right) - c^2\frac{\partial(-\eta'_{i0})}{\partial(ct)}, \tag{160}$$

where $\eta'_{\mu\nu}$ is the corresponding metric tensor of the non-inertial reference frame S_n.

If η'_{i0} are time-independent, the inertial acceleration $\mathbf{a}$ of the test point particle in Eqs. (160) simplifies to ([26], p. 280)

$$a_i = -c^2\frac{\partial}{\partial x^i}\left(\frac{1-\eta'_{00}}{2}\right). \tag{161}$$

Using Eqs. (161), the inertial force $\mathbf{F}_{iner}$ exerted on the test point particle can be written as

$$\mathbf{F}_{iner} = m\mathbf{a} = -mc^2\nabla\left(\frac{1-\eta'_{00}}{2}\right), \tag{162}$$

where m is the mass of the test point particle.

From Eqs. (162), the inertial force Lagrangian of a system of vacuum and the test point particle can be written as

$$L_{\rm iner1} = -mc^2\left(\frac{1-\eta'_{00}}{2}\right). \tag{163}$$

Therefore, the inertial force Lagrangian of a system of vacuum and continuously distributed particles can be written as

$$L_{\rm iner} = -\rho_m c^2\left(\frac{1-\eta'_{00}}{2}\right). \tag{164}$$

Noticing $T^{00}_m = \rho_m c^2$ and $\eta_{00} = 1$, the inertial force Lagrangian $L_{\rm iner}$ can be written as

$$L_{\rm iner} = f_0\psi^{\rm iner}_{00}T^{00}_m, \tag{165}$$

where

$$\psi^{\rm iner}_{00} = -\frac{1}{2f_0}(\eta_{00}-\eta'_{00}). \tag{166}$$

Inspired by Eqs. (165) and Eq. (166), we introduce the following assumption.

Assumption 27 *Suppose that the inertial force Lagrangian $L_{\rm iner}$ of a system of a free point particle and vacuum in the non-inertial reference frame S_n can be written as*

$$L_{\rm iner} = f_0\psi^{\rm iner}_{\mu\nu}mu'^{\mu}u'^{\nu}, \tag{167}$$

where m is the rest mass of the point particle, $u'^{\mu} \equiv dx'^{\mu}/d\tau_{\eta'}$, $\tau_{\eta'}$ is the proper time,

$$\psi^{\rm iner}_{\mu\nu} = -\frac{1}{2f_0}(\eta_{\mu\nu}-\eta'_{\mu\nu}). \tag{168}$$

Following similar methods in section VIII, we obtain the following result.

Proposition 28 *Suppose that Assumption 27 is valid. Then, the equations of motion of a free point particle can be written as*

$$\frac{d^2x'^{\mu}}{d\tau^2_{\eta'}} + \Gamma'^{\mu}_{\nu\sigma}\frac{dx'^{\nu}}{d\tau_{\eta'}}\frac{dx'^{\sigma}}{d\tau_{\eta'}} = 0, \tag{169}$$

where $\Gamma'^{\nu}_{\alpha\beta}$ are the corresponding Christoffel symbols in the non-inertial reference frame S_n.

Proof of Proposition 28. The Lagrangian of a free point particle in S_n can be written as ([23], p. 57;[24])

$$L'_0 = \frac{1}{2}m\frac{dx'^{\mu}}{d\tau_{\eta'}}\frac{dx'_{\mu}}{d\tau_{\eta'}} = \frac{1}{2}mu'^{\mu}u'_{\mu} = \frac{1}{2}m\eta'_{\mu\nu}u'^{\mu}u'^{\nu}, \tag{170}$$

where m is the rest mass of the point particle, $d\tau \equiv \frac{1}{c}\sqrt{dx'^{\mu}dx'_{\mu}}$ is the infinitesimal proper time interval, $u'^{\mu} \equiv dx'^{\mu}/d\tau_{\eta'}$.

Suppose that $T'^{\mu\nu}_{\Omega(1)} \approx 0$. Using Eq. (170) and Eq. (167), the total Lagrangian L_p of a system of the $\Omega(0)$ substratum, the $\Omega(1)$ substratum and the point particle can be written as

$$L_\mathrm{p} = L'_0 + L_\mathrm{iner} = \frac{1}{2} mu'^\mu u'_\mu + f_0 \psi^\mathrm{iner}_{\mu\nu} mu'^\mu u'^\nu. \quad (171)$$

The Euler-Lagrange equations for the total Lagrangian L_p can be written as ([33],p. 111)

$$\frac{\partial L_\mathrm{p}}{\partial x'^\mu} - \frac{d}{d\tau_{\eta'}} \frac{\partial L_\mathrm{p}}{\partial u'^\mu} = 0. \quad (172)$$

Putting Eq. (171) into Eqs. (172), we have

$$\frac{d}{d\tau_{\eta'}}\left[\left(\eta_{\mu\nu} + 2f_0\psi^\mathrm{iner}_{\mu\nu}\right)\frac{dx'^\nu}{d\tau_{\eta'}}\right] - f_0 \frac{\partial \psi^\mathrm{iner}_{\alpha\beta}}{\partial x'^\mu}\frac{dx'^\alpha}{d\tau_{\eta'}}\frac{dx'^\beta}{d\tau_{\eta'}} = 0. \quad (173)$$

Using Eq. (168), Eqs. (173) can be written as

$$\frac{d}{d\tau_{\eta'}}\left(\eta'_{\mu\nu}\frac{dx'^\nu}{d\tau_{\eta'}}\right) - \frac{1}{2}\frac{\partial \eta'_{\alpha\beta}}{\partial x'^\mu}\frac{dx'^\alpha}{d\tau_{\eta'}}\frac{dx'^\beta}{d\tau_{\eta'}} = 0. \quad (174)$$

Eqs. (174) represent a geodesic line in a Riemannian spacetime with a metric tensor $\eta'_{\mu\nu}$, which can also be written as Eqs. (169) ([34], p. 50). □

Eqs. (169) is a geodesic curve in a Minkowski spacetime. It is known that a geodesic curve is a straight line in a Minkowski spacetime ([36], p. 235). For instance, according to Newton's first law, a free particle moves along a straight line in the Galilean coordinates. Therefore, Assumption 27 may be supports by some experiments. Thus, inspired by the inertial force Lagrangian for a free point particle in Eq. (167), we introduce the following assumption for a matter system.

Assumption 29 *The inertial force Lagrangian L_iner of a matter system and vacuum in the non-inertial reference frame S_n can be written as*

$$L_\mathrm{iner} = f_0\psi^\mathrm{iner}_{\mu\nu}(T'^{\mu\nu}_m + T'^{\mu\nu}_{\Omega(1)}) + O[(f_0\psi^\mathrm{iner}_{\mu\nu})^2], \quad (175)$$

where $T'^{\mu\nu}_m$ and $T'^{\mu\nu}_{\Omega(1)}$ are the contravariant energy-momentum tensors of the system of the matter and the $\Omega(1)$ substratum respectively, $O[(f_0\psi^\mathrm{iner}_{\mu\nu})^2]$ denotes those terms which are small quantities of the order of $(f_0\psi^\mathrm{iner}_{\mu\nu})^2$.

XIII. FIELD EQUATIONS IN A SPECIAL CLASS OF NON-INERTIAL REFERENCE FRAMES

Suppose that the transformation equations between a non-inertial coordinate system $(x'^0,\ x'^1,\ x'^2,\ x'^3)$ and the Galilean coordinates (ct, x, y, z) are

$$x'^\alpha = f^\alpha(x^0,\ x^1,\ x^2,\ x^3). \quad (176)$$

Following V. Fock ([27],p. 370-373), we introduce the following definition of a special class of reference frames.

Definition 30 *Suppose that a coordinate system $(x'^0,\ x'^1,\ x'^2,\ x'^3)$ satisfies the following conditions: (1) every coordinates x'^α satisfies the d'Alembert's equation ([27],p. 369), i.e.,*

$$\Box_{\eta'} x'^\alpha \triangleq \frac{1}{\sqrt{-\eta'_0}}\frac{\partial}{\partial x'^\mu}\left(\sqrt{-\eta'_0}\eta'^{\mu\nu}\frac{\partial x'^\alpha}{\partial x'^\nu}\right) = 0, \quad (177)$$

where $\eta'_{\mu\nu}$ is the metric of the reference frame S_n, $\eta'_0 =$ Det $\eta'_{\mu\nu}$; (2) every coordinates x'^α converges to the Galilean coordinates (ct, x, y, z) at large enough distance, i.e.,

$$\lim_{r\to\infty} x'^\alpha = x^\alpha, \quad (178)$$

where $r = \sqrt{x^2+y^2+z^2}$; (3) $\eta'^{\mu\nu} - (\eta'^{\mu\nu})_\infty$ are outgoing waves, i.e., $\eta'^{\mu\nu} - (\eta'^{\mu\nu})_\infty$ satisfy the following condition of outward radiation: for $r\to\infty$, and all values of $t'_0 = t + r/c$ in an arbitrary fixed interval the following limiting conditions are satisfied ([27],p. 365)

$$\lim_{r\to\infty}\left[\frac{\partial[r(\eta'^{\mu\nu} - (\eta'^{\mu\nu})_\infty)]}{\partial r} + \frac{1}{c}\frac{\partial[r(\eta'^{\mu\nu} - (\eta'^{\mu\nu})_\infty)]}{\partial t}\right] = 0, \quad (179)$$

where $(\eta'^{\mu\nu})_\infty$ denotes the value of $\eta'^{\mu\nu}$ at infinity. Then, we call this coordinate system $(x'^0,\ x'^1,\ x'^2,\ x'^3)$ as a Fock coordinate system.

We use S_F to denote a Fock coordinate system. The Galilean coordinate system (ct, x, y, z) is a Fock coordinate system. V. Fock points out an advantage of Fock coordinate system:"When solving Einstein's equations for an isolated system of masses we used harmonic coordinates and in this way obtained a perfectly unambiguous solution." ([27],p. 369) Here the harmonic coordinates called by V. Fock are Fock coordinate systems.

According to a theorem of Fock about Fock coordinate systems ([27],p. 369-373), the transformation equations (176) from one Fock coordinate system to another can be written as

$$x'^\mu = a^\mu{}_\nu x^\nu, \quad (180)$$

i.e., a Lorentz transformation.

For convenience, we introduce the following noatons

$$\partial'_\mu \equiv \left(\frac{\partial}{\partial x'^0}, \frac{\partial}{\partial x'^1}, \frac{\partial}{\partial x'^2}, \frac{\partial}{\partial x'^3}\right), \quad \partial'^\mu \equiv \eta'^{\mu\nu}\partial'_\nu. \quad (181)$$

Proposition 31 *Suppose that the reference frame S_F is a Fock coordinate system and Assumptions 29 is valid, then the total Lagrangian L'_tot of a system of the $\Omega(0)$ substratum, the $\Omega(1)$ substratum, vacuum and matter in S_F can be written as*

$$\begin{aligned} L'_\mathrm{tot} = {} & \frac{1}{2}\partial'_\lambda\psi'_{\mu\nu}\partial'^\lambda\psi'^{\mu\nu} - 2\partial'_\lambda\psi'_{\mu\nu}\partial'^\mu\psi'^{\lambda\nu} - 6\partial'^\mu\psi'_{\mu\nu}\partial'^\nu\psi' \\ & -\frac{3}{2}\partial'_\lambda\psi'\partial'^\lambda\psi' + L'_\mathrm{more} + f_0\psi'_{\mu\nu}(T'^{\mu\nu}_m + T'^{\mu\nu}_{\Omega(1)}) \\ & + f_0\psi^\mathrm{iner}_{\mu\nu}(T'^{\mu\nu}_m + T'^{\mu\nu}_{\Omega(1)}) + O[(f_0\psi^\mathrm{iner}_{\mu\nu})^2] \\ & + O[(f_0\psi'_{\mu\nu})^2], \end{aligned} \quad (182)$$

at least the following differences between this theory and Einstein's theory of general relativity.

(1) In Einstein's theory, Einstein's equations (1) are assumptions [1, 2, 26]. Although A. Einstein introduced his new concept of gravitational aether ([37], p63-113), he did not derive his equations (1) theoretically based on his new concept of the gravitational aether. In our theory, the generalized Einstein's equations (114) and (230) are derived by methods of special relativistic continuum mechanics based on some assumptions.

(2) Although the theory of general relativity is a field theory of gravity, the definitions of gravitational fields are not based on continuum mechanics [1, 2, 26, 38–41]. Because of the absence of a continuum, the theory of general relativity may be regarded as a phenomenological theory of gravity. In our theory, gravity is transmitted by the $\Omega(0)$ substratum. The tensorial potential $\psi_{\mu\nu}$ of gravitational fields are defined based on special relativistic continuum mechanics.

(3) In Einstein's theory, the concept of Riemannian spacetime is introduced together with the field equations (1) [1, 2, 26]. The theory of general relativity can not provide a physical definition of the metric tensor of the Riemannian spacetime. In our theory, the background spacetime is the Minkowshi spacetime. However, the initial flat background spacetime is no longer physically observable. According to the equation of motion of a point particle in gravitational field (104), to the first order of $f_0\psi_{\mu\nu}$, the physically observable spacetime is a Riemannian spacetime with the metric tensor $g_{\mu\nu}$. The metric tensor $g_{\mu\nu}$ is defined based on the tensorial potential $\psi_{\mu\nu}$ of gravitational fields.

(4) The masses of particles are constants in Einstein's theory of general relativity [1, 2, 26]. In our theory, the masses of particles are functions of time t [15].

(5) The gravitational constant γ_N is a constant in Einstein's theory of general relativity [1, 2, 26]. The theory of general relativity can not provide a derivation of γ_N. In our theory, the parameter γ_N is derived theoretically. From Eq.(4), we see that γ_N depends on time t.

(6) In our theory, the parameter γ_N in Eq.(9) depends on the density ρ_0 of the $\Omega(0)$ substratum. If ρ_0 varies from place to place, i.e., $\rho_0 = \rho_0(t, x, y, z)$, then the space dependence of the gravitational constant γ_N can be seen from Eq.(4).

(7) In Einstein's theory, equations (1) are supposed to be valid in all reference frames [1, 2, 26]. In our theory, the generalized Einstein's equations (114) are valid only in inertial reference frames. The generalized Einstein's equations (230) are valid only in some special non-inertial reference frames.

(8) In Einstein's theory, equations (1) are rigorous [1, 2, 26]. However, in our theory, Eqs.(239) are valid approximately under some assumptions.

XVI. CONCLUSION

We extend our previous theory of gravitation based on a sink flow model of particles by methods of special relativistic fluid mechanics. In inertial reference frames, we construct a tensorial potential of the $\Omega(0)$ substratum. Based on some assumptions, we show that this tensorial potential satisfies the wave equation. Inspired by the equation of motion of a test particle, a definition of a metric tensor of a Riemannian spacetime is introduced. Generalized Einstein's equations in inertial reference frames are derived based on some assumptions. These equations reduce to Einstein's equations in case of weak field in harmonic reference frames. In some special non-inertial reference frames, generalized Einstein's equations are derived based on some assumptions. If the field is weak and the reference frame is quasi-inertial, these generalized Einstein's equations reduce to Einstein's equations. Thus, this theory may also explains all the experiments which support the theory of general relativity. In our theory, gravity is transmitted by the $\Omega(0)$ substratum. The theory of general relativity can not provide a physical definition of the metric tensor of the Riemannian spacetime. In our theory, the background spacetime is the Minkowshi spacetime. However, the flat background spacetime is no longer physically observable. According to the equation of motion of a point particle in gravitational field, to the first order, the physically observable spacetime is a Riemannian spacetime. The metric tensor of this Riemannian spacetime is defined based on the tensorial potential of gravitational fields.

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[1] S. Weinberg, *Gravitation and cosmology* (John Wiley & sons, New York, 1972).
[2] C. Misner, K. Thorne, and J. Wheeler, *Gravitation* (Freeman, New York, 1973).
[3] A. Hees and et al., Phys. Rev. Lett. **118**, 211101 (2017).
[4] J. Maddox, *What remains to be discovered* (The Free Press, New York, 1998).
[5] D. Marsh, Phys. Rev. Lett. **118**, 011302 (2017).
[6] A. Cho, Science **355**, 560 (2017).
[7] P. Dirac, *Directions in Physics* (John Wiley & sons, New York, 1978).
[8] J. Kim, P. Sikivie, and S. Weinberg, Nature **541**, 464 (2017).
[9] M. Farhoudi and M. Yousefian, Int. J. Theor. Phys. **55**, 2436 (2016).
[10] D. R. Long, Nature **260**, 417 (1976).

[11] A. Balcerzak, M. P. Dabrowski, and V. Salzano, Annalen der Physik **529**, 1600409 (2017).
[12] N. Cornish, D. Blas, and G. Nardini, Phys. Rev. Lett. **119**, 161102 (2017).
[13] D. Giovannini, J. Romero, V. Potocek, G. Ferenczi, F. Speirits, S. Barnett, D. Faccio, and M. Padgett, Science **347**, 857 (2015).
[14] J. R. Sambles, Science **347**, 828 (2015).
[15] X.-S. Wang, Progress in Physics **4**, 25 (2008).
[16] E. Whittaker, *A History of the Theories of Aether and Electricity, Revised and enlarged edtion, vol. 1* (Thomas Nelson and Sons Ltd., London, 1951).
[17] B. Riemann, *Collected Papers, Translated from the 1892 edition by Roger Baker, Charles Christenson and Henry Orde* (Kendrick Press, Heber, 2004).
[18] H. Poincare, *The science and hypothesis, translated from the French edtion, in Chinese* (Shangwu Press, Beijing, 1997).
[19] A. Einstein and L. Infeld, *The evolution of physics* (Cambridge University Press, London, 1938).
[20] I. G. Currie, *Fundamental Mechanics of Fluids* (Cambridge University Press, 2003), 3rd ed.
[21] X.-S. Wang, Progress in Physics **2**, 111 (2008).
[22] X.-S. Wang, Physics Essays **27**, 398 (2014).
[23] R. P. Feynman, *Feynman lectures on gravitation, foreword by John Preskill and Kip S. Thorne* (Addison-Wesley Publishing Company, New York, 1995).
[24] W. Thirring, Annals of Physics **16**, 96 (1961).
[25] S. Weinberg, Physical Review **138**, B988 (1965).
[26] C. Møller, *The theory of relativity, second edition* (Clarendon Press, Oxford, 1972).
[27] V. Fock, *The Theory of Space Time and Gravitation, 2nd Revised Edition, Translated from the Russian by N. Kemmer* (Pergamon Press, London, 1964).
[28] W. Rindler, *Introduction to special relativity* (Clarendon Press, Oxford, 1982).
[29] Z.-D. Chen, *Rational mechanics, in Chinese* (Chongqing Press, Chongqing, 2000).
[30] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics, 2nd Edition, translated from the Russian by J. B. Sykes and W. H. Reid* (Pergamon Press, Oxford, 1987).
[31] A. C. Eringen, *Elastodynamics* (Academic Press, New York, 1975).
[32] S. V. Babak and L. P. Grishchuk, Physical Review D **61**, 024038 (1999).
[33] H. C. Ohanian and R. Ruffini, *Gravitation and Spacetime, 3rd edition* (Cambridge University Press, New York, 2013).
[34] L. Liu, *The theory of general relativity, in Chinese, 2nd edition* (Advanced Education Press, Beijing, 2004).
[35] S. N. Gupta, Proc. Phys. Soc. **A65**, 608 (1952).
[36] V. Faraoni, *Special relativity* (Springer, New York, 2013).
[37] L. Kostro, *Einstein and the Ether* (Apeiron, Montreal, 2000).
[38] C. Truesdell, *The Elements of Continuum Mechanics* (Springer-Verlag, New York, 1966).
[39] Y. C. Fung, *A First Course in Continuum Mechanics* (Prentice-Hall, London, 1977).
[40] A. C. Eringen, *The Elements of Continuum Mechanics* (Robert E. Krieger Pub. Co., Huntington, 1980).
[41] L. D. Landau and Lifshitz, *Fluid Mechanics, translated from the Russian by J.B. Sykes and W.H. Reid.* (Pergamon, New York, 1987).