Dark Matter and Its Relationship with Gravity, Electromagnetism and the Two Nuclear Forces

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Summary
In a search for a solution of the galaxy rotation problem, a novel view on cosmological gravity is developed. In this view, the origin of gravity is traced back to the basic nuclear field of energy, spread by quarks as elementary pointlike sources. This gives a common basis for gravity, electromagnetism, the strong nuclear force and the weak nuclear force. After a review on gravimagnetism and the causality problem of gravity, a hypothetical concept for the gravity field is proposed, which solves the rotation problem without the need to accept dark matter as the deus ex machina.

Keywords: dark matter; cosmological gravity; galaxy rotation problem; gravimagnetism; antigravity

Introduction

Dark matter is a hypothetical physical massive substance, the existence of which is proposed to explain particular phenomena observed in cosmology that cannot be understood from common physical laws. An intriguing example of such a phenomenon is the experimental evidence that stars rotating at the edge of a galaxy show an orbital velocity that remains more or less constant as a function of their spacing from the galaxy center. Although this phenomenon can be explained for particular mass density distributions, the orbital velocity appears to be larger than allowed by the Newtonian law of gravity, which limits this velocity by the amount of enclosed mass. Therefore, "something else" should be responsible for the phenomenon of excessive orbital velocity.

Several mechanisms have been proposed to explain the unexpected. The hypothetical existence of mass within the galaxy that escapes from visible observation, is one of these. Such dark matter would explain an increase of the gravitational force, thereby making the orbital velocity higher than expected from the amount of visible mass. Another mechanism is the MOND hypothesis [1]. MOND stands for MOdified Newtonian Dynamics. In this approach, it is hypothesized that Newton's gravitational law is incomplete and that it can be adapted in a way that its influence becomes manifest at a cosmological scale and remains hidden in our daily world. This adaption, however, is not clearly physically justified, but is made on the basis of a curve fit to empirical results from observations. A third mechanism is proposed by Verlinde [2], who regards gravitation as an entropy phenomenon, emergent from other physical laws, rather than as a fundamental force of nature. He adopts the holographic principle of the string theory. According to this principle, the physical laws of our three-dimensional world, allows a (two-dimensional) holographic mapping on a shell around space. Verlinde hypothesizes fictitious molecules on this shell, which show the entropy as prescribed by the second law of thermodynamics. Physical mass objects in this fictitious world appear to show a motion behavior that can be described mathematically in a format that equals Newton's law of gravity. He claims that, similarly as with MOND, this mechanism gives a clue to understand the behavior of mass objects at a cosmological scale without the need to accept the existence of dark matter.

In this article, I wish to show that there might be fundamental physical reasons why mass objects in deep space behave differently from the behavior of mass objects in non-cosmological space. These reasons have to do with the relationship between mass, gravity, electromagnetism and the two basic nuclear forces. It all starts with the concept of quarks as described in previous work [3,4]. In this concept, a quark is a mass less pointlike source of an energetic flux. This flux produces two energetic fields, namely a vector type repelling "far field" and a scalar type attracting "near field". This enables the origin of stable bonds between two quarks, known as mesons, and between three quarks, known as baryons. These configurations behave as quantum mechanical oscillators. The energetic state of these oscillators becomes manifest as mass. A quark on its own, has no relevant mass attribute. Mass is the consequence of the quantum mechanical junction of quarks. Electric charge is a consequence of this junction as well. It is the manifestation of the (iso)spin condition of the quark junction. As a result, electric charge is quantized in integer units. Mass is quantized as well, albeit as a quantization level of the quantum mechanical oscillator state. This shows an irregular energy spacing. It is due to the characteristics of this oscillator, which is not purely harmonic as it would be in a quadratic field of energy. Instead, it is slightly anharmonic as a consequence of the Proca type of the quark's nuclear far field and the Yukawa type of the nuclear near field. There is, however, a clear parallel. Electromagnetism, which originates from the phenomenon of electric charge, and gravity, which originates from the phenomenon of mass, are indirect forces that follow from the existence of the energetic field of the two basic nuclear forces.

We know that the electromagnetic force has a vector potential next to a scalar potential. Taking into account the parallel between electromagnetism and gravity as just described, it would be reasonable to suppose that such would hold for gravity as well. So, where is the vector potential of gravity? It might well be that this potential escapes from our observation because of its weakness. Would it be possible that this vector potential may show its existence at the cosmological scale? It is the aim of this article, to explore if such could be true. If gravity would be incomplete, in the sense that next to its scalar field a hidden vectorial field would exist, it could well be possible that currents of mass particles would show similar properties as currents of electrically charged particles. In that case, mass currents would create the equivalent of a magnetic field and so could execute the equivalent of the Lorentz force on other mass currents. Such an effect may manifest itself at a cosmological scale as the force that enhances the Newtonian force, thereby suggesting that some dark matter increases the strength of the scalar gravity force.

The idea of gravimagnetism is not new. In fact, it traces back to Heaviside [5]. The present approach to it is based upon a relativistic formalism, derived from Einstein's Field Equation. The suggestion that the (equivalent) Lorentz force might be related with dark matter is not new either. Attempts to explain the dark matter phenomenon in terms of a gravimagnetic Lorentz force have remained unsuccessfully, because its strength is considered as being too weak. In this article, I wish to review the dark matter problem once more as well as the shortcoming of gravimagnetism to explain outstanding cosmological problems. After that, I'll propose a novel concept of Cosmological Gravity in an attempt to find a way out.

The galaxy rotation curve problem: dark matter or dark force?

Newtonian laws prescribe that the *transverse* velocity $v_\varphi(r)$ of a cosmic object revolving in a circular orbit with radius r in a gravity field is determined by

$$v_\varphi^2(r) = \frac{M(r)G}{r},$$ (1)

where $M(r)$ is the amount of enclosed mass and where G is the gravitational constant. This relationship is often denoted as Kepler's third law. As illustrated in figure 1, the velocity curve of cosmic objects in a galaxy, such as, for instance, the Milky Way, appears to be almost flat. It is tempting to believe that this can be due to a particular spectral distribution of the spectral density to compose $M(r)$. This, however, cannot be true, because $M(r)$ builds up to a constant value of the overall mass. And Kepler's law states in fact that a flat mass curve $M(r)$ is not compatible with a flat velocity curve.
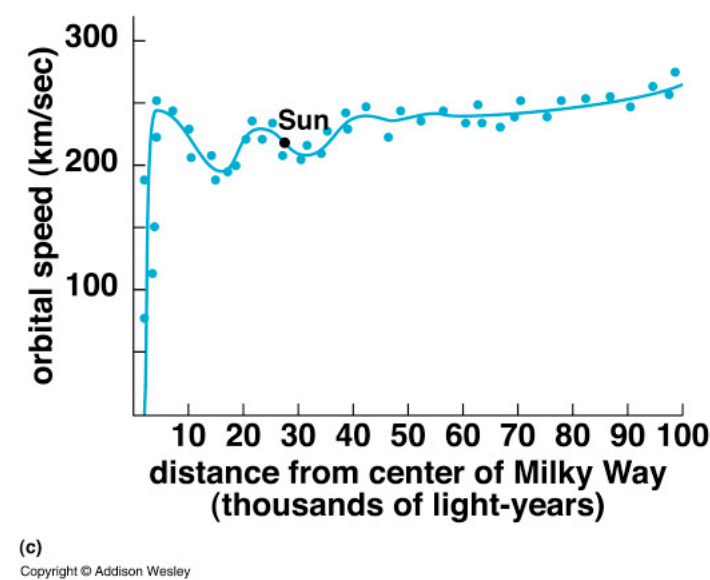
Figure 1: Rotation curves of solar objects in the Milky Way (From ircamera.as.arizona.edu)

Figure 2 illustrates the problem. It is one of the two: either some additional force on top of the Newtonian law is responsible for the phenomenon, or dark matter, affecting the mass distribution is responsible. Apart from this, it can be readily concluded from Table I that the mass data of the Milky Way are inadequate to explain the high value of $v_\varphi(r)$ at the edge of the galaxy.
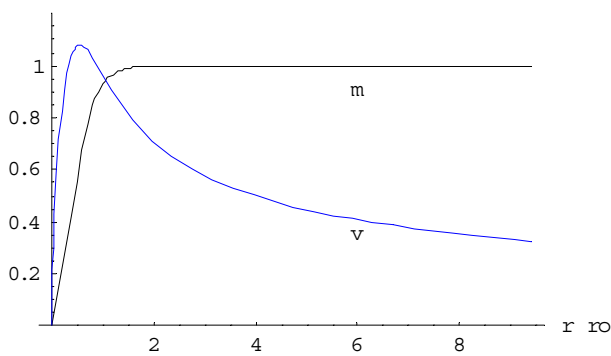
Figure 2. Incompatibility of a flat enclosed mass curve with a flat rotation curve.

Figure 1 shows that the revolution speed of the sun (at $R = 8$ kpc) in the Milky Way has about the same value of some 220 km/s as the one of a stellar at the edge of the Milky Way ($R = 20$ kpc). For a stellar object at the edge, all Milky Way mass can be considered as if it were centric. Using the values shown in Table I, it would result into a revolution speed of 132 km/s. The amount of missing mass can thus be estimated from

$$\frac{M_G + M_{dark}}{M_G} = \frac{1}{1-\alpha} = (\frac{220}{132})^2 \rightarrow \alpha = 0.64. \qquad (2)$$

It means that some 64% of the Milky Way would consist of dark matter.

Table I. Characteristics of the Milky Way

distance unit	1 kpc	3.06 10^{19} [m]
grav. constant	G	6.67 x 10^{-11} [m^{3}kg^{-1}s^{-2}]
solar mass	M_{sol}	1.99 10^{30} [kg]
mass of the bulge	M_{bulge}	2 x 10^{10} M_{sol}
mass of the disc	M_{disc}	6 x 10^{10} M_{sol}
Mass Milky Way	M_R	8 x 10^{10} M_{sol}
Radius Milky Way	R	20 kpc

Gravimagnetism as dark force?
As already stated, another possibility for explaining the anomalous rotation curve is the hypothetical existence of some dark force. Let us consider if the concept of gravimagnetism may reveal such. Many galaxies, such as, for instance, the Milky Way show a typical topological structure of the type as shown in figure 3.

Figure 3: Milky Way (from http://www.atlasoftheuniverse.com/milkyway2.jpg)

This structure is a collection of spiral arms in which solar objects show an orbital motion around the center of the galaxy. It will be clear that neighboring arms can be conceived as parallel currents of massive particles. In terms of the analogy between electromagnetism and gravity, such as summarized in Table II, one might hypothesize that these currents execute an attracting Lorentz force on each other of the type as shown in the last row of Table I. This would mean that, apart from the Newtonian gravity force that attract massive objects to the center of the galaxy, an additional centric force influences the equation of motion of such masses. This effect is formally expressed in the energy relationship

$$\frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 + V_G(r) + V_L(r) = E , \qquad (3)$$

where m is the mass of the particle, v its *radial* velocity, J its moment of inertia with respect to the galaxy center, $\omega\,(= \mathrm{d}\varphi/\mathrm{d}t\,)$ its angular frequency, V_G its potential energy due to gravity, V_L its potential energy due to the Lorentz force and E the total particle's energy. Next to conservation of energy as expressed by (3), the angular momentum is a conserved quantity, so

$$mr^2 \frac{\mathrm{d}\varphi}{\mathrm{d}t} = b , \qquad (4)$$

where b is a constant. It is well known that, by evaluating of this expression for the case that V_G is the potential energy associated with Newton's gravity law and that no Lorentz force is involved, under consideration of these conservation laws , elliptic orbits appear for the motion of the mass particle m. Among the variety of elliptic orbits possible, the circular motion, where the radial velocity $v = 0$, is a valid special case. As shown in figure 3, orbits in the Milky Way are more or less circular.

Table II: Analogy of gravimagnetism with electromagnetism.

	electric	gravitational	(gravi)dimension
current	$I = \Delta q / \Delta t$	$I = \Delta m / \Delta t$	[kg s^{-1}]
Coulomb/Newton	$F = q_1 q_2 / 4\pi\varepsilon_0 r^2$	$F = m_1 m_2 G / r^2$	[kg m s^{-2}]
Lorentz force	$qv \times B$	$mv \times B$	[kg m s^{-2}]
(gravi) magn. field	$B = \mu_0 I / 2r$	$B = \mu_G I / 2r$	[s^{-1}]
constant of nature	ε_0	$\varepsilon_G = 1/G$	[(m^3 kg^{-1} s^{-2})$^{-1}$]
propagation speed	$c = (\varepsilon_0 \mu_0)^{-1/2}$	c	[m s^{-1}]
gravimagnetic constant		$\mu_G = (c_G^2 \varepsilon_G)^{-1}$	[kg^{-1} m]
force between two l long current wires	$F = \mu_0 I_1 I_2 l / 2\pi r$	$F = \mu_G I_1 I_2 l / 2\pi r$	[kg m s^{-2}]

This picture shows a core (the bulge) and a number of spiral arms. Both the bulge and the massive objects (stellar) in the spiral arms rotate around the center. Experimental evidence shows that the tangential velocity of the stellars is different from the Keplian law as

expressed by (1). As discussed before and shown in figure 1, this value appears to approach a value independent of r, at remote distances from the center. A few number of spiral arms can be distinguished. These spiral arms have a certain width and a certain spacing between them. To keep the model simple, it will be supposed that the width Δ of the spiral arms is about the same as the spacing between them. The particle streams in the arms will be considered as gravitational currents I_i in parallel wires. In accordance with the hypothetical equivalence of electromagnetism and gravity, the currents in the arms execute a gravitational Lorentz force that can be calculated according to the formula as shown in the last row of Table I. This builds a certain potential energy $U_L(r)$ of the outer spiral arm with respect to the galaxy center, such that

$$U_L(r) = -\int_0^r F(r)\mathrm{d}r. = -\{\int_0^\Delta F(r)\mathrm{d}r + \int_\Delta^{2\Delta} F(r)\mathrm{d}r + ... \int_{r-\Delta}^r F(r)\mathrm{d}r\} \quad \text{, where} \tag{5}$$

$$\int_{i\Delta}^{(i+1)\Delta} F(r)\mathrm{d}r = \mu_G \int_{i\Delta}^{(i+1)\Delta} \frac{I_i I_{i+1}}{2\pi\Delta} 2\pi r_i \mathrm{d}r, \tag{6}$$

with $I_i = v_t \rho_i \Delta$ [kg s^{-1}],

where v_t is the *tangential* velocity of the massive objects and where $\rho(r)$ is the mass density (in kg/m^2) in the spiral arms. The observer's position here and elsewhere in this analysis, is assumed being in the center of the galaxy. In the case that the orbit of the massive object under consideration is circular, the tangential velocity v_t is the same as the transverse velocity v as defined in (1). In non-circular orbits, the value of the latter one decreases with r as a consequence of the conserved angular momentum as defined by (4). It has to be noted, though, that this conservation law only holds within an orbit under consideration. It does not apply when comparing massive objects in different orbits. Experimental observation reveals that v_t is more or less independent of r. So, let us continue under this assumption and let us investigate if a consistent theory can be built that is not in conflict with it.

 Further analysis reveals

$$\int_{i\Delta}^{(i+1)\Delta} F(r)dr = \mu_G \int_{i\Delta}^{(i+1)\Delta} \frac{I_i I_{i+1}}{2\pi\Delta} 2\pi r_i \mathrm{d}r = \mu_G \int_{i\Delta}^{(i+1)\Delta} \frac{(v_t \rho_i \Delta)^2}{\Delta} r_i \mathrm{d}r = \mu_G v_t^2 r_i \rho_i^2 \Delta^2 \tag{7}$$

Under use of the definitions $\varepsilon_G = 1/G$ and $c_G = (\sqrt{\varepsilon_G \mu_G})^{-1}$, it follows from (5) and (7),

$$U_L(r) = -\mu_G v_t^2 \Delta^2 \sum_{i=1}^N r_i \rho_i^2 = -\frac{v_t^2 G \Delta^2}{c_G^2} \sum_{i=1}^N r_i \rho_i^2 . \tag{8}$$

as the total potential energy of the Nth spiral arm due to the gravitational Lorentz force.

Per unit of mass, the contribution is

$$U_L'(r) = \frac{1}{2\pi r \rho_N \Delta} U_L(r) = -\frac{v_t^2}{c_G^2} \frac{G}{2\pi r} \Delta \sum_{i=1}^{N} \frac{r_i \rho_i^2}{\rho_N} . \tag{9}$$

It will be clear that the value of the sum will increase with the number N of the spiral arms within the radius r under consideration. In fact, it is reasonable to suppose that

$$\frac{\Delta}{2\pi r} \sum_{i=1}^{N} \frac{r_i \rho_i^2}{\rho_N} = \frac{M_{rs}}{r} \approx \frac{M_r}{r} , \tag{10}$$

where M_r the total amount of mass in the spiral arms up to r.

From (1) and (8), we have, without Lorentz force

$$\frac{1}{2} m v_t^2 = \frac{mGM_r}{r} , \tag{11}$$

and with Lorentz Force,

$$\frac{1}{2} m(1-\alpha) v_t^2 = \frac{mGM_r}{r} , \text{ where} \tag{12}$$

$$\alpha = \frac{1}{c_G^2} \frac{GM_{rs}}{r} . \tag{13}$$

It will be clear that if α would have a substantial value, it would have a substantial influence on the tangential velocity v_t of the stellar in the spiral arms rotating around the center of the galaxy. Note that GM_r / r has the dimension of a squared velocity. As long as $\alpha < 1$, equation (12) is consistent with a circular orbit equation. The circular motion would change into a spiral one if α exceeds 1. If α is about 0.9, it would seem as if the centric mass M_r is raised to a tenfold. It would seem as if 90% of its mass is "dark". Depending on the radial dependency of M_r and on the value $(1-\alpha)$, it might well be that v_t will not only show a flat behavior as a function of r, but that it will be substantial larger than expected from the amount of visible mass as well.

So, the crucial question is whether α may have a substantial value. It will be clear that if c_G would equate the light velocity in empty space, gravimagnetism cannot be the substitute for dark matter. So, the crucial question is if there is a reason to suppose that c_G can be different from the light velocity. This issue will be further addressed in the next paragraph. Before doing so, I wish to point to a seeming anomaly. In this analysis it has been taken for granted that the velocity curve of moving objects in the galaxy is flat, implying that the tangential velocity of orbiting objects is independent of the distance to the center. This corresponds with experimental evidence. It means, though, that the objects are revolving in a circle. The structure of the galaxy, however, shows spirals. The implication is that stellars cannot maintain their position in the spiral arms. If they did, their velocity should increase with the radius, because it would seem as if they were part of a rigid body. This seems being a paradox. In fact, it is not, because the spirals have to be considered as a pattern of mass density, which for some reason, is created in the dark

initial phase of the galaxy. So, where the spiral arms with their high mass density preserve their position in the galaxy, individual stellars don't. The spiral arms have to be considered as traffic jams, where stellars move in and move out. If this were not the case, the flatness of the velocity curve would demolish the spiral characteristic.

General Relativity and gravimagnetism
The resemblance between the electromagnetic field and the gravitational field does not necessarily mean that the one is a Chinese copy of the other. One thing is, for instance, the fact that where in electromagnetism charges are bipolar, charges in gravity are unipolar. Where the vectorial character of the electromagnetic field's potential function is without any doubt, it might still be that the potential function of the gravitational field is scalar. The observation that the weak field limit of the gravitational field, as expressed by Einstein's field equation, gives rise to a wave equation, does not necessarily imply that the gravity field's potential function is vectorial. To provide more insight in this, it might be useful to compare the origin of Maxwell's wave equation with the gravity wave equation that results from Einstein's Field Equation. Maxwell's wave equation is straightforwardly derived from the four well known Maxwell equations, written in SI notation as,

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \,;\; \nabla \cdot \mathbf{B} = 0 \,;\; \nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \text{ and } \nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \varepsilon_0 \partial \mathbf{E} / \partial t) \qquad (14)$$

Combining the time derivatives of these equations followed by elementary algebraic manipulation, results in the well known Maxwellian wave equations for the electric field strength $\mathbf{E}$ and the magnetic field strength $\mathbf{B}$, which are spatially and temporally of second order. These fields are created as a consequence of a spatial charge ρ (which may assume reduce to a Dirac type distribution) and a current density $\mathbf{J}$. The Maxwell equations as such are not sufficient for describing mechanical forces on charged particles. As soon as a force F is assigned on a particle with electrical charge q as $F = qE$, Einstein's description of Maxwell's laws in terms of special relativity prescribes the generalization of the force by including the Lorentz as a consequence of a moving charge q with velocity v as

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}. \qquad (15)$$

The Faraday induction $\partial \mathbf{B} / \partial t$ and the displacement current $\varepsilon_0 \partial \mathbf{E} / \partial t$, which mutually couple a magnetic field with an electric field, are the essential ingredients to generate the wave equation. This evokes the question in how far the bipolarity of electric charges in source terms ρ / ε_0 and $\mathbf{J}$ is responsible for these essential ingredients. Anyhow, where the displacement current in electromagnetism can be readily understood from the interaction between positive and negative charges in the interrupt of a charge conductor by a condenser, there is not such equivalence available with unipolar charges in gravity. Whether a one-to-one mapping of the electromagnetic Maxwellian equations to gravity is allowed is an open question still. This holds in particular for the dynamic terms $\varepsilon_0 \partial \mathbf{E} / \partial t$ (displacement current) and $\partial \mathbf{B} / \partial t$ (Faraday induction). It might well be that these ingredients are absent in gravimagnetism.

Let us now consider the gravitational wave equation as a consequence of the weak field limit of the Einsteinean Field Equation. The equation reads as,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{with} \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} . \tag{16}$$

where $T_{\mu\nu}$ is the stress-energy tensor, which describes the energy and the momenta of the source(s) and where $R_{\mu\nu}$ and R are respectively the so-called Ricci tensor and the Ricci scalar, which can be calculated if the metric tensor components $g_{\mu\nu}$ are known [6,7]. In the case that a particle under consideration is subject to a centric force only, the time-space condition shows a rotational symmetric isotropy. This allows to read the metric elements g_{ij} from a simple line element that can be written as

$$ds^2 = g_{00}(r,t)dq_0^2 + g_{rr}(r,t)dr^2 + r^2\sin^2\vartheta d\varphi^2 + r^2 d\vartheta^2 , \tag{17}$$

where $q_0 = ict$ and $i = \sqrt{-1}$.

It means that the number of metric elements g_{ij} reduce to a few, and only two of them are time and radial dependent. In the weak field limit, we have

$$g_{00}(r,t) = 1 + h_{00}(r,t), \text{ where } |h_{00}(r,t)| \ll 1, \text{ and}$$
$$g_{rr}(r,t) = 1 + h_{rr}(r,t), \text{ where } |h_{rr}(r,t)| \ll 1. \tag{18}$$

Under this condition, Einstein's Field Equation can be evaluated into a wave equation (see Appendix),

$$\frac{\partial^2 h_{00}}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 h_{00}}{\partial t^2} = 0 \text{ and } \frac{\partial^2 h_{rr}}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 h_{rr}}{\partial t^2} = 0, \text{ with } \frac{\partial h_{00}}{\partial r} = -\frac{\partial h_{rr}}{\partial r} . \tag{19}$$

Obviously, a centric force from the center of empty space is enough to create a gravitational wave function. This wave function propagates with the vacuum light velocity, thereby proving the causality of the gravitational force. It was Karl Schwarzchild, who calculated the numerical value for the metric elements for the case that the centric force is Newton's gravity force from a centric mass M . In that case

$$h_{rr} = \frac{2(GM/r)}{c^2} \text{ and } h_{00} = -\frac{2(GM/r)}{c^2} . \tag{20}$$

So far in these considerations, possible rotation energy of the source in the center, has not been considered. Inclusion of it will, of course, affect the metric tensor. It makes it more complex, because one might expect that the nice symmetric isotropy will be lost. Generically, the line element has to be expressed in terms of the Kerr metric [8], which in Mashhoon's gravimagnetic formalism is simplified to [9]

$$ds^2 = -c^2(1 - 2\frac{\Phi}{c^2})dt^2 - 4(\mathbf{A} \cdot .d\mathbf{x})dt + (1 + 2\frac{\Phi}{c^2})\delta_{ij}dx^i dx^j \,, \tag{21}$$

where δ_{ij} is the Kronecker delta and where

$$\Phi = \frac{GM}{r} \,, \quad \mathbf{A} = \mu_G \frac{\mathbf{J} \times \mathbf{x}}{r^3} \,, \quad r = |\mathbf{x}| \quad \text{and where } \mathbf{J} \text{ is the angular momentum of the source.}$$

Note that the angular momentum is expressed by the same symbol as the current source in Maxwell's equations (14). Although the semantics are different, the role is the same. Because we shall not make further use of Maxwell's equations, from now on $\mathbf{J}$ will be used as the symbol for angular momentum.

In the special case that a particle subject to this metric is revolving in the rotation plane of the source, the influence of the source's angular momentum becomes manifest as a quasi Lorentz force, which can be modeled as an add-on to the centric force. Under this condition the nice symmetric isotropy is restored. It will be clear that the value of the proportionality constant μ_G has a large impact on the strength of the quasi Lorentz force. It is quite common to establish this value from the following observation. First of all, Poisson's law for gravity is invoked, which states

$$\nabla^2 \Phi = 4\pi G \rho \,. \tag{22}$$

In analogy with Poisson's law for electromagnetism a gravity equivalent ε_G for electric permeability is defined, such that

$$\varepsilon_G = 1/4\pi G \,. \tag{22}$$

The next step is relating

$$c^2 = \frac{1}{\varepsilon_0 \mu_0} \Leftrightarrow c_G^2 = \frac{1}{\varepsilon_G \mu_G}? \tag{23}$$

The justification for equating c^2 with c_G^2, as is common practice, is the statement that the presence of a vector potential associated with the angular momentum brings the causality of the gravity field such that it propagates with the vacuum light velocity. Let me emphasize here that the presence of the angular momentum is not required for bringing the causality. As proven in the Appendix, the causality is implicitly provided by Einstein's Field equation even in the absence of an angular momentum of the source. Therefore, causality of the gravity field is no reason to equate c^2 with c_G^2.

 Gravimagnetism might be essentially different from electromagnetism in the sense that the mutual coupling between a gravitational equivalent of an electric field generated by a scalar potential and a gravitational equivalent of a magnetic field generated by a vector potential might be absent. The two type of fields might exist, however, maybe without mutual coupling. Without the coupling, the quantity c_G is just a constant that has nothing to do with

radiation or propagation of gravitational energy by a Poynting vector. Argumentation why there is reason to believe the absence of this coupling is given above, where the role of the difference in polarity of charges has been discussed. Let us continue by estimating the value of μ_G from known experiments. This will be done in the next paragraph.

Implications from the Gravity B Probe experiment

Probably the most relevant experiment for establishing the value of the gravimagnetic constant μ_G has been offered by the Gravity B Probe project [10,11]. This project has been set up to compare theoretical values of two gravitational effects with experimental evidence. These effects are, respectively, the *geodetic effect* and *frame dragging* (also known as the Lense-Thirring effect [12,13]). The geodetic effect is related with the phenomenon of the perihelium shift of planets in the solar system. This shift is predicted by Einstein's General Relativity. The perihelium of a planet in an almost circular loop with radius r_0 shifts each revolution by an amount of [7,14]

$$\Delta\varphi' = \frac{3\pi R_S}{r_0}, \text{ where } R_S = \frac{2M_{sol}G}{c^2}. \tag{24}$$

The quantity R_S is known as the Schwartzschild radius. A satellite orbiting around the earth will show the same effect. It can be measured by a gyroscope on board of a satellite with its axis pointing to a cosmological object in deep space. As a consequence of the curving of space-time due to the energetic gravitational field of the earth, the gyroscope on board of the satellite will show a precession motion. One might think that, per revolution of the satellite at a radius r_j' around the earth enter, the angle of the gyroscope axis will shift by an amount that equals the value of the perihelium shift. Actually, it is more complicated than that. The theoretical value of the perihelium shift is calculated from the line element of an isotropic non-rotating metric of the type as given by (17). More particularly,

$$ds^2 = -(1 - \frac{R_s}{r})dt^2 + (1 - \frac{R_s}{r})^{-1}dr^2 + r^2\sin^2\vartheta d\varphi^2 + r^2 d\vartheta^2. \tag{25}$$

The calculated value of the perihelium shift nicely fits with experimental evidence from orbiting planets indeed. However, as noted before, this line element is incomplete, because the influence of the rotation energy of the centric mass (of the sun for planets or the earth for orbiting satellites) is not included. Nevertheless, the perihelium shift behaves as predicted by (24). This can be understood as follows. The influence of the rotation energy can be accounted for by two different mechanisms. These mechanisms are physically equivalent, but are different in modeling. The first one is changing the line element of field metric followed by an analysis of the angular momentum of a mass object moving in this field. The second one is leaving the field metric unchanged and modeling the influence of the rotation energy as the add-on of a gravimagnetic field. In the latter model, a gravitational Lorentz force is associated with this field and as long as the revolving of a massive object takes place in the equatorial plane of a rotating centric mass, the gravitational Lorentz force is centric. This force is just a slight add-on on the Newtonian force, thereby hardly affecting the amount of perihelium shift of a revolving massive object in the equatorial plane. The angle of the axis of a gyroscope placed in a satellite orbiting in the equatorial plane of the

earth will show per orbit a slight additional amount of phase shift in the equatorial plane. Later in this article, I'll show that this additional amount is virtually indistinguishable from the amount as predicted by the perihelium shift of a non-rotating metric. For that reason the perihelium shift of planets is not different from the one as calculated from the Schwarzschild metric (25).

This picture will change in the case that the satellite with gyroscope(s) orbits in the polar plane. A gyroscope is a device that always keeps its orientation, irrespective of the bottom on which it is placed. If, for some reason whatever, the bottom tilts or turns, the orientation the gyroscope's axis is preserved. For an observer fixed to the bottom, however, it seems as if the gyroscope's axis shifts (precession). A particular example is Foucault's pendulum. The earth rotates under the plane of swinging. For the observer on earth it seems as if the pendulum plane rotates. The Lorentz force on a massive object orbiting in a polar plane is no longer oriented in the same direction as the gravitational force. The force is maximum and orthogonal to the gravitational force when the satellite passes the poles and the force is zero when the satellite passes the equator. As a consequence the orientation of the plane of orbit will be subject to a spatial phase shift orthogonal to the polar plane. This will be experienced by an observer in the satellite as a precession of the gyroscope next to the orbital perihelium phase shift. Figure 4 illustrates the process. Let us first consider the gravimagnetic view.
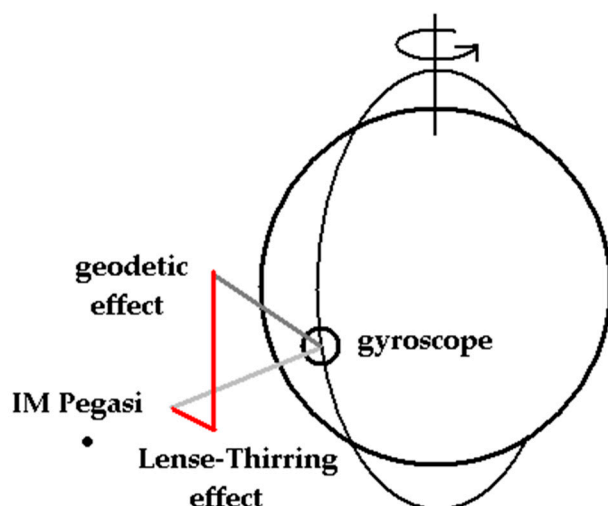
Figure 4. The two orbital precession influences (perihelium shift/geodetic effect and Lorentz force/Lense-Thirring effect) are coherent (i.e. in the same direction) for massive objects in an equatorial plane and non-coherent for massive objects in a polar plane. IM Pegasi is the guide star for the gyroscope. From: large.stanford.edu/courses/2007/ph210

The gravimagnetic view
In line with the reflections on gravimagnetism earlier in this article, I wish to develop the gravimagnetic view by conceiving mass particles as charged particles with charge m. A large massive volume will be modeled as a massive body with some internal mass density ρ. As a consequence of this mass density, a gravimagnetic field is generated, similar to the magnetic field of a volume with some electrical space charge. According to this picture, the earth will be modeled as a sphere with a uniform mass density ρ. The calculation of the generated

(gravi)magnetic field $\mathbf{B_G}$ is not trivial. The field for a sphere with uniform surface charge density σ can be found in textbooks [15]. It appears that,

$$\mathbf{B_G} = \frac{\mu_G \omega \sigma}{3} \frac{R^4}{r^3} \{2\cos\vartheta \hat{\mathbf{r}} + \sin\vartheta \hat{\vartheta}\} \text{, for } r \geq R \text{,} \tag{26}$$

($\cos\vartheta \hat{\mathbf{r}} = \cos\vartheta\sin\vartheta\mathbf{x} + \cos\vartheta\cos\vartheta\mathbf{z}$; $\sin\vartheta\hat{\vartheta} = -\sin\vartheta\cos\vartheta\mathbf{x} - \sin\vartheta\sin\vartheta\mathbf{z}$)

where ω is the angular velocity of the earth rotation and where $\hat{\mathbf{r}}$ and $\hat{\vartheta}$, respectively, are unit vectors in $r-$direction and in $\vartheta-$direction. This can be converted into the field of a sphere with uniform volume charge density ρ by integrating over shells $\rho dr = \sigma$, so that

$$\mathbf{B_G} = \frac{\mu_G \omega \rho}{3r^3} \frac{R^5}{5} \{2\cos\vartheta \hat{\mathbf{r}} + \sin\vartheta\hat{\vartheta}\} \text{, for } r \geq R \text{.} \tag{27}$$
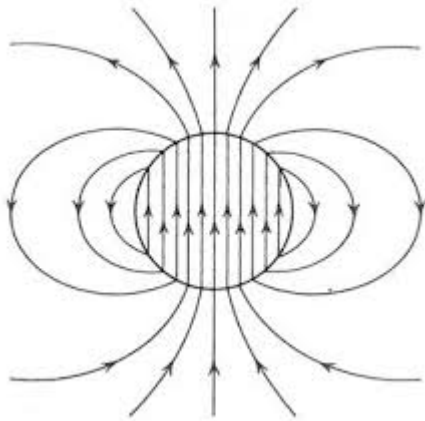
Figure 5. The rotational symmetry of the gravimagnetic field strength B.

In terms of the peripheral equatorial speed $v_e = \omega R_e$ and the mass $M_e = 4\pi\rho R_e^3/3$, the magnitude of the equatorial magnetic field ($\vartheta = \pi/2$) can be expressed as

$$B_G = \frac{\mu_G}{20\pi} \frac{R^3}{r^3} \frac{M_e}{R_e^2} v_e \tag{28}$$

As a consequence of this gravimagnetic field, a massive object with mass m, orbiting in the equatorial plane with tangential velocity v, is subject to a Lorentz force, the magnitude F_L of which is determined from $m\mathbf{v}\times\mathbf{B_G}$ as

$$F_L = m\frac{\mu_G}{20\pi}\frac{R_e^3}{r^3}\frac{M_e}{R_e^2}vv_e = m\frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e v}{c_G^2} \tag{29}$$

In the equatorial plane, this force is balanced by a centripetal force, so that

$$\frac{mv^2}{r} = m\frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e v}{c_G^2} \rightarrow v = r\frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e}{c_G^2} \rightarrow$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{M_e G}{5R_e^2}\frac{R_e^3}{r^3}\frac{v_e}{c_G^2} \tag{30}$$

This gives per orbit an additional shift on top of the perihelium shift to the amount of

$$\Delta\varphi = \frac{2\pi}{5}\frac{M_e G}{c_G^2 R_e}\frac{R_e^2}{r^2}\frac{v_e}{v} = \frac{2\pi}{5}\frac{M_e G}{c_G^2}\frac{R_e}{r^2}\frac{v_e}{v} = \frac{\pi}{5}\frac{R_{SG}}{r}\left(\frac{R_e}{r}\frac{v_e}{v}\right), \tag{31}$$

with $R_{SG} = \dfrac{2M_e G}{c_G^2}$.

If c_G is equated with the vacuum light velocity c, R_{SG} is the same as the Schwarzschild radius.

It will be clear that if this is the case, this additional shift as a consequence of the rotation of the centric mass is negligible for planets in orbits around the sun, because of the large mismatch between the radius of the sun and the average radius of the orbit. This is different for a satellite orbiting in the periphery of the earth.

In the Gravity Probe B project the distance of the satellite to the earth center amounts to $r = 7000$ km. The peripheral speed of the earth is $v_e = 2\pi$ x 6400/(24 x 60 x 60) km/s. From these expressions is straightforwardly calculated that, under assumption of $c_G = c$, this additional precession over a year amounts to 50 macs, while the perihelium shift amounts to 13188 macs/yr (milliarcsec per year). Note that the perihelium shift is not dependent on c_G, but on c instead.

Let us now suppose that the satellite orbits in the polar plane. Now the satellite experiences a Lorentz force orthogonal to the orbit plane. From (27) it is obvious that the Lorentz force while passing the poles is twice as large as in the equatorial case. It is zero while passing the equator. In other positions, the strength of the Lorentz force is composed as

$F_L = B_z v_x + B_x v_z$, where $\tag{32}$

the two components of the gravimagnetic field strength are given by,

$B_z = B_G(2\cos\vartheta\cos\vartheta - \sin\vartheta\sin\vartheta)$ and $B_x = B_G(2\cos\vartheta\sin\vartheta - \sin\vartheta\cos\vartheta)$,

and where the tangential velocity v is decomposed as

$v_z = -v\sin\vartheta$ and $v_x = v\cos\vartheta$.

Figure 6 shows these quantities as a function of ϑ.

The Lorentz force is orthogonal to the plane of motion. Averaged over the orbit, the effective Lorentz force is 0.42 x in magnitude as in the equatorial case (thereby resulting in a frame dragging effect of 21 macs/yr). Now, however, the phase shift is orthogonal to the perihelium phase shift. The labor executed by the Lorentz force is converted into the energy of a precession motion orthogonal on the plane of motion. In the equatorial case the two precessions are coherent, i.e. pointing in the same direction, while they are incoherent in the polar case. In the latter case observation of the gyroscope allows to measure the two effects independently.
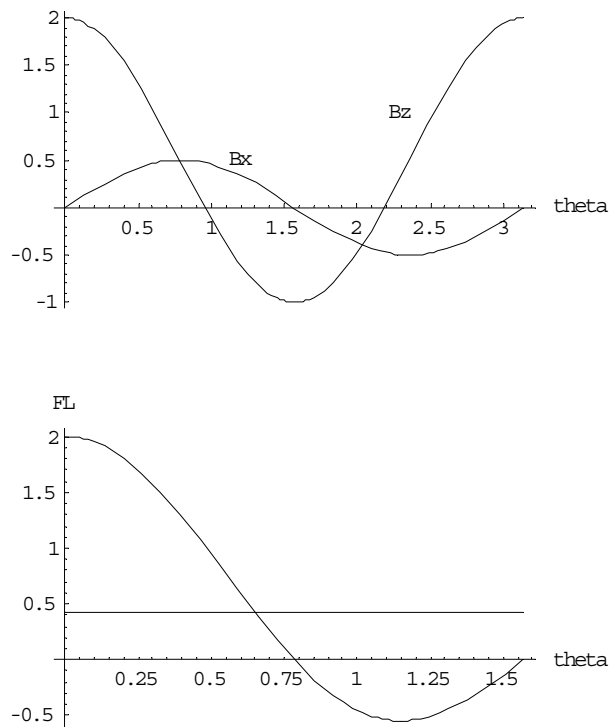
Figure 6. Upper part: The z and x component of the gravimagnetic fieldstrength B as a function of ϑ (the y component is zero). Lower part: The Lorentz force as a function of ϑ and its average value (0.42).

The reported measurement data from the Gravity B probe project are, respectively, for the geodetic effect, 6606.1 macs/yr (milliarcsec per year), and for the frame dragging effect 37.2 macs/yr. Curiously, the figure for the geodetic effect is almost exactly a factor 2 smaller than results from the analysis above for the perihelium shift, while it is about a factor 2 larger for the gravimagnetic effect.

Schiff's view
The gravity B probe project has been set-up for testing the gravity theory as documented by L.I. Schiff in his classic article [16] . This theory describes the influence of a rotatic metric, such as produced by the earth, on the spin vector (= angular momentum vector) **S** of a gyroscope placed in an orbiting satellite. The conclusion of the theory is that the time behavior of the spin vector from the perspective of an observer in the satellite and corrected for the earth time t, can be expressed as

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}t} = \mathbf{\Omega} \times \mathbf{S}, \tag{33}$$

where

$$\mathbf{\Omega} = \mathbf{\Omega_G} + \mathbf{\Omega_{FD}} = \frac{3GM}{2c^2r^3}(\mathbf{r} \times \mathbf{v}) + \frac{GI}{c^2r^3}\{\frac{3}{r^2}(\mathbf{\omega}\cdot\mathbf{r})\mathbf{r} - \mathbf{\omega}\}.$$

where I is the earth's moment of inertia moment. The first term is independent of the earth's rotation, while the second term determines its impact. In terms of the earth's angular momentum $\mathbf{J} = I\mathbf{\omega}$, this can be rewritten as,

$$\mathbf{\Omega} = \mathbf{\Omega_G} + \mathbf{\Omega_{FD}} = \frac{3GM}{2c^2r^3}(\mathbf{r} \times \mathbf{v}) + \frac{G}{c^2r^3}\{\frac{3}{r^2}(\mathbf{J}\cdot\mathbf{r})\mathbf{r} - \mathbf{J}\}. \tag{34}$$

The energy E_G associated with the first term amounts to

$$E_G = \mathbf{S}\cdot\mathbf{\Omega_G} = mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{3GM}{2c^2r^2}v. \tag{35}$$

This energy is converted into a rotational precession energy E_p, so that

$$E_p = E_G \rightarrow \frac{1}{2}J\omega^2 = \frac{1}{2}r^2m(\frac{\mathrm{d}\varphi}{\mathrm{d}t})^2 = mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{3GM}{2c^2r^2}v = mr^2\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{3}{4}R_s\frac{1}{r^2}v, \tag{36}$$

and so

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{3R_s}{2r^2}v = \frac{3R_s}{2r^2}(\frac{2\pi r}{\Delta T}). \tag{37}$$

Per orbit of the satellite, it results in a phase shift $\Delta\varphi_G$ to the amount of

$$\Delta\varphi_G = \frac{3\pi R_s}{r}. \tag{38}$$

This is the same as the value of the perihelium shift defined in the previous paragraph. For the second term precession energy of a gyroscope in the equatorial plane, we have from (34)

$$E_G = \mathbf{S}\cdot\mathbf{\Omega_{FD}} = mr\frac{\mathrm{d}\varphi}{\mathrm{d}t}\frac{G}{c^2r^3}J. \tag{39}$$

For the angular momentum of the earth, interpreted as a spherical object, we have

$$J = M_e\frac{4\pi R_e^2}{5T_e} = M_e\frac{4\pi R_e^2}{5T_e} = \frac{2}{5}M_eR_ev_e, \tag{40}$$

where T_e is the rotation period of the earth.

With these expressions the second term of the gyroscope precession in the equatorial plane results from

$$E_p = E_G \rightarrow \frac{1}{2} r^2 m (\frac{d\varphi}{dt})^2 = mr^2 \frac{d\varphi}{dt} \frac{G}{c^2 r^3} \frac{2}{5} M_e R_e v_e = \frac{1}{5} mr^2 \frac{d\varphi}{dt} R_s \frac{R_e v_e}{r^3}, \qquad (41)$$

so that

$$\frac{d\varphi}{dt} = \frac{2}{5} R_s \frac{R_e v_e}{r^3}, \qquad (42)$$

and therefore

$$\Delta \varphi_G = \frac{2}{5} R_s \frac{R_e v_e}{r^3} (\frac{2\pi r}{v}) = \frac{4\pi R_s}{5r} \frac{R_e v_e}{rv} \quad \text{per orbit of the satellite.} \qquad (43)$$

This gives 200 macs/yr. It is 4x larger than in the gravimagnetic view for $c_G = c$. A gyroscope in a polar plane shifts (200 x 0.42 =) 84 macs/yr. Similar as the perihelium shift, it is 2x larger than quoted from the Gravity B probe project. Most probably, this is factor is due to tracing back the measurement data to Ω or, instead, to shifts of the gyroscope axis per orbit.

Comparing Schiff's theory with gravimagnetism
The result of the Gravity B probe experiment confirms Schiff's conclusion about the weak limit of Einstein's Field Equation. Compared with the gravimagnetic view, there is no difference in the geodetic part, which, in a proper interpretation of the reported data, is the same as the well known perihelium shift of planetary objects in the solar system. The frame dragging result in the weak limit is exactly 4x larger than follows from a one-to-one transposition of Maxwellian laws to gravity. Repairing this discrepancy requires a modification of the Lorentz force by a factor 4 by adapting the Maxwellian set accordingly. This may be done, either by accepting $\mu_G = 8\pi G / c$, or by introducing a gyromagnetic ratio as a fudge [17].

Cosmological Gravity
The experimental results of the Gravity B project have shown that gravimagnetism exists, but that it is inadequate to explain the origin of black matter, because of the weakness of the frame dragging effect. So, something else is required to explain the phenomenon of flat rotation curves in a galaxy. Unusual problems ask for unusual explanations. In this paragraph, I wish to propose an unusual explanation, albeit that use will be made of common concepts in field theory. To this end, the Lagrangian density of the cosmological gravity field will be derived from the generic expression,

$$\otimes = -\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + U(\Phi) + \rho \Phi, \qquad (44)$$

where $U(\Phi)$ is the potential energy of the field and where $\rho\Phi$ is the source term. Let us compare three different options for $U(\Phi)$, respectively $U(\Phi)=0, U(\Phi)=\lambda^2\Phi$ and $U(\Phi)=-\lambda^2\Phi$, where $\lambda>0$ and real. Application of the Principle of Action as embodied in the Lagrange-Euler equation yields a differential equation for the spatial behavior of the field's potential energy. The homogenous equations are respectively

$$\text{(a)} \ \frac{1}{r}\frac{d^2}{dr^2}(r\Phi)=0 \ ; \ \text{(b)} \ \frac{1}{r}\frac{d^2}{dr^2}(r\Phi)+\lambda^2\Phi=0 \ \text{and} \ \text{(c)} \ \frac{1}{r}\frac{d^2}{dr^2}(r\Phi)-\lambda^2\Phi=0. \quad (45)$$

The non-trivial solutions for the first and second case are

$$\Phi=\frac{\Phi_0}{\lambda r} \ \text{and} \ \Phi=\Phi_0\frac{\exp(-\lambda r)}{\lambda r}. \quad (46)$$

The first case applies to electromagnetism (for $\Phi_0=Q\lambda/4\pi\varepsilon_0$) and Newtonian gravity (for $\Phi_0=MG\lambda$). The second case applies to Proca's generalization of the Maxwellian field. It reduces to the first case if $\lambda\rightarrow0$, while keeping Φ_0/λ constant. Generically, it represents a field with a format that corresponds with the potential as proposed by Yukawa to explain the short range of the nuclear force. It has been used by the author of this article for the purpose to express the Gravitational Constant in quantum mechanical quantities[3].

Let us now consider the third case. It can be readily verified from (45c) that a non-trivial solution for this case is,

$$\Phi=\Phi_0\frac{\cos\lambda r+\sin\lambda r}{\lambda r}. \quad (47)$$

In accordance with the concepts of classical field theory, the field strength can be established as the spatial derivative of the potential Φ. Identifying Φ_0/λ as MG and λ as a range parameter, we may identify this field strength as a cosmological gravity force F_{CG}. Let us compare this force with the Newtonian force F_N. To do so more explicitly, we compare $F_N r^2$ with $F_{CG} r^2$. The comparison is shown in figure 7.
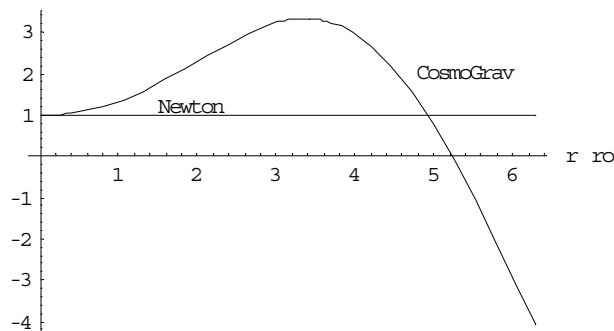
Figure 7: The cosmological gravity force compared with the Newtionian force (λ=0.7).

This figure shows that, for relative small values of r, the cosmological gravity force behaves similarly as the Newtonian force, but that its relative strength over the Newtonian force increases significantly for large values of r. This is a similar behavior as heuristically implemented in MOND. The effective range is determined by the parameter λ. It might therefore well be that the cosmological gravity force manifests itself only at cosmological scale. Figure 8 shows that under influence of this force, the rotation curves in the galaxy may assume a flat behavior.
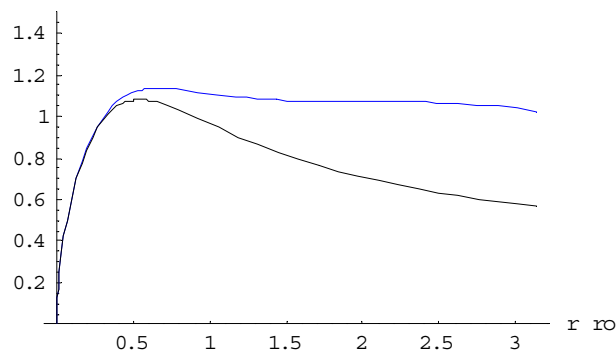
Figure 8: boost of the rotation curve under influence of cosmological gravity.

This hypothetical cosmological gravity shows an intriguing phenomenon. At very far cosmological distance, the attraction of gravity is inverted into repulsion. There is some
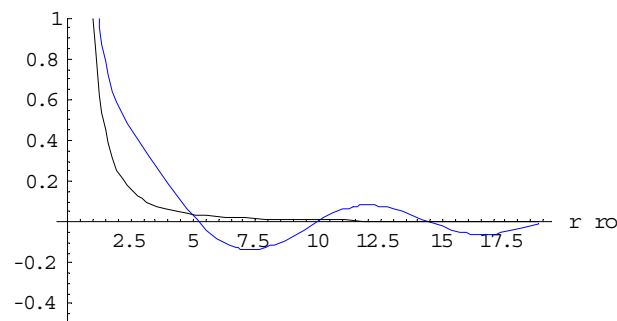
Figure 9: Inversion of the gravity force to antigravity at large cosmological distances. Black: Newtonian. Blue: Cosmological Gravity.

speculation reported in literature that such antigravity is required to explain the phenomenon of dark energy, responsible for the accelerated expansion of the universe [18]. Exploration of this phenomenon is a subject outside the scope of this article. It requires further research.

This view on gravity has an exciting beauty. It unifies the four forces of nature in a single expression for the scalar part of their Lagrangian density, because of

$$\odot = -\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + U(\Phi) + \rho\Phi \text{ , where}$$

$U(\Phi) = -\lambda^2 \Phi$ for gravity,

$U(\Phi) = \lambda^2 \Phi$ for the nuclear forces [3,4],

$U(\Phi) = 0$ for electromagnetism.   (48)

A remaining challenge now is, to harmonize this view of symmetry with a generalization of the space-time metric [8], by interpretation of the classical field concept of potential energy $U(\Phi)$ in terms of elements of the metric tensor in Einstein's Field Equation.

Discussion

The search for relationships between the four basic forces of nature is an on-going challenge. In this respect, the gravitational force is the most problematic one. Even to the extent that the question if gravity is a basic force indeed, or just emergent from the other basic forces, is still subject to debate. Where electromagnetism and the two nuclear forces are unified in the context of quantum physics, gravity is still on its own. This, in spite of the beauty of Einstein's General Relativity, which has revealed and explained so many gravity related cosmological phenomena. In this article, I have described gravity from a point of view inherited from my earlier work on quantum physics, in which I have positioned quarks as pointlike sources spreading a composite field of energy, identified by me as the Higgs field. It gives rise to bonds of two quarks (mesons) and three quarks (baryons), to which attributes can be assigned that we know as electric charge and mass. In this view, the fields of the strong nuclear force and the weak nuclear force are basic and the fields of electromagnetism and gravity are sourced by the attributes of the quark bonds. In earlier work, this view has enabled me to express the Gravitational Constant in terms of quantum mechanical quantities [3], probably so far considered as just a curiosity. It appeared being possible by combining views of General Relativity with the views of classical field theory. One of the instruments used, was applying Proca's generalization of Maxwell's laws to the nuclear fields of energy. In the context of this article, I have done something similar for gravity. Just by changing the polarity of the mass term in Proca's generalization. In this article gravity has been modeled as a scalar field, i.e., as a field with a scalar potential. Whether this field can be generalized to a vectorial one (that therefore radiates) depends on the question if the gravitational equivalents of the Faraday induction and the displacement current exist in the gravimagnetic view on gravity. If so, my hypothesis could possibly meet serious problems with respect to gauging the field under a gauge invariance constraint of the proposed modified Proca description for the gravity field. It is for that reason why I have included the Appendix, in which I have shown that the causality of the gravity field does not need a vectorial bosonic description. This might also be the answer on the factor 4 discrepancy between the weak field limit view of Einstein's Field Equation for a rotational metric and the gravimagnetic analogon of full electromagnetism.

 According to the hypothesis developed in this article, there is no dark matter. There is no dark force either. There is just a generalization of gravity laws. This generalization gives an

answer to the anomaly of the rotation curves in galaxies. One of the intriguing consequences is the potential existence of antigravity. This, however, does not mean that there is a domain for it. It is relative. Those who are living in that domain, do not experience antigravity. They just see us living in an antigravity domain instead.

 Let me conclude by expressing that my hypothesis is still in a state of immaturity. It gives a challenge, though, for further exploration. The ultimate challenge is proving or disproving it by experiment.

Appendix: Causality of gravity
The objective in this appendix is to show how the causality of gravity, such as shown in the wave equation (19) in the main text, is implicitly contained in Einstein's wave equation, even in the case that the source of gravity does not show an amount of rotation energy. This objective implies that we have to solve the weak limit of Einstein's Field Equation for a space-time metric that is given by the line element (17),

$$\mathrm{d}s^2 = g_{00}(r,t)\mathrm{d}q_0^2 + g_{rr}(r,t)\mathrm{d}r^2 + r^2\sin^2\vartheta\mathrm{d}\varphi^2 + r^2\mathrm{d}\vartheta^2 , \qquad\qquad (A\text{-}1)$$

where $q_0 = \mathrm{i}ct$.

Note: The space-time ($\mathrm{i}ct, r, \vartheta, \varphi$) is described on the basis of the "Hawking" metric (+,+,+,+).

The weak field limit condition is defined as (18),

$$g_{00}(r,t) = 1 + h_{00}(r,t), \text{ where } |h_{00}(r,t)| \ll 1, \text{ and}$$
$$g_{rr}(r,t) = 1 + h_{rr}(r,t), \text{ where } |h_{rr}(r,t)| \ll 1. \qquad\qquad (A\text{-}2)$$

In empty space, the Einstein Field Equation (16) under this symmetric spherical isotropy, reduces to a simple set of equations for the elements $R_{\mu\mu}$ of the Ricci tensor,

(a): $R_{00} = 0$; (b): $R_{rr} = 0$; (c): $R_{\vartheta} = 0$ and (d): $R_{\varphi\varphi} = 0$. \qquad\qquad (A-3)

The hard task is to calculate these elements. As I wish to show later in this appendix, the calculation gives the result as shown in Table A1.
 (Note: h' and h'' means differentiation, respectively double differentiation of h to r ;
 $\dot{h}$ and $\ddot{h}$ means differentiation, respectively double differentiation of h to t)

Adding (A-3b) to (A-3a) results in

$$h_{00}'' - \frac{\ddot{h}_{rr}}{c^2} + 2\frac{h_{00}'}{r} + 2\frac{h_{rr}'}{r} . \qquad\qquad (A\text{-}4)$$

From (A-3c) and (A-3d) it follows that

$$h'_{00} = -h'_{rr}. \tag{A-5}$$

From (A-4) and (A-5) then follows,

$$h''_{00} - \frac{\ddot{h}_{00}}{c^2} = 0 \quad \text{and} \quad h''_{rr} - \frac{\ddot{h}_{rr}}{c^2} = 0, \tag{A-6}$$

or written differently,

$$\frac{\partial^2 h_{00}}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 h_{00}}{\partial t^2} = 0 \quad \text{and} \quad \frac{\partial^2 h_{rr}}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 h_{rr}}{\partial t^2} = 0, \text{ with } \frac{\partial h_{00}}{\partial r} = -\frac{\partial h_{rr}}{\partial r}. \tag{A-7}$$

Table A1: metric tensor and Ricci tensor

metric tensor	Ricci tensor
$g_{00} = 1 + h_{00}$	$R_{00} = -\dfrac{h'_{00}}{2} + \dfrac{\ddot{h}_{rr}}{c^2} + \dfrac{h'_{00}}{4}(h'_{00} - h'_{rr}) - \dfrac{\dot{h}_{rr}}{4}(\dot{h}_{00} - \dot{h}_{rr}) - \dfrac{h'_{00}}{r}$
$g_{rr} = 1 + h_{rr}$	$R_{rr} = -\dfrac{h'_{00}}{2} + \dfrac{\ddot{h}_{rr}}{c^2} - \dfrac{h'_{00}}{4}(h'_{00} - h'_{rr}) + \dfrac{\dot{h}_{rr}}{4}(\dot{h}_{00} - \dot{h}_{rr}) + \dfrac{h'_{00}}{r}$
$g_{\vartheta\vartheta} = r^2$	$R_{\vartheta\vartheta} = -\dfrac{r}{2}(h'_{00} + h'_{rr})$
$g_{\varphi\varphi} = r^2 \sin^2(\vartheta)$	$R_{\kappa\varphi} = -\dfrac{r}{2}\sin^2(\vartheta)(h'_{00} + h'_{rr})$

It will be clear that (A-7) is the equation of a wave that propagates in the direction of r with a velocity c. This equation is identical in format as Maxwell's wave equation for electromagnetism. It proves the causality of gravity without requiring an equivalent of magnetism in the source of the field.

Calculation of the Ricci tensor
The Ricci tensor is described in expanded form by

$$R_{ij} = \sum_{k=0}^{3}\left(\frac{\partial \Gamma_{ij}^{k}}{\partial q_k} - \frac{\partial \Gamma_{ik}^{k}}{\partial q_j}\right) + \sum_{i=0}^{3}\sum_{k=0}^{3}\left(\Gamma_{ij}^{i}\Gamma_{ki}^{k} - \Gamma_{ik}^{i}\Gamma_{ji}^{k}\right). \tag{A-8}$$

The Christoffel symbols Γ_{ij}^{k} represent functions of the metric elements, such that

$$\Gamma_{ij}^{k} = \frac{1}{2}\sum_{m=0}^{3} g^{km}\left\{\frac{\partial g_{jm}}{\partial q_i} + \frac{\partial g_{im}}{\partial q_j} + \frac{\partial g_{ji}}{\partial q_m}\right\}. \tag{A-9}$$

Under symmetric spherical isotropy, only diagonal terms remain, so that the expression reduces to

$$R_{ii} = \sum_{k=0}^{3}\left(\frac{\partial \Gamma_{ii}^{k}}{\partial q_k} - \frac{\partial \Gamma_{ik}^{k}}{\partial q_i}\right) + \sum_{i=0}^{3}\sum_{k=0}^{3}\left(\Gamma_{ii}^{i}\Gamma_{ki}^{k} - \Gamma_{ik}^{i}\Gamma_{ii}^{k}\right), \tag{A-10}$$

and the Christoffel symbols reduce to

$$\Gamma_{ij}^{k} = \frac{1}{2g_{kk}}\left\{\frac{\partial g_{kj}}{\partial q_i} + \frac{\partial g_{ki}}{\partial q_j} + \frac{\partial g_{ji}}{\partial q_k}\right\},$$ (A-11)

such that only three different forms remain,

$$\Gamma_{kk}^{k} = \frac{1}{2g_{kk}}\frac{\partial g_{kk}}{\partial q_k}; \quad \Gamma_{ii}^{k} = \frac{1}{2g_{kk}}\frac{\partial g_{ii}}{\partial q_k} \ (k \neq i) \text{ and } \Gamma_{ik}^{k} = \frac{1}{2g_{kk}}\frac{\partial g_{kk}}{\partial q_i}.$$ (A-12)

Table A2 shows the Christoffel elements different from zero.

Table A2: Christoffel elements and affine connections of the isotropic non-rotating metric

Γ_{tt}^{t}	Γ_{tr}^{r}			Γ_{tt}^{r}	Γ_{tr}^{r}		
Γ_{rt}^{t}	Γ_{rr}^{t}			Γ_{rt}^{r}	Γ_{rr}^{r}		
						$\Gamma_{\vartheta\vartheta}^{r}$	
						$\Gamma_{\kappa\varphi}^{r}$	
		$\Gamma_{r\vartheta}^{\vartheta}$				$\Gamma_{r\varphi}^{\varphi}$	
	$\Gamma_{\vartheta r}^{\vartheta}$					$\Gamma_{\vartheta\varphi}^{\varphi}$	
			$\Gamma_{\varphi\varphi}^{\vartheta}$		$\Gamma_{\varphi r}^{\varphi}$	$\Gamma_{\varphi\vartheta}^{\varphi}$	

where,

$$\Gamma_{tt}^{t} = \frac{1}{2ic}\dot{h}_{00} \quad \Gamma_{rr}^{t} = -\frac{1}{2ic}\dot{h}_{rr} \quad \Gamma_{tr}^{t} = \Gamma_{rt}^{t} = \frac{1}{2}h'_{00}$$

$$\Gamma_{tt}^{r} = -\frac{1}{2}h'_{00} \quad \Gamma_{rr}^{r} = \frac{1}{2}h'_{rr} \quad \Gamma_{tr}^{r} = \Gamma_{rt}^{r} = \frac{1}{2ic}\dot{h}_{rr}$$

$$\Gamma_{\vartheta\vartheta}^{r} = -\frac{r}{1+h_{rr}} \quad \Gamma_{\varphi\varphi}^{r} = -\frac{r\sin^2\vartheta}{1+h_{rr}}$$

$$\Gamma_{r\vartheta}^{\vartheta} = \Gamma_{\vartheta r}^{\vartheta} = \frac{1}{r} \quad \Gamma_{\varphi\varphi}^{\vartheta} = -\sin\vartheta\cos\vartheta$$

$$\Gamma_{r\varphi}^{\varphi} = \Gamma_{\varphi r}^{\varphi} = \frac{1}{r} \quad \Gamma_{\vartheta\varphi}^{\varphi} = \Gamma_{\varphi\vartheta}^{\varphi} = \cot\vartheta$$ (A-13)

Application of (A-13) on (A-10) gives the Ricci tensor as listed in Table A1.

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