

1 Article

2 Positioning method of weighted centroid aided 3 inertial measurement

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12 **Abstract:** Aiming at the defects of low precision and time cumulative error, an external wireless
13 signal weighted centroid localization algorithm aided inertial positioning method is designed in
14 this paper. According to the signal strength of each anchor node received at the test point, the
15 distance between the anchor node and the anchor node is obtained by using the attenuation model
16 of the wireless signal. Three anchor nodes are used to measure the distance between the anchor
17 node and the measured point. We can obtain the area to be measured according to the actual
18 situation, the position of the measured point is obtained by the weighted centroid localization
19 algorithm and a combined model of wireless signal aided inertial navigation system is established.
20 The simulation results show that the method can greatly improve the positioning accuracy and
21 restrain the divergence of the longitude error and latitude error.

22 **Keywords:** weighted centroid; signal intensity; attenuation model; combined model

23

24 1. Introduction

25 Due to the development of the Internet and the popularity of mobile devices and personal
26 devices, LBS (Based Services Location) is becoming increasingly important, users get positioning
27 information [1] and use it for navigation, tracking, monitoring, pushing information and other
28 services. GPS can easily provide outdoor personal location information, but because the GPS needs
29 to receive at least 4 satellites to achieve positioning. Therefore, the influence of the satellite signal has
30 a great influence on the positioning effect, and it has a serious impact on the positioning effect in the
31 indoor environment due to the occlusion of the satellite signal.

32 Inertial navigation system has the advantages of strong autonomy, high output frequency,
33 short time accuracy, especially the rapid development of MEMS IMU in recent years makes it
34 become small and the cost is lower. It often uses the Pedestrian Dead Reckoning method with
35 MEMS IMU for Indoor positioning. At present, most of the Pedestrian Dead Reckoning methods
36 are to fix the inertial sensor on the human torso [2] or foot [3]. The lack of flexibility of this approach
37 has greatly affected the user experience. Zero Velocity Update Colman filtering algorithm [4] can
38 limit the error in the range of linear growth, it can also help Colman filter to eliminate the
39 accumulated error of inertial navigation direction with gyroscopes, electronic compass and
40 magnetic measurement meter value [5]. To a certain extent, this method improves the precision of
41 navigation, but it will produce a certain cumulative error with the increase of time.

42 Wireless positioning has been widely used due to the popularity of Wi-Fi hotspots, wireless
43 sensor networks, Bluetooth and other wireless signals. In order to solve the problem of long time
44 accumulation of inertial positioning errors, this paper uses the wireless signal weighted centroid
45 algorithm to assist the inertial positioning, and derive the state equation and the measurement
46 equation in detail. Colman filter is used to estimate the optimal combination system, and simulate

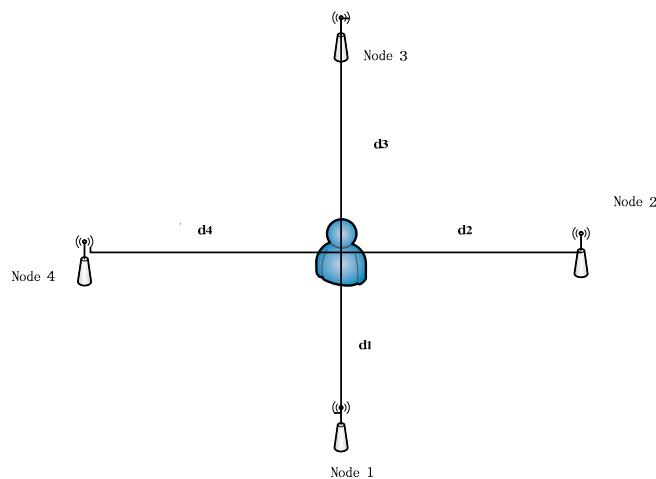
47 the effect of the combination. The simulation results show that the combined system can restrain
 48 the divergence of the error, and greatly improve the positioning accuracy. At the same time, it is
 49 compared with the traditional centroid location algorithm aided inertial positioning. The results
 50 show that the auxiliary effect of the weighted centroid localization is better, and the positioning
 51 accuracy is higher.

52 **2. Weighted centroid localization method for wireless signals**

53 Some methods are commonly used to measure the distance between nodes, such as RSSI
 54 (Received Signal Strength Indicator) [6], TOA (Time of Arrival), TDOA (Time Difference of Arrival),
 55 AOA (Angel of Arrival) and other positioning algorithms.

56 TOA and TDOA introduce the concept of time and add additional hardware requirements.
 57 AOA positioning error is relatively large. The localization algorithm based on RSSI is easy to
 58 implement and does not require additional hardware overhead. But RSSI positioning algorithm
 59 signal is vulnerable to the impact of the environment in the propagation process, such as reflection,
 60 diffraction, nonvisual range, etc. In order to solve this problem, this paper uses the method of
 61 adjusting the signal transmission factor to adjust the model parameters of the path loss dynamically,
 62 so as to reduce the positioning error. The distance between the measured point and the anchor node
 63 is measured according to the wireless signal, so as to determine the area where the measurement
 64 point is located, and further the position of the measured point is obtained according to the
 65 weighted centroid localization algorithm.

66 *2.1. Basic principle of signal strength ranging*
 67



68 **Figure 1.** Wireless Location Network Structure

69 RSSI ranging is very easy to be affected by the environment, therefore, how to further improve
 70 the range accuracy is the key to ranging technology [7]. The research shows that the transmission
 71 loss can be converted to a distance by using the propagation loss model, which is known to be the
 72 power of the transmitting node and the receiving node, the traditional wireless signal attenuation
 73 model is:

$$74 \quad \text{RSSI}(d) = \text{RSSI}(d_0) - 10n \lg(d/d_0) + \zeta_\sigma \quad (1)$$

75 Where $\text{RSSI}(d)$ indicates the received signal strength value, which goes to launch anchor
 76 nodes at a distance of d . $\text{RSSI}(d_0)$ indicates the received signal strength value, which goes to
 77 launch anchor nodes at a distance of d_0 . d_0 is generally used as the reference distance. Because
 78 the distance is farther, the attenuation is more serious, so we generally select the distance from the
 79 launch anchor node closer. Generally set it to 1m. n represents path attenuation factor(range
 80 between 2 and 4). ζ_σ represents normal random variable and σ represents standard deviation.

81 When d_0 is 1, equation (1) can be converted to:

82

$$RSSI(d) = RSSI(1) - 10n \lg(d) + \zeta_\sigma \quad (2)$$

83 Because the traditional signal model has the disadvantage of being influenced by the
 84 environment easily. In this paper, we use the method of adjusting the model factor to improve the
 85 signal attenuation model. Select three anchor nodes randomly in Figure 1. Assuming that the
 86 selected anchor node is 1, 2, 3, and 1 is anchor node, 2 and 3 are points to be located. Assuming that
 87 is 1, according to the signal attenuation model, we can get:

$$RSSI(d_{12}) = RSSI(1) + 10n_1 \lg(d_{12}) + (\zeta_\sigma)_1 \quad (3)$$

$$RSSI(d_{13}) = RSSI(1) + 10n_1 \lg(d_{13}) + (\zeta_\sigma)_1 \quad (4)$$

90 Assuming that $\lambda_1 = RSSI(1) + (\zeta_\sigma)_1$, and we can get propagation parameters λ_1 and n_1 of
 91 anchor node 1, from equation (3) and equation (4).

92 Usually, there will be an anchor node nearest to the anchor node in the large number of anchor
 93 nodes. Because the closer to the anchor node, the less affected by the environment, then the anchor
 94 node is the optimal anchor node. Therefore, the parameter adjustment factor is introduced:

$$95 \quad e_i = \frac{RSSI(d_i) - RSSI(d_1)}{\sum_{i=2}^n [RSSI(d_i) - RSSI(d_1)]} \quad (5)$$

96 Where n is number of anchor nodes, d_1 is the distance between the optimal anchor node
 97 and the location node, $RSSI(d_i)$ is the signal strength of the $i(i=1, 2, 3..)$ node received by the
 98 anchor node.

99 Then the adjustment model propagation parameters are:

$$100 \quad \lambda = \sum_{i=1}^n \frac{1 - e_i}{n-1} \times \lambda_i \quad (6)$$

$$101 \quad n = \sum_{i=1}^n \frac{1 - e_i}{n-1} \times n_i \quad (7)$$

102 The adjustment factor is introduced to make the propagation parameters of the nodes close to
 103 the positioning node get better. The propagation path loss model is:

$$104 \quad RSSI(d) = \lambda + 10n \lg(d) \quad (8)$$

105 Before wireless location, we should get the propagation parameters λ , n , e of the
 106 corresponding propagation environment. Therefore, it can improve the accuracy of the propagation
 107 loss model, so as to improve the accuracy of wireless location.

108 *2.2. Weighted centroid localization algorithm*

109 The signal intensity of the transmitting node and the transmitting node are known, RSSI can
 110 calculate the propagation loss of the signal according to the signal strength received by the receiving
 111 node. Then the propagation loss is transformed to the distance by using the theory and empirical
 112 model, and the position of the node is calculated according to the distance [8]. In theory, circles are
 113 drawn based on the distance from the propagation loss model at each anchor node, and these circles
 114 intersect at a point. This is the point of theory to be located, but in the actual situation all circles do
 115 not intersect at one point. Choose 1, 2, 3 node as an example, the actual situation may be the
 116 following:

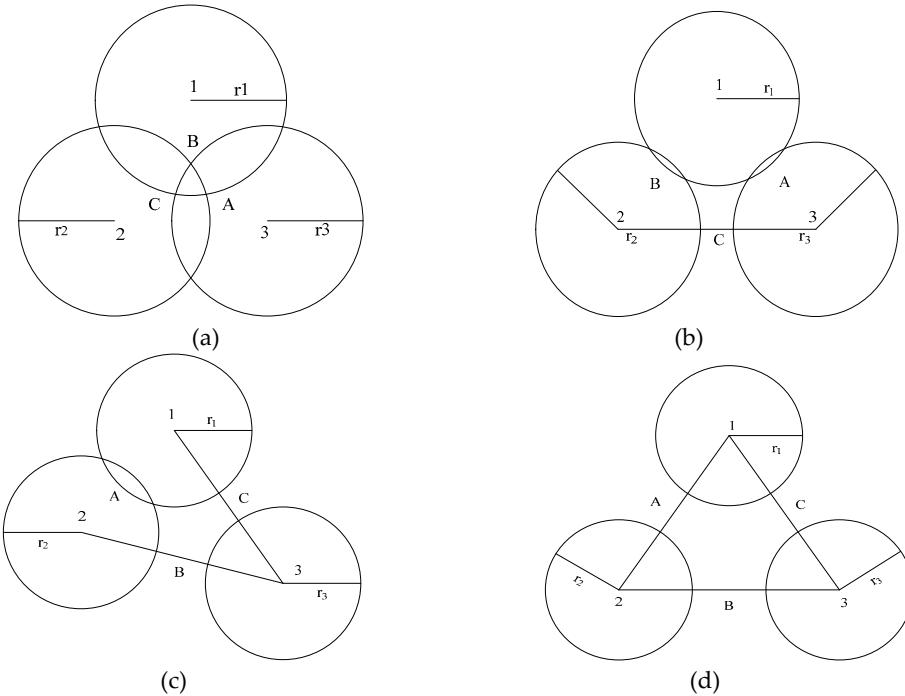


Figure 2. Wireless Location of The Actual Situation

Take two equations randomly and go simultaneous, and the intersection point of the two circle is obtained. When two circles have two points of intersection, choose the point closer to the third circle, which is outside the two circles. If two circles have no intersection, then take the midpoint of the connecting line of the center, so we get three points A, B, C, as shown in Figure 2. And the unknown point must be in the region of A, B, and C. Obviously, with the increase of the number of the communication anchor nodes, the possible area of the unknown node is gradually reduced [9]. Take Figure 2 (a) as an example, the centroid of A, B, C is used as the position of the unknown point:

$$x = (x_A + x_B + x_C) / 3 \quad (9)$$

$$y = (y_A + y_B + y_C) / 3 \quad (10)$$

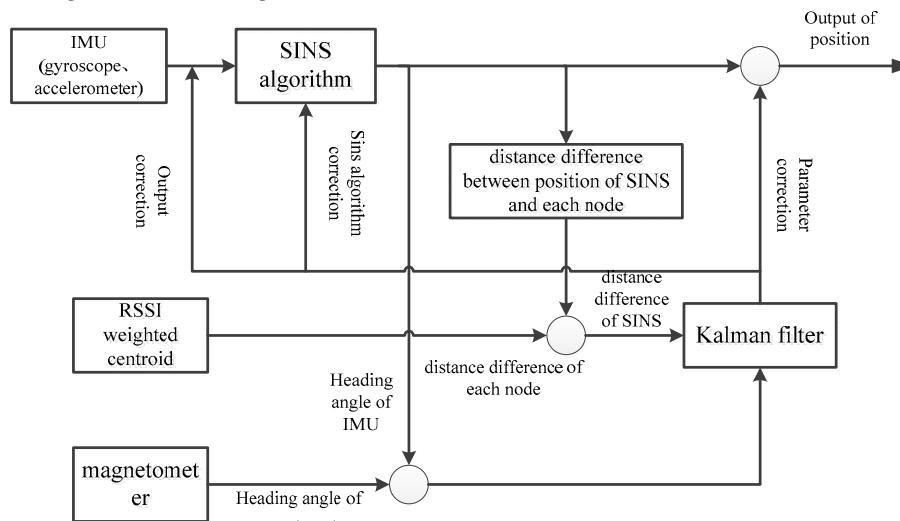
Because the coordinate of the unknown point is located through the A, B, C points, traditional centroid localization algorithm does not consider A, B, C to the extent of the impact of the unknown point, so the traditional centroid localization algorithm has a certain error in positioning. In this paper, a weighted method is used to reflect the contribution of different vertices to the unknown point. The farther away from the anchor node, the smaller the proportion of the corresponding node coordinates, which is inversely proportional to the distance, each vertex is determined by two distances. As is shown in Figure 2 (a), A point is determined by circle 1 and circle 2, then the weight of the corresponding A point should be $1/(r_1 + r_2)$. Similarly, the weight of the B point is $1/(r_2 + r_3)$ and the weight of the C point is $1/(r_3 + r_1)$. Then the unknown point coordinates determined by weighted centroid localization algorithm are:

$$x = \frac{\frac{x_A}{r_1 + r_2} + \frac{x_B}{r_2 + r_3} + \frac{x_C}{r_3 + r_1}}{\frac{1}{r_1 + r_2} + \frac{1}{r_2 + r_3} + \frac{1}{r_3 + r_1}} \quad (11)$$

$$y = \frac{\frac{y_A}{r_1 + r_2} + \frac{y_B}{r_2 + r_3} + \frac{y_C}{r_3 + r_1}}{\frac{1}{r_1 + r_2} + \frac{1}{r_2 + r_3} + \frac{1}{r_3 + r_1}} \quad (12)$$

143 3. Wireless signal assisted SINS positioning principle

144 Due to the development of MEMS technology, now the mobile terminal used for positioning of
 145 the inertial sensors are mostly MEMS IMU, because the gyro drifts very large and the earth's rotation
 146 rate is very small, so the earth's rotation speed is completely submerged in noise. So it will be
 147 relatively large to calculate the heading error by using a gyroscope. Today, most of the MEMS IMU
 148 integrated magnetic meter, method for calculating heading angle by magnetic force meter has also
 149 been gradually got the research and application [10], this paper adopt the method of using magnetic
 150 force to assist MEMS IMU heading information. The principle block diagram of wireless signal aided
 151 SINS positioning as shown in Figure 3.

152
153 **Figure 3.** Principle of Wireless Signal Aided Inertial Positioning

154 3.1. State equation and measurement equation of RSSI weighted centroid localization aided inertia

155 The error state equation of SINS system is

156
$$\dot{X}_{SINS} = F_{SINS} X_{SINS} + G_{SINS} W_{SINS} \quad (13)$$

157 The state equation for wireless signal localization is:

158
$$\dot{X}_{RSSI} = F_{RSSI} X_{RSSI} + G_{RSSI} W_{RSSI} \quad (14)$$

159 Combine equation (13) and (14), then we can get the state equation of the system:

160
$$\begin{bmatrix} \dot{X}_{SINS} \\ \dot{X}_{RSSI} \end{bmatrix} = \begin{bmatrix} F_{SINS} & 0 \\ 0 & F_{RSSI} \end{bmatrix} \begin{bmatrix} X_{SINS} \\ X_{RSSI} \end{bmatrix} + \begin{bmatrix} G_{SINS} & 0 \\ 0 & G_{RSSI} \end{bmatrix} \begin{bmatrix} W_{SINS} \\ W_{RSSI} \end{bmatrix} \quad (15)$$

161 Where \dot{X}_{SINS} , \dot{X}_{RSSI} represent state variable of SINS and wireless location respectively.162 F_{SINS} , F_{RSSI} represent system transfer matrix of SINS and wireless location respectively. G_{SINS} ,163 G_{RSSI} represent process noise transfer matrix of SINS and wireless location respectively. W_{SINS} ,164 W_{RSSI} represent system noise of SINS and wireless location respectively.

165
$$X_{SINS} = [\delta V_E \ \delta V_N \ \delta V_U \ \varphi_E \ \varphi_N \ \varphi_U \ \delta_L \ \delta_\lambda \ \delta_h \ \nabla_{bx} \ \nabla_{by} \ \nabla_{bz} \ \varepsilon_{bx} \ \varepsilon_{by} \ \varepsilon_{bz}]^T \quad (16)$$

166 Where δV_E , δV_N , δV_U are eastward velocity error, north direction velocity error and the
 167 velocity error of the sky direction respectively. φ_E , φ_N , φ_U are east to the misalignment angle,
 168 north to the misalignment angle and the misalignment angle of the direction of the sky respectively.
 169 δ_L , δ_λ , δ_h are Latitude error, longitude error and height error respectively. ∇_{bx} , ∇_{by} , ∇_{bz} are

170 three axial offset error of accelerometer. ε_{bx} , ε_{by} , ε_{bz} are three axial drift of gyroscope. F_{SINS}
 171 can be determined by the SINS error equation.

172
$$X_{RSSI} = [\delta R_1 \ \delta R_2 \ \delta R_3]^T \quad (17)$$

173 RSSI aided inertial positioning calculates the position by inertia, and calculate the distance from
 174 each wireless node according to the position. According to the distance we can further get the
 175 distance difference, and it is measured as the measurement information with the distance difference
 176 calculated by the wireless signal. The measurement equation is described as:

177
$$Z = \rho_{SINS} - \rho_{RSSI} = \begin{bmatrix} \rho_{SINS1} - \rho_{RSSI1} \\ \rho_{SINS2} - \rho_{RSSI2} \\ \rho_{SINS3} - \rho_{RSSI3} \end{bmatrix} = HX + V \quad (18)$$

178 Assuming that the position of the SINS calculation is (x, y, z) , then:

179
$$\rho_{SINSi} = \sqrt{(x_{SINS} - x_i)^2 + (y_{SINS} - y_i)^2 + (z_{SINS} - z_i)^2} - \sqrt{(x_{SINS} - x_0)^2 + (y_{SINS} - y_0)^2 + (z_{SINS} - z_0)^2} \quad (19)$$

180 Assuming that the position of the wireless signal calculated by the weighted centroid location
 181 algorithm is, then

182
$$\rho_{RSSIi} = \sqrt{(x_{RSSI} - x_i)^2 + (y_{RSSI} - y_i)^2 + (z_{RSSI} - z_i)^2} - \sqrt{(x_{RSSI} - x_0)^2 + (y_{RSSI} - y_0)^2 + (z_{RSSI} - z_0)^2} \quad (20)$$

183 Where i represents nodes except node 0.

184 *3.2. State equation and measurement equation of SINS/ magnetometer*

185 The equation of state for the SINS/magnetometer is

186
$$\begin{bmatrix} \dot{X}_{SINS} \\ \dot{X}_{magnetometer} \end{bmatrix} = \begin{bmatrix} F_{SINS} & 0 \\ 0 & F_{magnetometer} \end{bmatrix} \begin{bmatrix} X_{SINS} \\ X_{magnetometer} \end{bmatrix} + \begin{bmatrix} G_{SINS} & 0 \\ 0 & G_{magnetometer} \end{bmatrix} \begin{bmatrix} W_{SINS} \\ W_{magnetometer} \end{bmatrix} \quad (21)$$

187 The selection of SINS state quantity \dot{X} and system transfer matrix are the same as 2.1. The
 188 error of heading angle is chosen as the state of

189
$$X_{magnetometer} = \delta\psi_{magnetometer} \quad (22)$$

190 The measurement equation is:

191
$$Z = H \begin{bmatrix} X_{SINS} \\ X_{magnetometer} \end{bmatrix} + V \quad (23)$$

192 Where

193
$$Z = \hat{\psi}_{SINS} - \hat{\psi}_{magnetometer} \quad (24)$$

194 **4. Simulation and analysis**

195 *4.1 Simulation condition setting*

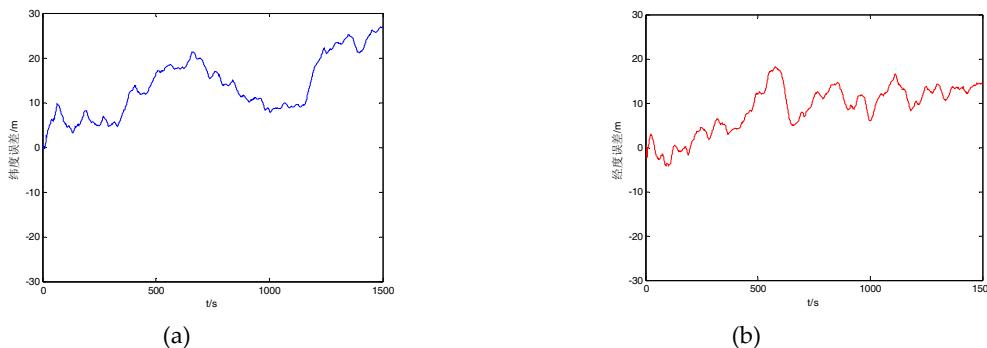
196 In order to verify the effect of the weighted centroid location aided inertial positioning of the
 197 wireless signal, the simulation experiment of 1500s is carried out by using C++. The simulation

198 conditions are set as follows: gyro constant drift is $0.04^\circ/\text{h}$, gyro random drift is $0.04^\circ/\text{h}$,
 199 accelerometer zero bias is $100\mu\text{g}$, accelerometer random bias is $50\mu\text{g}$. The initial parameters of the
 200 carrier are as follows: pitching angle is 0° , roll angle is 0° , heading angle is 0° , initial pitch angle error is
 201 135° , initial roll angle error is 0.02° , and heading angle error is 0.02° . The initial position error is set
 202 as follows: latitude error is 1m , longitude error is 1m . The position of the four wireless nodes are as
 203 follows: node 1 ($32^\circ, 118.01^\circ$), node 2 ($32.01^\circ, 118.02^\circ$), node 3 ($32.02^\circ, 118.01^\circ$) and node 4 ($32.01^\circ,$
 204 118°). Set the carrier to travel at a constant speed, the speed is 1m/s , the starting position is ($32^\circ,$
 205 118°), then carry out the simulation experiment of 1500s.

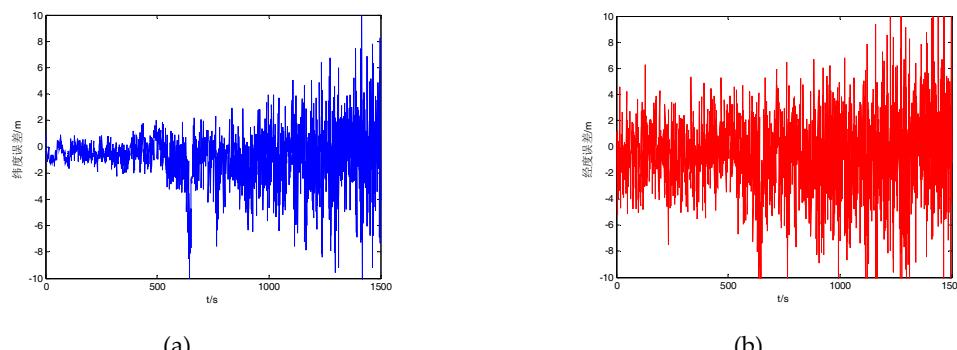
206 *4.2 Simulation results and analysis*

207 Based on the simulation conditions, we can obtain the simulation results of SINS without
 208 auxiliary and wireless assistant. Because it is mainly to study the positioning effect, it is important to
 209 study the information of longitude and latitude. Figure 4 is the traditional centroid algorithm
 210 assisted SINS's latitude and longitude error in the traditional transmission model of wireless signal,
 211 it can be seen from the figure that the error of SINS can be restrained, and it is gradually spread into
 212 convergence, but the errors on the way show great fluctuation. This is mainly due to the traditional
 213 communication model itself vulnerable to the impact of the environment, which brings greater
 214 errors. On the other hand, because the position of the measured point is only determined by the
 215 centroid algorithm, when the measured point is located near the center of mass of the defined area,
 216 there is a high precision, but when the measured point is in the boundary position of the defined
 217 area, the error will be relatively large. Figure 6 is the latitude and longitude error of the auxiliary
 218 SINS of the weighted centroid location algorithm, which is based on the signal propagation model in
 219 this paper. This algorithm can restrain the divergence of SINS error very well and compared to the
 220 algorithm in Figure 5, this method introduces the weight value to determine the position of the
 221 measured point, and the high precision can be obtained when the measured point falls into the
 222 boundary.

223 The algorithm can reduce the error of latitude and longitude and greatly improve the auxiliary
 224 effect of wireless signal assistant SINS; Table 1 is the simulation results.



225 **Figure 4.** The latitude and longitude error of pure SINS location. (a) Latitude error; (b) Longitude error.



226 **Figure 5.** The error of latitude and longitude in the traditional propagation model assisted SINS.

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(a) Latitude error; (b) Longitude error.

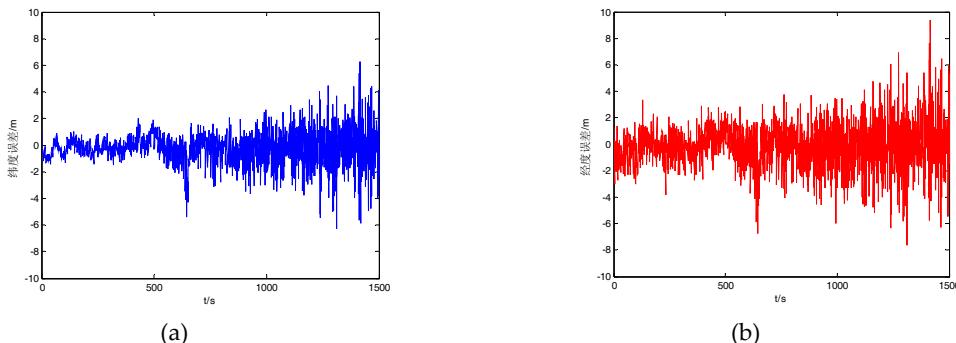
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Figure 6. The auxiliary SINS location error of latitude and longitude in the propagation model in this paper. (a) Latitude error; (b) Longitude error.

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Table 1. Comparison of simulation results of three kinds of positioning

	Latitude average deviation (m)	Longitude average deviation (m)	Maximum deviation of latitude (m)	Maximum deviation of longitude (m)	Standard deviation of latitude (m)	Standard deviation of longitude (m)
Single SINS	13.5977	9.1389	26.9316	18.2245	6.5798	7.2239
Traditional method	1.6007	2.6412	10.8778	16.9741	2.1908	3.4389
Method in this paper	0.9511	1.3758	6.3075	9.3784	1.3024	1.8247

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By comparing Figure 4, Figure 5, Figure 6 and Table 1, we can see that: the weighted centroid localization algorithm based on the signal propagation model can effectively restrain the divergence of the SINS positioning error, and the steady-state error is smaller compared to the traditional method.

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By analyzing the standard deviation of latitude and longitude, it can be seen that the stability is more stable in this paper.

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5. Conclusions

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Aiming at the defects of low positioning accuracy for MEMS IMU and time cumulative error, we use the wireless signal aided inertial algorithm in this paper, which is widely used to improve the positioning accuracy. By selecting the optimal anchor node, then calculate the adjustment factor, and adjust the propagation model coefficient. By the position of the initial fixed point of each anchor node, the method of weighted centroid is used to find the location of the measured point. Finally, a model of wireless signal assisted SINS is established. The results show that the method can effectively restrain the divergence of SINS, and improve the steady-state accuracy and positioning stability compared to the traditional method.

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Acknowledgments: This work was financially supported by the National Natural Science Foundation of China (51175082, 51375088, 61473085), NSFC-Zhejiang Joint Fund for the Integration of Industrialization and Informatization (U1509219), and Scientific Research Fund of Zhejiang Provincial Education Department (Y201533572).

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Author Contributions: Q.L. designed the system structure and experiments, wrote the paper. X.X. performed the simulation. T.Z. wrote the paper.

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Conflicts of Interest: The authors declare no conflict of interest.

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